Mapping of Sub-Seabed Anomalies in Seismically Heterogeneous Marine Soils

Jacques Yves Guigné  
University of Bath, Physics Department, Claverton Down, Bath, UK  
Acoustic Zoom Inc., 685 St. Thomas Line, Paradise, NL, Canada

Adam Gogacz  
Acoustic Zoom Inc., 685 St. Thomas Line, Paradise, NL, Canada

1 INTRODUCTION

There remains a technology gap for dealing with marine sub-seabed site investigations. The limited sampling scales of geotechnical probes and corers typically hold little spatial knowledge. Acoustic based 2D profiling technologies map the continuity of reflective boundaries. Hence the question is posed - what is to be believed to be the true sedimentary state of the sub-seabed?

Three-dimensional determinations of (geophysical) structural complexities of the near subsurface require exact accounting of specular and non-specular returns with spatial (lateral and depth) accuracies that are more reliable than is currently attainable using conventional seismic site survey procedures (towed streamer based methods). Increased accuracy in detecting acoustic transmission properties allows for more refined correlations between acoustic and geotechnical properties of near-surface soils to be made. To successfully image the true sedimentary character and depositional (e.g. rock fragments, small-scale sand/shale lenses), internal composition variances require the preserving of the full energy redistribution of signals, in particular the capture of the diffuse diffracted signals along with the more dominant reflective energy. To facilitate this use of dense source/receiver spacing and millimeter accurate location calibrations is required. This can be accomplished by interrogating the sub-seabed through a stationary transmitter and receiver spatial centimeter spaced network with horizontal dimensions greater than 5 meters.

The approach called ‘Acoustic Sub-seabed Interrogation’ (ASI) was first introduced by Guigné in 1986 at University of Bath under the supervision of Prof. Nicholas Pace and since has been developed and applied to study near surface geo-hazards in the sub-seabed for mitigating risks in major offshore engineering geotechnical projects. The approach is specifically designed to acquire the backscattering response of targets such as boulders, discontinuities and inclusions within the sub-seafloor through the use of a dense data acquisition grid with its processing of signals akin to constant-offset prestack Kirchhoff time migration.

2 CREATING A PROTOTYPE DESIGN

2.1 The First Design

The first ASI design was experimented with in 1990 to 1993. The experimental design involved a platform that supported sixteen planar sparker transmitters in an octagonal polyethylene framework held by an aluminum outer structure. A twelve-meter-long rotating boom at the apex of the instrument provided support to twelve equally spaced calibrated hydrophones. Figure 1 shows the assembly and presents the positions of the sparker transmitters and receivers. The 12 receivers along the boom were rotated during data collection and aligned with four transmitters to form a transmitter-receiving row called a “beam.” This data acquisition configuration delivered four linear “beams” of data. The resulting data would then be processed and subsequently translated into a 3D volumetric imagery having a minimum ten-meter diameter with a depth of penetration in the sub-seabed of over ten meters.

This stationary acoustic acquisition ‘lens’ provided an “Acoustic Core” solution involving a 3-dimensional determination of geophysical parameters of the subsurface. Through such a stationary acoustic platform, signal coherence between repeated echoes is maintained. These transmitted
signals and multiplicity of acquisition locations provide the necessary coherency and density to statistically evaluate (in three dimensions) the character of glacial till type sediments and their distribution.

![Figure 1: First embodiment of the ASI using sixteen sparker sources as transmitters. Source: Author's archived photographs, circa 1990 from personal photo library.](image)

2.1 The Next Generation

The early 1991 prototype ASI embodiment led the way to various offshore seabed and laboratory-based investigations from 1992 to 2005 to better understand the acoustic complexities of buried objects (i.e. rock fragments or man-made) and to capture the scattering nature of these targets within the structural framework of layered geological formations. In 2006, a more sophisticated ASI engineering development was initiated. This product developed by PanGeo Subsea Inc. and trademarked as “Acoustic Corer™,” consists of: sonar hardware, data collection software, advanced (digital) data processing software, and interpretation software. The approach is designed to acquire full wavefield (both the specular and non-specular) response of the first 30 meters in complex sub-seabeds. Whenever available, emphasis in processing the data is to fuse it with geotechnical and geological datasets.

