RESONANCES AND COUPLING COEFFICIENTS OF PZT4 BARS AND DISCS

J.R. Dunn

School of Electronic & Electrical Engineering, University of Birmingham, Edgbaston, Birmingham B15 2TT, England.

1. INTRODUCTION AND METHODS

This paper is a contribution towards the design of high frequency transducer elements and arrays, where the elements are simply pieces of piezoelectric ceramic material without resonant loading or matching sections and the radiating faces have dimensions in the range of 0.5 to 4 wavelengths in water. The frequency range of interest is from around 200 kHz upwards; at lower frequencies resonant sections of passive materials are easily added to improve the characteristics, the bandwidth in particular. Much effort has gone into those designs, but another look has been made at the simplest structures. In the design of an array the starting point is usually the radiating face, the size being determined by the requirements on beamwidth and subdivision for purposes such as beam-steering, and the dependent variable is then the thickness of the element to give the correct resonant frequency with the maximum coupling coefficient. There are usually a number of modes of vibration which could give the design frequency, but the correct one needs to be chosen for optimum performance; even so there may be ranges of sizes of the radiating face with modes of vibration for which the coupling coefficient is too low to be useful and where the pattern of vibration on the surface is inappropriate for the application. This problem was investigated purely experimentally many years ago for bars (1,2) with some observations on discs (limited by the availability of specimens); it was apparent that for bars a width of two water wavelengths, which was required for the particular application, was far from ideal and two closely spaced bars, each one wavelength wide, had a much better performance, albeit with a more complicated construction.

The present investigation was purely analytical using Finite Element methods to find the spectrum of the resonances of bars and discs of varying width-to-thickness ratio, and also to find the associated electromechanical coupling coefficients. The bars are modelled in their cross-section as two-dimensional structures rigidly clamped in the third dimension, and the discs as axisymmetric structures, again so that a two-dimensional analysis could be done, but in both cases neither internal losses nor radiation loading are included. Because of the symmetry only one quarter of the cross-section has to be modelled, with the internal (opposing) faces blocked normally. Full piezoelectric and anisotropic mechanical properties were used, the material being PZT-4 and standard values taken (3). The first eight resonances for each model were fully investigated for resonant frequency and motional capacitance; higher resonant frequencies were found to a lower resolution, but the FE meshes were not fine enough for confidence to be placed on the results. The free (at zero frequency) and fully mechanically blocked (normally on the external surfaces) capacitances were also found; the blocked capacitance, used in the calculation of the coupling coefficients at the resonances, is in fact independent of frequency, since vibrations within mechanically blocked surfaces can generate no nett charge on the electroded surfaces. The highest possible coupling coefficient is set by the ratio of the free and blocked capacitances, the book data gives a value of 0.715 and the present FE methods give 0.664 for the bars and 0.666 for the discs; there is a very small dependence on shape for the discs (not for the bars), but this is probably due to the approximations inherent in the axisymmetric FE analysis. The results are presented in normalised form, the common variable being the size (width or diameter) in wavelengths in water (the velocity of sound is taken as 1500 m/sec.), since this is the starting point for the design, and the shape is measured by the ratio of width or diameter to thickness. The coupling coefficient is already a normalised parameter, being a measure of the fraction of energy input in one form that is stored in the other form.

2. RESONANT CHARACTERISTICS OF BARS

2.1 Resonances

The bars analysed had their ratio of width to thickness over the range 0.2667 to 3.7500 and were poled in the thickness direction. The overall spectrum of their resonances is shown in fig.1, with an expanded version of the low frequency end in fig.2. Lines corresponding to the pure thickness mode for large plates and its third overtone are also shown (the overtone is not an integer factor times the fundamental frequency where the coupling coefficient is high). The slope of the calculated mode lines is always steeper than that of the pure modes. Where in a few places the mode lines appear to come very close to crossing, they should not actually meet but change direction in close proximity. The lowest mode at small values of w/t is clearly a thickness mode (but with vibrations blocked in the third, length, dimension), the constant slope being a measure of the frequency constant, 1640 kHz.mm, compared with the large plate thickness constant of 2000 kHz.mm. For large values of w/t the normalised resonant width becomes independent of shape, and therefore this is a width mode with a frequency constant 1720 kHz.mm. For intermediate shapes the vibration is a mixture of thickness and width displacements. For the higher modes the vibration becomes very complex, and detailed analysis of loaded models is required for determining which modes would be useful in real transducer arrays. Also shown in fig.2 are some points from practical experiments carried out previously (2); these bars were not very long, the ratio of length to thickness being about 4 except for one case where it was 8, this being the one with a resonance shown for the lowest mode only. The agreement between theory and practice is remarkably good, considering the limited length to thickness ratio and the manufacturing tolerances on material properties. Also in the cross-over region it appears as if there is interference between a longitudinal mode with the resonant frequency determined mainly by the thickness and a transverse mode with the frequency controlled mainly by the width.

