SUPPORT MATHEMATICS FOR ACOUSTICS RESEARCH TRAINING

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1 INTRODUCTION

Mathematical methods are important for research in many aspects of acoustics. Currently, fundamental mathematical methodologies taught at undergraduate level are often advanced through independent learning by individual researchers. They develop their mathematical skills as appropriate rather than being made aware of the potential of advanced mathematical tools at the onset of their research career. Furthermore, most researchers in acoustics do not have access to masters level courses to broaden their postgraduate study. A first attempt to remedy this was made at a Summer School held at the University of Southampton 14-18 July 2003. The Summer School aimed to address the mathematical needs of UK post-graduate research students in engineering and physical acoustics. It took the form of an intensive residential week for 40 post-graduate students drawn from across the UK supported by a high staff/student ratio. The School benefited from lecturing contributions by leading mathematicians and acousticians from several Universities including Chris Linton, Maureen McIver (Loughborough) Chris Howls, Rod Self (Southampton) Trevor Cox , Philip Duncan, Yui Wei Lam (Salford), John Chapman (Keele) Nigel Peake (Cambridge), John Elliott (Hull) and Trevor Esward (National Physical Laboratory). It was coordinated by Professor Keith Attenborough, on behalf of the Institute of Acoustics, Research Coordination Committee, and by Dr. Matthew Wright at ISVR. Although publicity was distributed widely, analysis of the students indicates that they were recruited mainly from the pre-1992 universities and that newer universities were relatively under-represented. The course introduced important mathematical techniques in acoustical contexts using lectures, examples classes, tutorials and one-to-one 'surgeries'. The objective of the course was to convey the essential mathematical tools and concepts that will enable the attending research students in physical and engineering acoustics to make a rapid impact in their research. The emphasis was on how mathematics is used in acoustics rather than on mathematics for its own sake.

During the Summer School preparation, Lyn Leventhall (Learning Technology Group, School of Computing and Information Systems, Kingston University) approached the organizers concerning her PhD research on mathematical communication. At the School she took video and audio recordings during some of the lectures and held focus group evaluations.

A relatively short amount of time was available between confirmation of the funding of the summer school and its proposed date. The lectures were arranged by balancing a number of constraints, *viz:*

- What topics were felt to be important to current acoustical research.
- What subjects could be taught in the time available without unbalancing the programme.
- What subjects fell into suitable themes so that lectures could be arranged in a suitable order.
- Who had volunteered, or could be asked, to teach particular topics.

The week started by revising important mathematical techniques (vector calculus, complex variables, integral transforms) during the first day. The other four days were devoted to the mathematics of waveguides, signal processing and aeroacoustics. These were augmented by tutorials, computer (MATLAB) sessions and evening 'surgeries' in which there was opportunity for individual interaction with tutors on topics (both mathematical and acoustical) chosen by the participants. A distinction was drawn between 'lectures' which took place during the day and fell within the days theme, often being linked and having associated tutorial questions, and 'talks' which were given at the end of the day on a subject of the speaker's choice, with no tutorials. The final morning consisted entirely of such talks.

Apart from competence in algebra, trigonometry, and geometry it was assumed that registrants would have some familiarity with topics listed in Table 1. Potential attendees were requested to identify any topics from the list for which they would appreciate assistance. In the event not many of the registrants responded to this. Moreover the range of previous mathematical ability among the registrants was very wide; from a degree in Music Technology to the second year of a PhD in mathematics! Some *ad hoc* arrangements were made for extra tutorials on topics in demand. These included matrices and Z-transforms.

The students worked and ate together each day, from 9:00 to 18:30 in the Maths building, followed by a communal meal at the University Staff Club, and stayed together in rooms at one of the University Halls of Residence. An additional benefit to the summer school beyond the imparting of mathematical education was the opportunity for networking among future acoustics researchers, and exposure to ideas outside their own projects.

Table 1 Prerequisite topics

TOPIC
Differentiation and Integration
Basic definition of rate of change and slope of graph
Derivatives of simple algebraic functions, polynomials, exponentials, logarithms and trigonometric functions
Rules including product, quotient and function-of-a-function
Maxima and Minima
functions of more than one variable
elements of partial differentiation
Integration as the limit of a sum
Integration as the reverse of differentiation
Indefinite and definite integrals
Numerical integration including rectangular, trapezium and Simpson's rules
First order ordinary differential equations and linear second order with constant coefficients, solution methods including integrating factor
Definition of imaginary unit (square root of minus 1)
Argand diagrams
Complex conjugate and modulus
Addition and multiplication of complex numbers
Polar representation including De Moivre's theorem
Complex roots and functions
Matrices, Determinants and Vectors
Matrix addition and Multiplication
Matrix inverse
Determinants
Matrix representation of Systems of equations
Vector algebra including vector addition and scalar and vector products
Intersection of lines and planes
Vectors in three dimensions

2 FEEDBACK

2.1 Student Feedback

Thirty responses to a feedback questionnaire that was circulated and completed on site have been analysed. Post-school surveys have produced a further seven responses. Student response concerning the extent to which the School had met their needs was mainly positive. Some of the student responses to the question "*To what extent did the summer school meet your needs*?" were as follows

'Well, ..now aware of maths in acoustics in a general sense', 'to a great extent', 'helped quite a lot', 'needs met to a highly satisfactory level', 'fairly well', 'Monday p.m., parts of Tuesday and Wednesday excellent, Thursday particularly good'. Quite well, revision and extension'. 'Learned some techniques useful for my work', 'it has been good to remind me of what I studied before'. 'very good event, informative and useful' 'I have lots of new ideas on my project now' 'has given a broad idea of what kind of maths might be required in the future',
Comment supplied via supervisor three months after the School: 'has prepared (me) well for questions in recent (acoustics-related) job interviews'

A selection of comments responding to "*Which sections did you find particularly useful and not useful*? Is as follows:

'Very Useful', 'Very useful (DSP and Monday/Tuesday)', 'All particularly DSP', 'Maths of Generalised Functions', 'All but particularly waveguides and DSP', 'Good reference base'. 'Aeroacoustics very useful'. 'Monday not useful.' 'Most useful aeroacoustic theory but general maths useful'. 'Evening tutorials on matrices and Z transforms were the most useful.'