![Acoustic Packages](image)

Figure 2: The Acoustic Corer™ has two booms with each boom 6m in length, 180° rotational freedom holding an acoustic ‘package’ on each boom with each acoustic packages 3.5m above ground.
The package houses a high frequency (4.5 kHz – 12.5 kHz) chirp transducer, a low frequency (1.5 kHz – 6.5 kHz) chirp transducer and a hydrophone matrix; the acoustic packages move along the boom.

Processing of the collected data ‘renders’ the data into a volume whereby a layer-by-layer (whenever such stratification is observed) data analysis is made to examine both the specular nature of stratigraphic layers and the non-specular responses from (discontinuous) features such as boulders. In contrast, conventional sub-bottom profiling relies on the specular-reflected energy to form a sectional image of the geology. Indeed, associated data processing techniques emphasize only the laterally extensive seismic horizons while local discontinuities are not differentiated and become part of the ‘background noise’.

In all of the sub-bottom profilers on the market today including the more sophisticated 3D acoustic profiling acquisitions with long receiver offsets and inversion processing protocols, these geophysical mapping approaches discriminate against the diffuse, non-specular backscattered energy often treating these signals as ‘background noise’. In many instances, the very detail of the geological structure under investigation is too small (sub-wavelength scale) to yield laterally continuous events as observed in specular reflections. The textural character and detail in the seabed is thus not resolvable.

This is in contrast with the volumetric acoustic interrogation technique that enables high resolution imaging of the backscattering, subtle, and complex geological structures using beamforming via dense, multi-aspect illumination. The steerable (phased) source and receiver arrays are specifically designed to coherently use the backscattered response as its primary source of signals. It is these very small, but often dominant, diffused, non-specular, and diffracted reflections that illustrate the truer sedimentary and geotechnically relevant character of complex sub-seabed where these are characterized by the presence of internal discontinuities that diffuse the incident elastic wave. In view of geohazard, acoustic sub-seabed interrogations deliver a volumetric map of the internal roughness of the sub-seabed, which is reflective of its laterally and vertically disrupted depositional and/or erosional histories. The resultant imagery presents a more realistic picture of cavities, of fissures, and of cobble or boulder discontinuities embedded within a geological formation. As exemplified by the boulder shown in Figure 3, current acoustic profiler based imaging technologies do not handle well signals that are backscattered and this is why the detection in complex seaboards involving potential geohazards such as cobbles and boulders are so problematic.

![Figure 3: Typical buried boulders off the East Coast of Canada of similar diameters to the standard piles used offshore. Source: PanGeo Subsea Inc. marketing archive 2010.](image)

The strength of volumetric sub-seabed interrogations is founded on its pursuit to focus the backscattered wavefield (coherent summation). The focusing methodology relies on spatially variable approximation of the subsurface with an effective medium (homogenization) to back-project the diffusely scattered wave to the point of origin of scattering. That is, for each voxel (small computational volume) the total backscattered contribution is calculated, where each transducer-receiver pair observes the platform location’s specific total travel time to-and-from the scattering volume. If an actual scatterer exists within a computational volume, such as a boulder, the
The contribution is high due to coherent summation. On the other hand, if no scatterer (boulder) is present within the specified volume, the total contribution registers values that are very low due to incoherent summation of the ambient noise present in the data. Moreover, because the size of the (synthetic) aperture is much larger than the wavelength, the scattering at 30m or less occurs within the near-field of the source/receiver antennas. The entire volume rendering/interrogating process involves successive interrogations of individual cells (voxels) within a volume of influence. The answer product derived from using such densely collected and beam-formed synthetic aperture sonar (SAS) application delivers a volumetric acoustic core product.