2.2 Electromechanical coupling coefficients
These are shown in fig.3 as coupling vs. resonant width in wavelengths. The highest coupling is shown by the lowest mode, and this is to be expected since the mechanical stresses and strains have the greatest coherence in direction and sense over the whole cross-section; the maximum value shown is 0.642, compared with the static value of 0.664. The higher modes show lower but potentially useful values over limited ranges of w/ , the second mode from about 1.3 to 2.8, the third mode in a narrow range around 2.1, and the fourth mode around 2.8 to 3.8, and so on for higher modes; so for a coupling coefficient greater than 0.25 some mode can be found for any frequency. However these results are not necessarily a good guide to effective radiation, and the practical measurements suggested that elements which are two water wavelengths wide at the second mode resonance are in fact not good radiators; a detailed study of the pattern of the particle displacements on the radiating surface is needed to resolve this question. An interesting point is that the higher modes generally show better coupling in regions where the predicted mode lines cross the lines calculated simply from the theoretical frequency constants. Another point is that where two mode lines converge but do not cross the coupling coefficients for the two modes change rapidly with the shape, one being high when the other is low.

3. RESONANT CHARACTERISTICS OF DISCS

3.1 Resonances
The discs analysed have the same shape ratios as the bars (the same basic coordinate data is used in the FE models) and are poled and driven axially. The overall spectrum of their resonances is shown in fig.4. Lines corresponding to the pure thickness mode for large plates and its third overtone are shown as before. The general pattern of the resonances is very similar to that for bars, thus the lowest mode approaches a pure length mode as D/t becomes very small with a frequency constant of 1510 kHz.mm. (this is close to the value expected from the material parameters), and for large values of D/t it becomes a radial mode with a frequency constant 2200 kHz.mm. Modes 2 and 3 approach each other very closely without crossing just as they do for the bars, and modes 3 and 4 show similar behaviour.

3.2 Coupling coefficients
These are shown in fig.5, with similar characteristics as those for the bars. Mode 1 has a good coupling coefficient over the whole range of sizes between longitudinal and radial resonances, and this would be expected as the stresses and strains are in phase over the whole quadrant which was modelled. The coupling coefficient for small D/t is 0.642, exactly the same as for the bars, and for large D/t it tends towards 0.500, smaller than the theoretical value of 0.580 for the ideal planar case. Mode 2 has useful coupling for the diameter ranging between 1.5 and 3.25 wavelengths; mode 3 shows a peak over a narrow range around 2.5 wavelengths, this is the region where the frequency constant (frequency times thickness) is close to that for the ideal pure

thickness mode. Mode 4 has high coupling for sizes in the range 3.0 to 4.2, again this is around the region where the frequency constant is near the ideal; there are indications that the same rule applies for higher modes, although the mesh in the FE model is getting too coarse on the scale of wavelengths in the piezo-electric material for good accuracy at these higher frequencies.

4. CONCLUSIONS

In so far as this has been a study of unloaded resonant behaviour, it has confirmed the practical observations made many years ago on the practical realisation of transducer elements only a few wavelengthsd across that the manufacturer's data for resonant frequency in terms of a single dimension is not sufficient for obtaining the desired frequency and a good coupling coefficient. However the study needs to be continued with investigations into the radiation characteristics, including suitable mountings for any intended use.

