UNDERSTANDING THE FLEXTENSIONAL TRANSDUCER
THE B. B. BAUER EQUIVALENT CIRCUIT

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INTRODUCTION

Flextensional transducers are not as amenable to equivalent circuit analysis as piston transducers which may be treated as one-dimensional structures, but there have been various attempts to produce equivalent circuits [1-3] and computer models have been developed from this work [4].

A more complex analysis technique was developed and widely used in the United States for similar types of transducer [5] but it is not easy to use. An alternative technique developed by B. B. Bauer [6-8] and re-introduced by Smith [9] has been found useful and is used here to provide a simplified transducer model which is not rigourous but which provides an intuitive understanding of the transducer mechanisms and is useful for parametric studies.

In recent years, finite element and boundary element (FE/BE) methods have been used extensively [10-11] and has been shown to give good results both electrically and mechanically [12]. However, the technique analyses the exact data given to it. There may be iteration techniques which allow FE/BE to perform parametric studies but this does not provide good intuition into the mechanisms controlling the transducer behaviour. It also relies implicitly on standard sets of piezoelectric materials data derived in the 1960's [13] and this can lead to significant divergence between theory and measurement.

The technique presented here is not as rigourous as other equivalent circuit methods and is not adequate for full design of flextensional transducers. Neither is FE/BE. The two techniques are complementary and together provide a powerful design tool.

THE B B BAUER EQUIVALENT CIRCUIT

Practical measurements of admittance of transducers can be used to determine a simple equivalent circuit for any transducer such as shown in figure 1 and this can be used to estimate the acoustic performance of the transducer. It does not, however, tell us anything about the contributions of the individual components to that transducer performance.

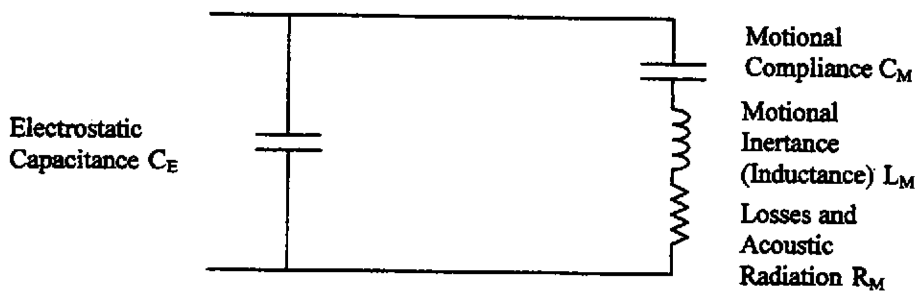

Figure 1. Simple Equivalent Circuit

The B B Bauer equivalent circuit enables each of the individual components to be represented as a simple equivalent circuit, and each of these can be linked together via a transformer. Then the various components can be added via the transformer to form a simple equivalent circuit as in figure 1. Smith [9] advocates a 1:1 transformer, but Bauer [7] allows any value. In this paper, a ratio of unity will be used. The B B Bauer equivalent circuit of a flextensional transducer is shown in figure 2.

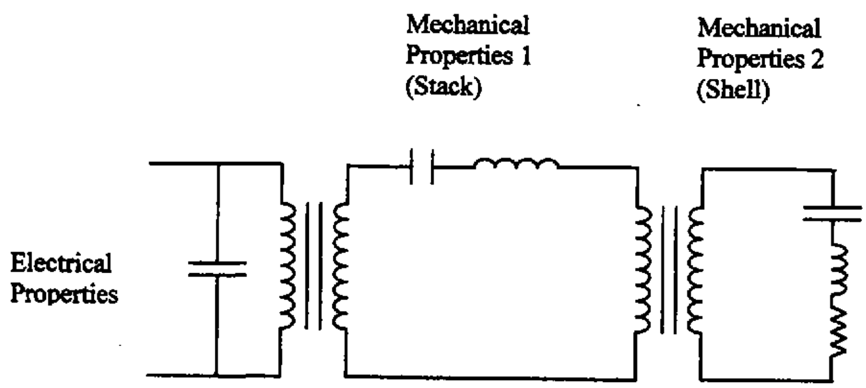

Figure 2. B B Bauer Equivalent Circuit

A further modification of this is to render the shell as a number of parallel resonant circuits, each representing a different resonance. The principal three are the flextensional, the second flextensional and the breathing resonance. Adding radiation components to these is unwise as each will have its own radiation law. It is better to consider this as an in-air circuit.