Responses to "*What changes do you suggest we make in future*?" included:

'Advance circulation of lecturers' notes'. 'More applications and computer sessions.' 'More time for exercise sessions at cost of fewer lectures (e.g. Wiener Hopf).' 'Better accommodation.' 'More engineering and computational aeroacoustics.' 'More practical examples.' 'Reduce scope.' 'More realistic exercises.' 'Make clear the minimum level of background knowledge required.' 'More computer simulations, paper copy of every presentation.' 'More (optional) basic lectures.' 'Less flexible timetable.' 'Streaming.' 'Social event earlier.' 'More and better-planned surgeries.'
'Knowing that this is not necessarily feasible, I believe that a more "modular" schedule with one or two days with parallel course-sessions could help the school be more attractive to a wider range of students (for example the mathematics of the first day could have one "entry" and one "advanced" version).

2.2 Staff Feedback

Several tutors found it rewarding to work with such excellent students. Tutors have suggested also that periods for private study followed by graded tutorials would be desirable. Lyn Leventhall has suggested that a future school should include 5-minute presentations on their research by each registrant. This would help participants identify useful colleagues and help with their training in presentational skills.

3 SUGGESTIONS FOR FUTURE ACTIVITY

3.1 Future Summer Schools

In respect of the minimum level of background knowledge required, it is suggested that the HELM[1] workbooks and CAL be made available to applicants for any future 'maths for acoustics' school and that applicants be required to complete a pre-test based on these materials. In view of the wide range of mathematical background encountered this year and the obviously overburdened first day, a possible way forward would be a 'streamed' summer school with a common first day on basic techniques (significantly reduced content compared with this year) and subsequently two parallel sessions; one more advanced than the other. This would give scope for more 'basic techniques' for some students, graded tutorials and inclusion of more topics such as statistics (including handling experimental data) and computational acoustics (CFD, BEM, FEM). Overall one week with parallel sessions is preferred to a two week School. A proposed structure for a future School is shown in Table 2.

3.2 Book of Lecture Notes

Several students expressed a wish that they be given the lecture notes in advance. This need should be met for a future school by a book currently being edited by Dr. Wright and based on this year's lectures to be published by Imperial College Press. Each author has been invited to revise and expand where appropriate their material and notation conventions have been standardised as far as reasonable among chapters. This was an issue that there was not time to address in the relatively short preparation time for the summer school itself. It is hoped that the paperback edition will be available at a price that will put it within the reach of graduate students, and that copies could be provided to future students on registration.

The planned chapters of the book are:

Part I: Mathematical Methods

1. Vector Calculus – *J. W. Elliott*
2. Functions of a Complex Variable – *J. W. Elliott*
3. Integral Transforms – *J. W. Elliott*
4. Asymptotic Evaluation of Integrals – *R. H. Self*

Part II: Wave Motion

5. The Wiener–Hopf Technique – *M. C. M. Wright*
6. Waveguides –*M. McIver & C. M. Linton*
7. Wavefield Decomposition – *M. C. M. Wright*
8. Acoustics of Rigid–Porous Materials – *K. Attenborough & O. Umnova*

Part III: Aeroacoustics

9. Generalised Functions in Aeroacoustics – N. Peake
10. Monopoles, Dipoles, and Quadrupoles – C. J. Chapman
11. Corrugated Pipe Flow – J. W. Elliott

Part IV: Signal Processing

12. Digital Filters – P. J. Duncan
13. Measurement of LTI Systems – T. J. Cox
14. Numerical Optimisation –T. J. Cox

4 CONCLUSIONS

The experiment made in July 2003 at Southampton has confirmed the need for additional training in mathematics for early researchers. A plan for a future school in 2005 is underway. The contributors to the 2003 School would be grateful for suggestions and support for the organization of future similar events.

Table 2 Proposed format for future maths for acoustics school

	Stream 1	Stream 2	All
Day 1	Basic techniques session 1	Basic techniques session 1	Tutorials & Surgeries
Day 2	Basic Techniques session 2	Waveguides	Tutorials, Surgeries & lectures
Day 3	Signal Processing & Statistics	Maths of Digital Signal Processing	Computer session 1, Surgeries & lectures
Day 4	Computational Acoustics 1	Computational Acoustics 1	Computer session 2 & guest lecture
	Lectures on applications	Aeroacoustics (Matched Asymptotic Expansions or Green's functions)	
Day 5	Revision tutorials	Computational Acoustics II	

5 ACKNOWLEDGMENT

The Summer School was made possible by EPSRC GR/S47021/01

6 REFERENCE

1 FDTL4 project "Helping Engineers Learn Mathematics" consortium lead by Loughborough