5. REFERENCES

1. J.R. Dunn, A review of problems in high-frequency transducers, paper no.7, Transducer Workshop, Institute of Acoustics/Underwater Acoustics Group, December 1976.
2. J.R. Dunn, Arrays for a 500 kHz electronically scanned sonar, ibid. paper no.8
3. W.P. Mason (ed.), Physical Acoustics vol.1A, pp.202-204, Academic Press, New York, 1964

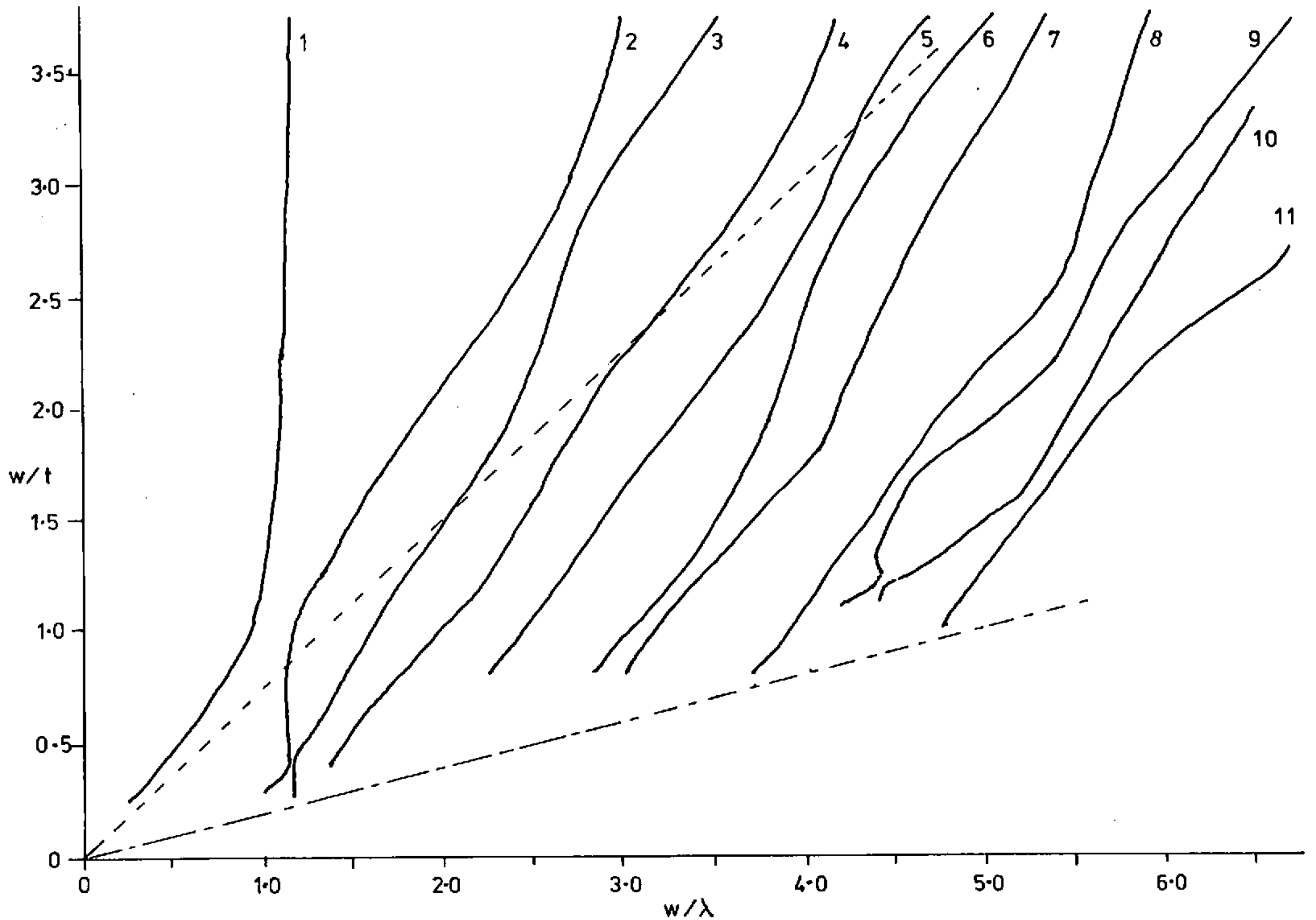

Fig.1 Resonances of bars : full spectrum of modes 1 to 11
- - - - f.t = 2000 kHz.mm ——·—— f.t = 7440 kHz.mm