The analysis of the transducer can then start with the ceramic stack. In a real transducer, there may be several stacks within one shell, but these can be represented in simple one-dimensional analysis as a single stack of height equal to the summed heights of the stacks. The dimensions of the stack are taken as length Lg, height H and width W.

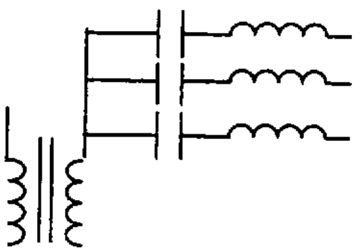

Figure 3. Modification of Shell to Show Several Resonances.

The most useful function in describing the electroacoustic operation of ceramics and stack is the coupling coefficient k, and its most useful definition is that of the proportion of energy stored mechanically to the total energy storage. Then the ratio of mechanical to electrostatic energy, E is given by:

$$E = \frac{C_m}{C_e} = \frac{k^2}{1-k^2} \tag{1}$$

For the ceramic, k_{33} coupling is assumed throughout. The coupling of the other stack components, the joints and end insulators is taken as zero, so they contribute no electroacoustic energy to the stack although they have lengths and compliances which ensure that they absorb some energy. The proportion of electroacoustic energy in the ceramics is then in the ratio of the total stack compliance to that of the ceramics. If n ceramics each have length d and insulators I, joints j, and each has Young's modulus $Y_c = 1/S_{33}^D$, Y_i and Y_j, then the stack modulus Y_{stk} is:

$$\frac{1}{Y_{stk}} = (nd + 2i + (n+1)j)^{-1}\left(\frac{nd}{Y_c} + \frac{(n+1)j}{Y_j} + \frac{2i}{Y_i}\right) \tag{2}$$

The ratio Y_{stk}/Y_c is equal to E_{stk}/E_c so the coupling coefficient of the stack can be calculated by reversing equation 1:

$$k_{stk} = \sqrt{\frac{E_{stk}}{1+E_{stk}}} \tag{3}$$

For a typical stack of 20 ceramics of 5mm thickness with joints each 0.083 mm thickness, Y_c = 120 GPa, Y_j = 3 GPA (the insulators usually contribute little change) the stack modulus is reduced to about 72 GPa. The effect of the joints, particularly their compliance, can be readily seen. The ratio: (length/Y) is similar for ceramics and joints. An average density ρ_{stk} can also be calculated from materials and lengths and this can be used to calculate a stack resonant frequency f_{stk}, from which an inductance value L_{stk} can be calculated:

UNDERSTANDING FLEXTESIONALS

$$f_{stk} = \frac{1}{2Lg}\sqrt{\frac{Y_{stk}}{\rho_{stk}}} \ldots\ldots\ldots\ldots L_{stk} = \frac{1}{4\pi^2 f_{stk}{}^2 C_{stk}} \qquad (4)$$

After the C and L values have been calculated for the stack, those for the shell can next be calculated. It is simplest to start with the breathing resonance. This occurs where the circumferential length equals a wavelength in the shell material. The circumference of an ellipse is not easy to calculate, but it can be approximated as four times the chord length of each quadrant, as shown in figure 4. Note that the semi-major and semi-minor axes a and b are measured to the centre of the wall thickness.

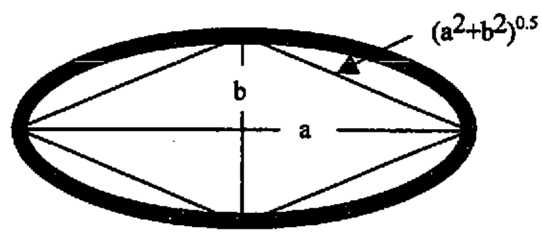

Figure 4. Approximation to Circumference of Ellipse.

$$f_B = \frac{1}{4\sqrt{a^2+b^2}}\sqrt{\frac{Y_{sh}}{\rho_{sh}}} \qquad (5)$$

Note that this frequency is independent of shell thickness t.
The compliance of the shell can be calculated from:

$$\frac{1}{C_B} = N_B = \frac{Y_{sh}\, th}{4\sqrt{a^2+b^2}} \qquad (6)$$

and the value of L_B can be calculated in a manner analogous to equation 4. Shell thickness is t and its height is h. The mass of the shell can be estimated also from its circumferential length times height times thickness times density.