2.2 Theoretical Background: Forward Problem

The assumed framework for the modeling that underpins the imaging of boulders is that of constant density medium with propagation of disturbances governed by the acoustic wave equation. This assumption ensures that in the frequency domain the acoustic wave equation is transformed to the Helmholtz operator and is thus self-adjointed, which simplifies the exposition. Boulders are modeled as discrete, compactly supported perturbations of a global background sound speed function defined in the three dimensional Cartesian space $\mathbb{R}^3$.

Suppose the full sound speed function is decomposed as,

$$c(x) = c_0(x) + \Delta c(x)$$

where $c_0: \mathbb{R}^3 \to \mathbb{R}_+$ represents a (smooth) background sound speed and the compactly supported (non-zero in a region of finite volume) $\Delta c: \mathbb{R}^3 \to \mathbb{R}$ represents the totality of all boulders, then the pressure wave due to a point source at $x_s$ is described by the following equation,

$$\left(\nabla^2 - \frac{1}{c^2(x)} \frac{\partial^2}{\partial t^2}\right) u(x, x_s, t) := \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2(x)} \frac{\partial^2}{\partial t^2}\right) u(x, x_s, t) = \delta(x - x_s)\delta(t)$$

and in the frequency domain the above acoustic wave equation takes on the following form,

$$\left(\nabla^2 - \frac{\omega^2}{c^2}\right) u(x, x_s, \omega) = \delta(x - x_s)$$

Suppose further that we decompose the inverse of the squared sound speed as,

$$\frac{1}{c^2(x)} = s_0^2(x) + \frac{1}{c_0^2(x)} \text{higher order terms}$$

and define the incident wave $u_i$ as a function satisfying (Green function for the smooth background acoustic wave),

$$\left(\nabla^2 - \omega^2 s_0^2(x)\right) u_i(x, x_s, \omega) = \delta(x - x_s)$$

then, the total wave is decomposed as,

$$u(x, x_s, \omega) := u_i(x, x_s, \omega) + u_s(x, x_s, \omega)$$

where $u_s(x, x_s, \omega)$, the scattered wave, arises as the disturbance due to the presence of the perturbations (boulders). That is, the perturbations act as secondary sources generating the scattered wave whereas the point source generates the incident wave.

Let the Helmholtz operator

$$L := \nabla^2 - \frac{\omega^2}{c^2}$$

be decomposed as,

$$L := L_0 + L_p$$

where,

$$L_0 := \nabla^2 - \omega^2 s_0^2$$

and

$$L_p := \omega^2 \Delta^2 s$$

then,

$$L_0^{-1} \delta(x - x_s) = u_i + L_0^{-1} L_p u_i + u_s + L_0^{-1} L_p u_s$$

and thus,

$$u_s = -L_0^{-1} L_p u_i - \lim_{s \to 0} L_0^{-1} L_p u_s$$

The second term in the above equation corresponds to “multiply” scattered pressure field and will be ignored in further discussion (Born approximation) where it is assumed that the perturbation $\Delta c$ is small.
Under Sommerfeld radiation condition (energy is radiated away from the source and none is radiated from infinity) assumption for the scattered, \( u_s \), and incident, \( u_i \), and Green’s second identity the (singly) scattered pressure wave is represented as,

\[
u_s(x, x_s, \omega) = -\omega^2 \int_{\Omega} u_i(\xi, x, \omega) u_i(\xi, x_s, \omega) \Delta^2 s(\xi) d\xi \]

where the region \( \Omega \) completely encloses the support of \( \Delta^2 s \).