RESONANCES AND COUPLING COEFFICIENTS OF PZT4 BARS AND DISCS

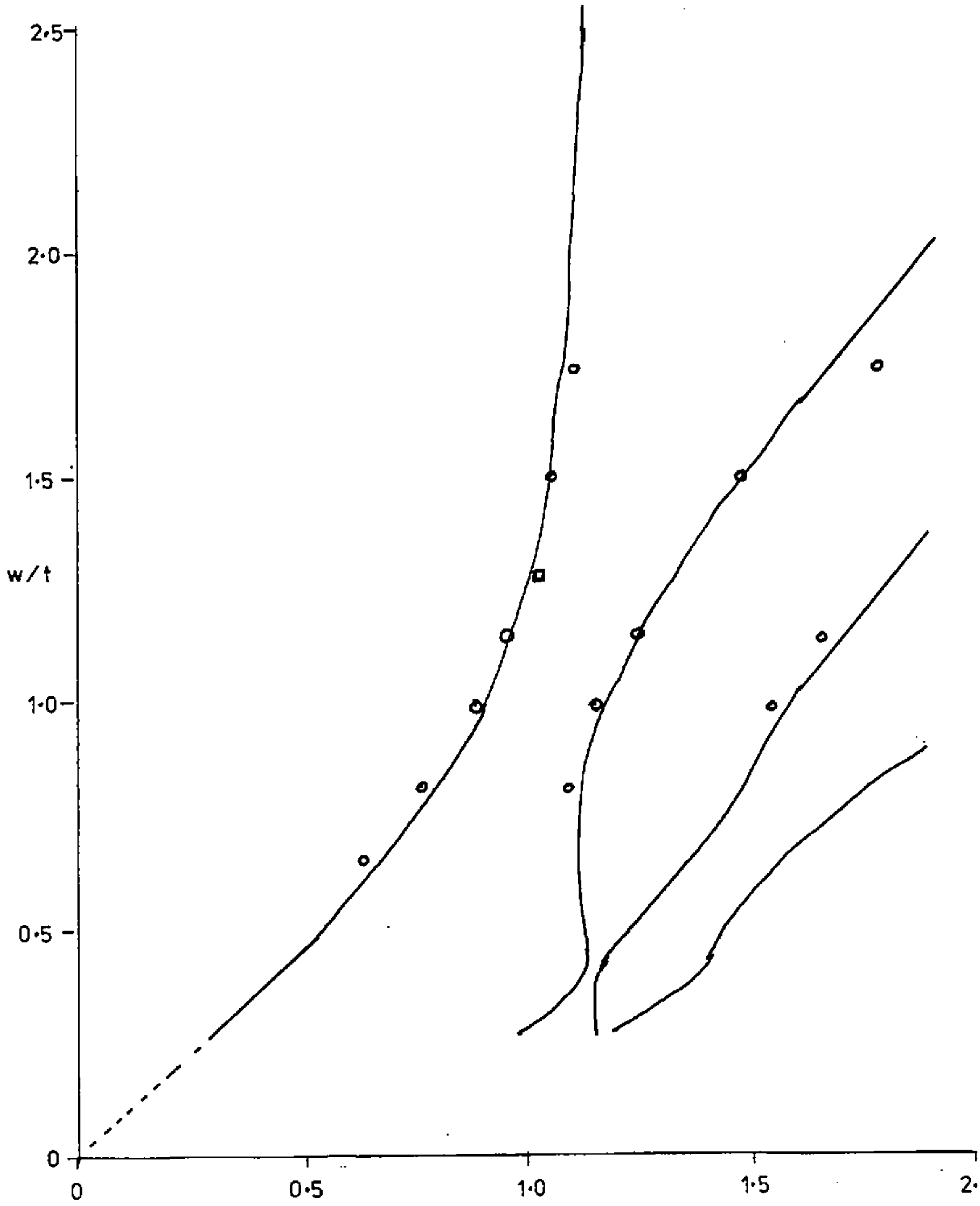

Fig.2 Resonances of bars : modes 1 to 4 in the low frequency region
o Measured values for short bars
□ Measured value for a long bar