The flextensional frequencies and stiffnesses are most easily calculated assuming the shell comprises two thick bars each of radius of curvature $(a^2+b^2)^{0.5}$, as done in Kinsler and Frey[14].

$$f_{bar} = \frac{\pi}{8}(S)^2\frac{\kappa}{l^2}\sqrt{\frac{Y}{\rho}} \ldots\ldots\ldots \;\ldots \kappa = \sqrt{12} \qquad (7)$$

$$l = barlength = 2\sqrt{a^2+b^2} \ldots\ldots\ldots (S) = series\ ,3.0112\ ,5.7 \ldots\ldots$$

UNDERSTANDING FLEXTESIONALS

Note that for the second flextensional resonance, the next even mode appears at $(7/3.0112)^2$ = 5.4 times the main flextensional resonance. $f_{fl1} = f_{bar}$ in equation 7. Noting that $(3.56)^2$ is approximately 12:

$$f_{fl1} = \frac{3.56t}{4(a^2+b^2)\sqrt{12}}\sqrt{\frac{Y_{shl}}{\rho_{shl}}} = \frac{f_B t}{\sqrt{a^2+b^2}} \tag{8}$$

This is of particular use as the frequency of the shell alone in air has been found to be close to that of the complete transducer in water and so can be used for in-water power calculations.

An estimate of stiffness (and hence compliance) can also be gained using formulae from reference [14]. The force of the ceramic stack acts on the stiffness moment at a point b.(b/a) from the node. The stiffness N_{fl1} can be estimated as equation 9, where N_B was calculated in equation 6. Once compliance and frequency are known, a value for inductance can be calculated.

$$N_{fl1} = \frac{-4YAt^2}{12\sqrt{a^2+b^2}}\left(\frac{a}{b^2}\right)^2 = N_B\frac{4}{3}\left(\frac{ta}{b^2}\right)^2 \tag{9}$$

Noting the earlier statement that $f_{fl2} = 5.4\ f_{fl1}$, an approximation is made for the second flextensional resonance that stiffness is increased by a factor of 5.4 and mass is decreased by a factor of 5.4 compared to the first flextensional resonance.

In the case often met where the wall thickness t = 0.2a, the breathing frequency and the second flextensional resonance will be close together, and may not be distinguishable in practice as two separate resonances. Note also in this case that the above expression (equation 9) is close to unity. The stiffness and so the coupling coefficient (to be shown later) of flextensional and breathing mode will be well matched, while the second flextensional stiffness will be much higher, resulting in a lower coupling coefficient. In thin-wall designs, the coupling coefficient of the second flextensional resonance is higher, but at the expense of the other resonances.

All mechanical and electromechanical values of L and C are known in the B B Bauer circuit as shown in figure 2, and these can be summed to gain values for the complete transducer to render a simple circuit as in figure 1. each shell resonance is taken in turn to provide a separate resonant circuit.

Note that stack and shell reactive components are in series. The inductance values can just be added. The compliance values are added by inverse, so the total compliance is always less than that of either component. Furthermore, only the energy stored in the stack can be observed electrically so there is a further multiplier of $C_{stk}/(C_{stk}+C_{shl})$.

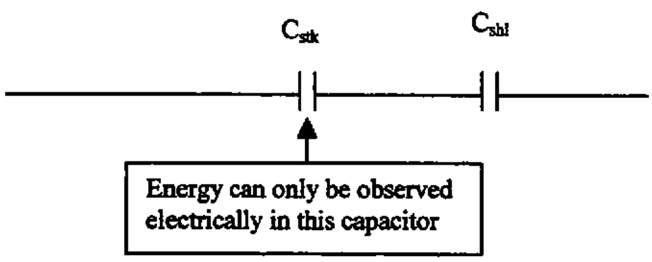

Figure 5. Energy Storage Observed electrically

The electromechanical energy stored in the complete transducer is given by:

$$C_T = \frac{C_{stk} C_{shl}}{\left(C_{stk} + C_{shl}\right)} \cdot \frac{C_{stk}}{\left(C_{stk} + C_{shl}\right)} = \frac{C_{stk}^2 C_{shl}}{\left(C_{stk} + C_{shl}\right)^2} \tag{10}$$

Applying values to this expression will show that the maximum electromechanical energy storage in the complete transducer occurs when $C_{stk} = C_{shl}$ and then this gives a value only one quarter of that of C_{stk}. This determines the upper limit of coupling coefficient obtainable for a flextensional transducer for a given ceramic performance. Values fall off gently at first for a compliance mismatch by a factor of 2 and then deteriorate significantly, as shown in figure 6.