The smooth background assumption lends itself to further approximation under the umbrella of ray theory (asymptotic series) where the incident wave is formally given as,

\[ u_i(x, y, \omega) \sim \sum_{i=0}^{\infty} u_i^i(x, y) e^{i\omega \tau(x, y)} \]

\( \tau(x, y) \) denotes the travel-time between the points \( x \) and \( y \) through the (acoustic) medium defined via \( c_0 \). The smoothness condition on the background sound speed allows us to make the following high-frequency approximation,

\[ u_i(x, y, \omega) = u_i^0(x, y) e^{i\omega \tau(x, y)} \]

Substitution (and some additional high frequency assumptions) of this approximate solution into the wave equation defined by the operator \( L_0 \) leads to the Eikonal and Transport equations where the travel-time function \( \tau(x, y) \) satisfies the Eikonal equation the amplitude function \( u_i^0(x, y) \) satisfies the Transport equation.

Hence, Born approximation and geometric optics approximation yield the following expression for the scattered wave,

\[
u_s(x, x_s, \omega) = -\omega^2 \int_{\Omega} u_i^0(\xi, x_s) u_i^0(\xi, x) e^{i\omega(\tau(\xi, x_s) + \tau(x, \xi))} \Delta^2 s(\xi) d\xi \]

Note that in the special case of constant background sound speed medium the amplitude and travel-time functions are given as,

\[ u_i^0(x, y) = \frac{1}{||x - y||} \quad \text{and} \quad \tau(x, x_s) = \frac{||x - x_s||}{c_0} \]

### 2.3 Theoretical Background: Inverse Problem

In order to abbreviate the notation define,

\[
\phi(x, x_s, \xi) := \tau(\xi, x_s) + \tau(x, \xi)
\]

and

\[
a(x, x_s, \xi) := a_i^0(\xi, x_s) + a_i^0(\xi, x)
\]

then, the scattered wave in the frequency domain is expressed as,

\[
u_s(x, x_s, \omega) = -\omega^2 \int_{\Omega} a(x, x_s, \xi) e^{i\omega(\phi(x, x_s, \xi))} \Delta^2 s(\xi) d\xi
\]

and in the time domain as,

\[
u_s(x, x_s, t) = -\frac{\partial^2}{\partial t^2} \int_{\Omega} a(x, x_s, \xi) \frac{\delta(t - \phi(x, x_s, \xi)) \Delta^2 s(\xi) d\xi}{r(\delta_s)(x, x_s, t)}
\]

The operator \( R(\delta_s)(x, x_s, t) \) is related to the generalized Radon transform as defined by Beylkin (1985). Suppose the values of \( x \) in the above scattered wave equation are restricted to the boundary \( \partial \Omega \) of the region \( \Omega \) and \( t > 0 \), then the generalized backprojection imaging operator \( R^* \) is defined as,

\[
R^*(u_s)(x) := \int_{\partial \Omega} u_s(\xi, x_s, \phi(x, x_s, \xi)) \frac{h(x, \xi)}{a(x, x_s, \xi)} \frac{d\xi}{w(x, x_s, \xi)}
\]

where the Beylkin determinant \( h(x, \xi) \) is,
The generalized back-projection operator is shown in Beylkin (1985) to be a pseudo-differential operator, which up to a smooth function is a stable inverse of the (approximate Born + ray theory) modeling operator describing the scattered acoustic wave as defined above, that is,
\[
\Delta^2 s(x) \approx R^*(u_s)(x)
\]

The imaging operator,
\[
R^*(u_s)(x) : = \int_{\Omega} u_s(\xi, x, \phi(x, x_s, \xi)) w(x, x_s, \xi) \, d\xi
\]
is readily recognized as a weighted diffraction summation (integration) operator over the aperture of the recorded data and more specifically the summation is carried out over the diffraction curve defined by \( \phi(x, x_s, \xi) \) and scaled by \( w(x, x_s, \xi) \). The set \( \Omega \) is assumed to sufficiently smooth and necessarily enclosing the support of the perturbation field \( \Delta c \) and \( x_s \); however, in most applications the source and receiver positions only comprise a limited portion of the entirely ‘surrounding’ shell. This aperture limitation results in spatial smoothing of the final image; it is an artifact observed in migrated data and is known a priori and can only be suppressed by careful acquisition design.