RESONANCES AND COUPLING COEFFICIENTS OF PZT4 BARS AND DISCS

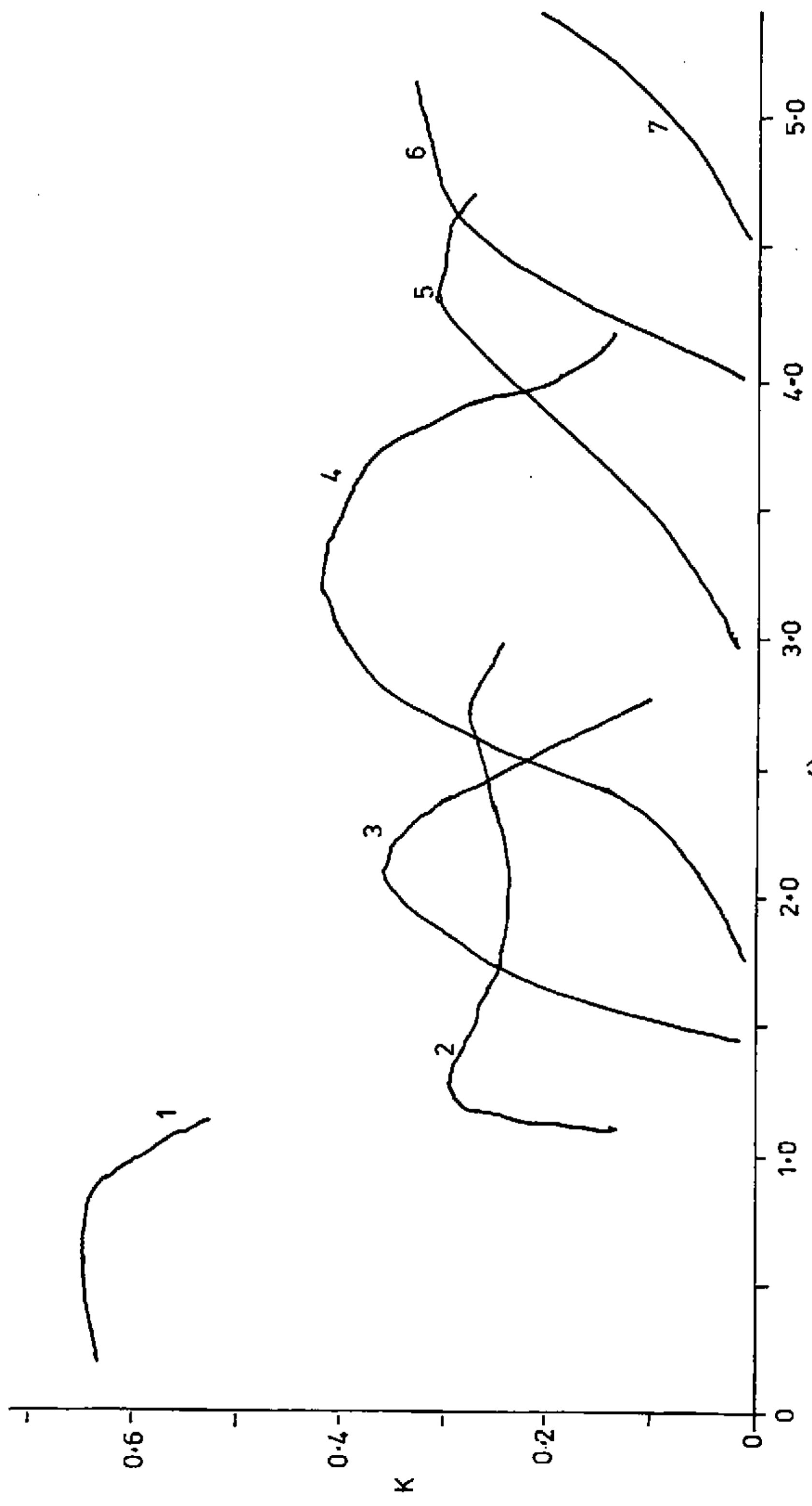

Fig.3 Coupling coefficients of bars, modes 1 to 7