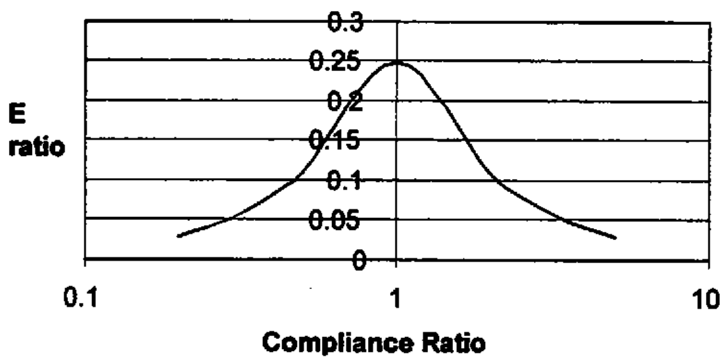

Figure 6. Energy Ratio vs. Stack/Shell Compliance

This demonstrates the need to match shell and stack stiffness as closely as possible to obtain the highest output from the transducer. There may be other constraints, such as need to stress the device for deep operation or use of a very compliant shell for low frequency resonance, but over-stiff stacks are extra mass and cost for less benefit.

The model brakes down in calculation of transducer resonant frequency if the stack is much stiffer than the shell. The stack stiffness dominates all other parameters and too high a frequency is obtained. However, the paragraph above indicates that this condition does not represent good transducer design and the condition is best avoided.

The discussion has centred on in-air measured performance. It is possible to estimate in water performance by empirical formulae for mass loading and Q can be estimated by formulae such as are given in reference [15].

CASE STUDY

Use of the simple model is best illustrated by considering a transducer design and demonstrating performance variations on it. The case considered here will be an aluminium shell transducer of dimensions semi-major axis 75mm, semi-minor axis 30mm, wall thickness 15 mm. Depth of shell 100mm. Stack comprises 20 plates NAVY III ceramic (k_{33} = 0.6, ε_r = 1000) each 5mm thick, cross section 20 by 60 mm. The stack has an insulator (Y=80GPa, ρ=2600) 3mm thick each end. There are 21 joints each 0.083mm thick, Y=3 GPa, ρ=1000. For the ceramics, $1/S_{33}^D$= 120GPa, ρ=7500. For the shell, Y=69GPa, ρ=2700kg/m^3.

The performance will be calculated. A number of variants will then be made to the design and the effects of these will be shown. Note that the variation applies only for that variant. Variations are individual, not cumulative.

The effective modulus of the stack is, according to equation 2 (note that the joints have nearly as much effect as the ceramics in the calculation):

$$\frac{5 \times 20}{120} + \frac{2 \times 3}{80} + \frac{21 \times 0{,}083}{3} = \frac{107.74}{Y} \ldots\ldots\ Y = 72.4\,GPa$$

From equation 1, k=0.6 is equivalent to E=0.56, so for the stack,
E = 0.56 x (72.4/120), E_{stk}=0.34, so from equation 3 k_{stk}=0.50.
The density of the stack ρ_{stk} = 7100 kg/m^3. By equation 4, F_{stk} = 14800 Hz.

The compliance of the stack is 1.45 10^{-9} and L is 0.08. Note that these are effectively mechanical units used as electrical analogues.

Looking next at the shell, using equation 5, the breathing frequency is found to be 15600 Hz and the compliance (equation 6) 0.78 10^{-9}. Applying the multipliers given in equations 9 and 10, f_{fl1}=2900 Hz, C_{fl1}=1.56 10^{-9}. Then L=1.93.

Series addition of the L values and C values gives a resonant frequency in air of the complete transducer of 4100 Hz. However, calculation of the coupling coefficient requires the extra terms, as given in equation 10. Note that the stiffness of stack and shell are well matched, so the energy multiplier is 0.25, the maximum value. The E value for the total transducer is 0.25 x 0.34 = 0.085, so the coupling coefficient k of the total transducer is (equation 3) is 0.28.