2.4 Diffraction Focusing Analysis: Field Data Examples

If we assume the boulders to be discretely distributed (sufficiently sparse to produce discernible diffractions) and sufficiently small in relation to the wavelength of the source wavelet, then we may approximate the boulder perturbation field \( \Delta^2 s(x) \) as a (countable) sum of scaled Dirac distributions (secondary point sources),
\[
\Delta^2 s(x) \approx \sum_{s=0}^{\infty} s_v \delta(x - x_v)
\]
This representation of the perturbation \( \Delta^2 s \) allows for the scattered wave to be expressed as,
\[
u_s(x, x_s, t) = -\frac{\partial^2}{\partial t^2} \sum_{s=0}^{\infty} s_v a(x, x_s, x_v) \delta(t - \phi(x, x_v, x_v))
\]
which is a sum of individual diffractions.

The discrete distribution of the boulders forms the basis of the diffraction-focusing method as expressed by the imaging operator \( R^* \). That is, the boulders give rise to diffractions and the imaging operator ‘back projects’ (focuses) the diffractions to the point of origin. By nature of being discrete the boulders in the migrated data appear as high amplitude spatially localized events in an otherwise ‘transparent’ background. The diffraction-focusing method is a process whereby the background sound speed is obtained from the recorded data by adjusting the background sound speed (e.g. inversion, trial and error) so as to maximize the amplitude (in the migrated data) of the event against the neighboring background. The four panels in Figure 4a show constant time slices through recorded data collected on the Grand Banks. The scattering wave is captured in the raw data in time slices as a series of expanding circular events.

Figure 4a: Four panels showing signature of a boulder, constant time slices in raw recorded data.
The panels in Figure 4b show the process of arriving at the optimal background sound speed estimation. The vertical axis in the three leftmost panels represents the two-way travel-time from the recording surface to the voxel under investigation, the horizontal axis in the leftmost panel represents a spectrum of background sound speeds, the horizontal axis in the second and third leftmost panels represent grid easting and northing directions. The rightmost panel shows a constant time/depth (the imaging operator is akin to prestack time migration) slice through the migrated data.

Figure 4b: Example of a signature of a boulder in migrated datasets.

The panels shown in Figure 5, 6 and 7 exemplify the process of imaging of boulders and estimation of the background sound speed field. Whenever the boulders are small (size on the order of the wavelength or smaller), then the expected signature in the migrated data is a high intensity maximally localized event. However, if the boulder is sufficiently large, then the point-scatterer assumption no longer holds and imaging proceeds in a manner similar to imaging of reflectors where resolution of the boundary becomes focal.

Figure 5: Example of a boulder where maximal focus intensity is reached.
2.5 Discussion

An important objective of interrogating the sub-seabed in a stationary manner is to image geotechnical anomalies and geo-hazards such as discontinuous layers and boulders. Because boulders tend to scatter acoustic energy the resulting imagery is a diffused response and in typical sub-bottom profiling will be noted as part of the background noise. With the volumetric acoustic imaging interrogation approach multi-aspects views of the returns become coherent and part of the true seismic response thus raising these diffuse signals out of the noise background as coherent signal. With optimal velocity focusing boulders and other anomalies reveal themselves with focused
maximal intensity. Detection of boulders (rock fragments > 0.3 m) in glacial and post-glacial soils requires utmost care in data processing as unconsolidated soils prove highly absorptive in the 4kHz+ bandwidth regime leading to diminished penetration and ‘target’ resolution. Buried boulders must be treated as a 3D problem (conventional sub-bottom profiling methods are too coarse); absence of laterally extensive seismic horizons requires diffraction/scattering focusing methods to be employed for acoustic speed estimation, subsequently used in imaging/detection.

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