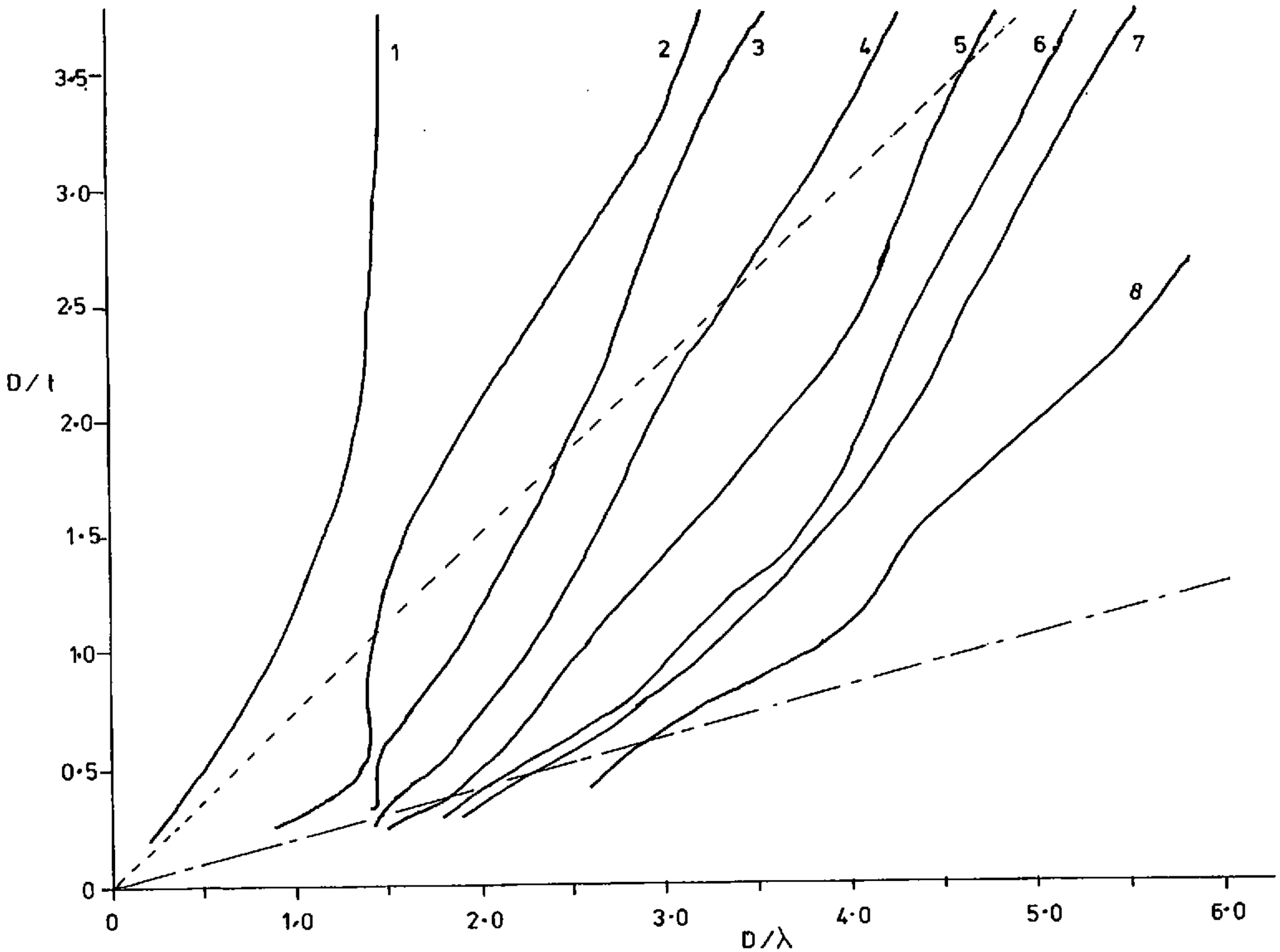

Fig.4 Resonances of discs : modes 1 to 8
- - - - f.t = 2000 kHz.mm ——- · —— f.t = 7440 kHz.mm