Variant 1. Make shell out of GRP (Y=35GPa, ρ=1800).
The resonant frequencies f_b and f_{fl1} are multiplied by the change in material sound speed and become 13650 Hz and 2300 Hz respectively. The flextensional shell stiffness is modified by the changed value of Y, so C_{fl1}= 3.08 10^{-9} and consequently L_{fl1}=3.07. The combined values of C and L are 0.99 10^{-9} and 3.15 respectively, giving a resonant frequency of the transducer in air of 2850 Hz. Note that the compliance of stack and shell are now not so well matched and the energy multiplier is now 0.22. The coupling coefficient of the transducer is now 0.26.

Variant 2. Increase joint stiffness in stacks to 10GPa (Aluminium shell).
The stack stiffness is increased to 99GPa. The E value of the stack becomes
0.56 x (99/120) = 0.46 so the coupling coefficient of the stack increases to 0.56. The stack compliance is reduced to 1.06 10^{-9}. The L value is unchanged as the resonant frequency increases. The series added values of L and C become 2.05 and 0.63 10^{-9} giving a resonant frequency of 4430 Hz. Note that the E multiplier is reduced slightly to 0.24 giving a combined E value of 0.11 and a so coupling coefficient of 0.32. This shows how critical joint properties are to transducer performance.

Variant 3. Increase ceramic performance to k=0.66 while retaining other properties.
The ceramic E value is now 0.77, so the stack E value is 0.77 x 72.4/120. k=0.56. The E multiplier for the shell remains at 0.25, so the coupling coefficient for the transducer becomes k=0.32.

Variant 4. Half thickness of shell.
The shell breathing resonance remains the same, but the flextensional resonance is halved (equation 8) to 1450 Hz. The stiffness is halved for the breathing resonance (equation 6) and further quartered for the flextensional resonance (equation 9). So C_{fl1} is increased by 8 to 12.48 10^{-9}. Then L = 0.97.

Combining components gives C= 0.98 10^{-9} and L=2.9, so f_{fl1}=2990 Hz. Note that is probably higher than reality, but it was stated earlier that this is an area where the model breaks down. However, it is still good for calculation of coupling coefficient. It is seen that the compliance of shell and stack are now badly mis-matched, so the E multiplier (equation 10) is 0.068, well down from the maximum of 0.25. E for the transducer is 0.068 x 0.34 = 0.023. k=0.15. It might even be worth reducing the ceramic dimensions to increase power!

UNDERSTANDING FLEXTESIONALS

These results are summarised in table 1 below. The change in coupling may not seem great, but the power output is determined by its square, and when the volume of ceramic is limited by small transducer dimensions, it is important to maximise k. Note that the equations work linearly in Y and E for determining performance, so they are readily useable for parametric studies. That is the advantage of this simplified model.

Variant	Y_{stk}	k_{stk}	f_b	f_{fl1}	$C_{fl1}\ 10^{-9}$	f_t	k
0	72.4	.50	15600	2900	1.56	4100	0.28
1	72.4	.50	13650	2300	3.08	2850	0.26
2	99	.56	15600	2900	1.56	4430	0.32
3	72.4	.56	15600	2900	1.56	4100	0.32
4	72.4	.50	15600	1450	12.48	2990	0.15

Table 1 . Variation Study. Results Summary

COMMENT AND CONCLUSION

The purpose of this paper is to estimate values of coupling coefficient and resonant frequency for insertion in the electroacoustic power equation:

$$Power\ (acoustic\ ) = 2\pi f \varepsilon_0 \varepsilon_r Vol.E^2 k^2 Q \eta \tag{11}$$

where Vol = ceramic volume, E = electric field, ε's the ceramic permittivity. Q can be found by empirical formula noted elsewhere [15] and η, the radiation efficiency has been found in practice to be 50-60% [16]. The radiation efficiency is determined by comparing sensitivity and directivity with conductance.

The formulae provide a means of estimating f and k. Ideally, equations linear in k would be easiest to use. However, they are linear in E which is simply related, and k values can be derived at any point in the calculation after multiplying E values. They are not rigourously derived expressions, but they provide a rapid means of estimating a transducer's electroacoustic performance from its dimensions. They can be used particularly effectively to observe the variation in response as the dimensions or properties are varied, so that suitable design dimensions can be established before detailed analysis is undertaken.

Finite Element/Boundary Element analysis provides the most rigourous and complete means of determining the transducer's performance and modern codes can even be set to iterate to find the optimum dimensions for the required performance. However, that is an automated process and it does not provide intuition into dimensions and properties which most affect electroacoustic performance in the same way that manual calculations do. Providing that intuition through easy calculation is the prime role of this simplified model.

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