memo082

RESONANCES AND COUPLING COEFFICIENTS OF PZT4 BARS AND DISCS

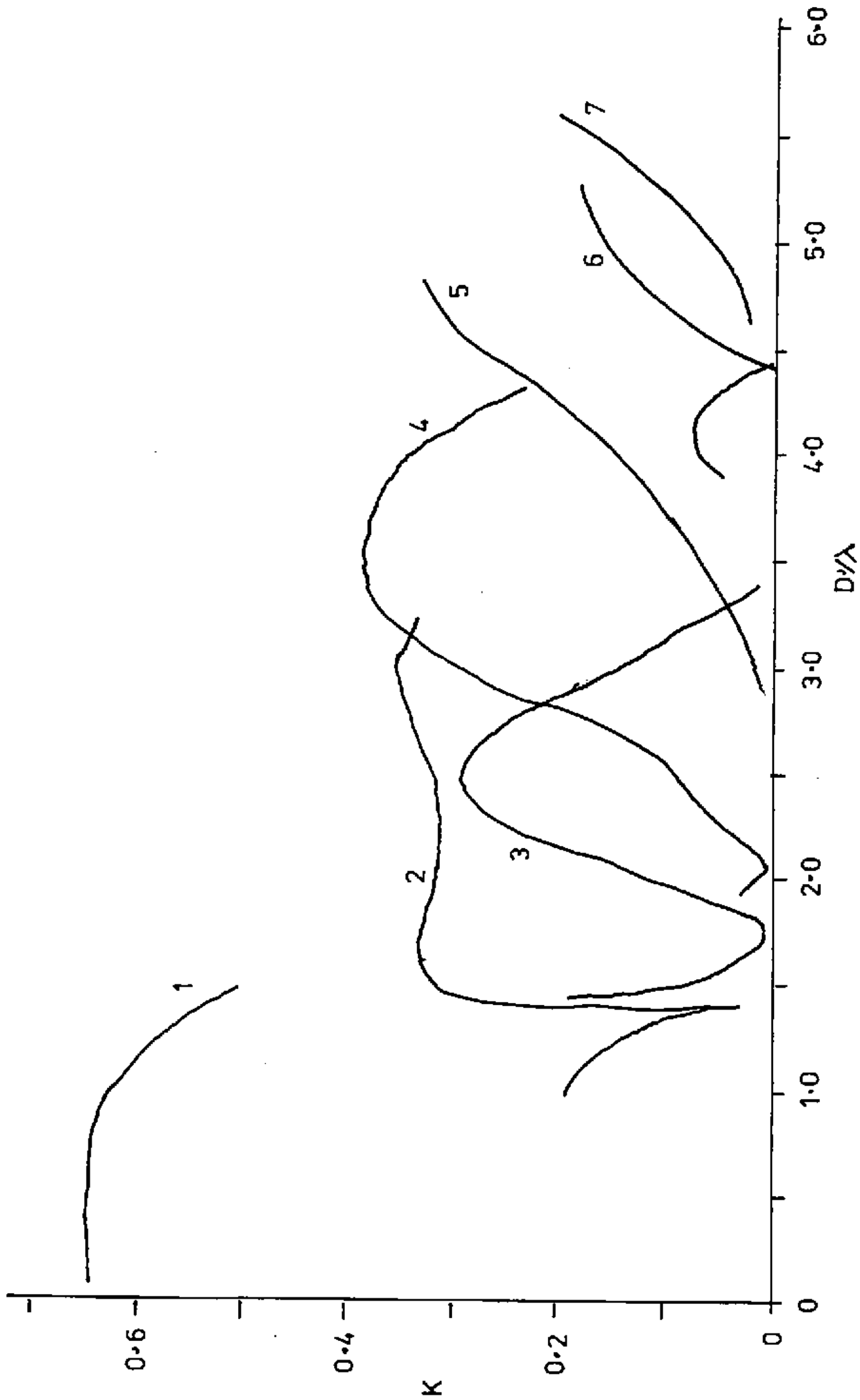

Fig.5 Coupling coefficients of discs, modes 1 to 7