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THE APPLICATION OF FINITE ELEMENT MODELS TO STUDY THE ENERGY FLOW PROPERTIES OF STRUCTURAL JUNCTIONS

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1. INTRODUCTION

One of the foremost drawbacks of the state-of-the-art computational techniques with respect to the evaluation of coupling loss factors and transmission coefficients deals with the impossibility to take into account vital structural details which nonetheless become more and more critical (from the vibration transmission characteristics viewpoint) for higher frequencies. For this reason, it is meaningful to establish pertinent methods able of evaluating the transmission characteristics in an accurate way, other than the inept existing computational methods.
The method which is developed in here allows to include essential geometrical details and therefore allows to fully optimize the vibration transmission characteristics of the junction. The approach is based on the finite element method and consists basically of building an appropriate local finite element model of the junction, applying suitable boundary conditions and performing a consistent dynamic analysis in order to obtain the transmission coefficients of the junction.

2. THEORY

The procedure will be outlined in the case of a beam-beam junction. For reasons of simplicity, attention will be focused on a beam-beam junction comprising *three* beams. The junction is modeled based on finite elements and is schematically pictured in figure *1*.
Let *[K]* be the global finite element stiffness matrix of the junction. *[M]* denotes the global mass matrix of the junction. Without taking into account damping phenomena, it holds that :

$$([K]-\omega^2[M])\cdot\{U\}=\{F\} \tag{1}$$

The matrix $[K]-\omega^2[M]$ is denoted $[K^d]$ further on.
The dynamic stiffness matrix of the junction $[K^d]$ is reduced to the nodes lying at the ends of the beams. (Node b_1, b_2 and b_3). A classical, static condensation of the internal degrees of freedom is carried out. Let subscript b denote the degrees of freedom at the boundaries of the beams. Subscript i denotes the internal degrees of freedom. Equation (1) can be partitioned to yield :

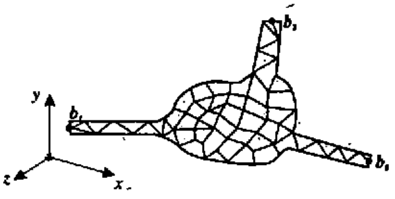

Fig. 1

$$\begin{bmatrix} K^d_{bb} & K^d_{bi} \\ K^d_{ib} & K^d_{ii} \end{bmatrix}\cdot\begin{Bmatrix} U_b \\ U_i \end{Bmatrix}=\begin{Bmatrix} F_b \\ 0 \end{Bmatrix} \tag{2}$$

The dynamic stiffness matrix of the junction, reduced to it's boundaries becomes :

$$[K^d_{reduced}]=[K^d_{bb}]-[K^d_{bi}]\cdot[K^d_{ii}]^{-1}\cdot[K^d_{ib}] \tag{3}$$

Consequently, the relation between the external forces acting upon the junction at the boundaries and the displacements at the boundaries is given by :

$$\{F_b\}=[K^d_{reduced}]\cdot\{U_b\} \tag{4}$$

The forces and displacements of the beams, acting at the boundaries of the beam can be partitioned into forces and displacements associated with positive and associated with negative going waves. Therefore, equation (4) becomes :

$$\{F_b^+\}+\{F_b^-\}=[K^d_{reduced}]\cdot\{U_b^+\}+[K^d_{reduced}]\cdot\{U_b^-\} \tag{5}$$

Where $\{F_b^+\}$ represents the vector of beam forces at the end-nodes associated with the positive going waves. $\{U^+\}$ represents the vector of displacements associated with positive going waves. A similar notation holds for negative going waves.
The internal forces of the beams which are assumed to be associated with a specific wave field (positive or negative) can be replaced by the corresponding product of the appropriate semi-infinite stiffness and displacement. Hence, equation (5) becomes :

$$[K_\infty^+]\cdot\{U_b^+\}+[K_\infty^-]\cdot\{U_b^-\}=[K]\cdot\{U_b^+\}+[K]\cdot\{U_b^-\} \tag{6}$$

by rearranging the terms of equation (6), one obtains the following equation :

$$\{U_b^+\}=[A]\cdot\{U_b^-\} \tag{7}$$

Thus, equation (7) expresses the relation between the displacements of the incidence waves and the displacements of the outgoing waves at the boundaries of the junction. From here on, the procedure to calculate the transmission matrix is straightforward. DE LANGHE [2] covers this procedure more in detail.

3. EXAMPLE : RIGHT ANGLE INCORPORATING A ROUND OFF

The following example has been chosen to demonstrate the superiority of the finite element wave approach compared to the classical computational approach in the case of a simple round off contained in the junction. Within the framework of the classical approach, based on the standard work to-date, it is not possible to take into account round offs at the junction as the classical approach models the round off as a right angle assuming a *rigid point connection*, eventually taking into account an additional off-set. The finite element wave approach encompasses this problem because the round off can be modeled in detail by finite elements.

The example described in here consists of two right angled semi-infinite beams (constructed of rigid PVC), incorporating a round off at the connection between both beams. A square section of $10 \times 10\ mm^2$ is taken. The structure which is analyzed is schematically pictured in figure 2 (left side). The finite element model is pictured on the right hand side of figure 2. Note that for the finite element model, only the curved part is modeled as it is redundant to incorporate the straight parts into the finite element model. The semi-infinite stiffnesses are directly applied at the ends of the curved parts of the junction.

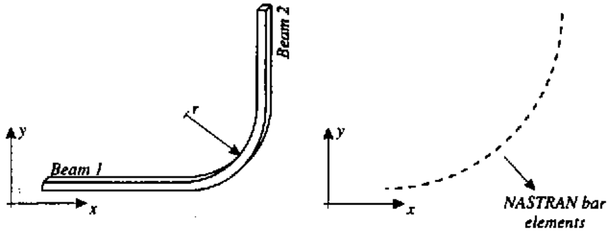

Fig. 2

Figure 3 shows the bending transmission coefficient as a function of the parameter *radius / bending wavelength.*

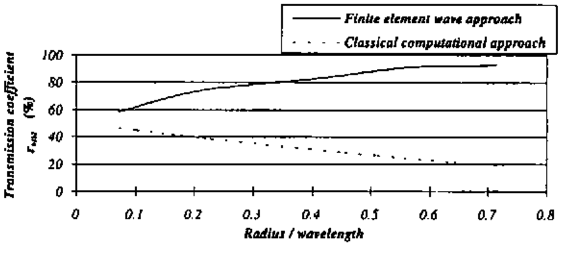

Fig. 3

The example illustrates clearly that if the radius becomes larger than *1/5* of the wavelength, then the effect of the finite dimensions of the radius becomes critical and therefore, the classical approach fails for the determination of the transmission coefficients. The latter conclusion can be extended in such a way that one may state that structural details of the junction which are of the same order of magnitude compared to the wavelength do become important and the effects should definitely be taken into account.

4. CONCLUSION

When applying SEA to complex structures in an accurate way, it is necessary to obtain good values of the transmission coefficients of the structural junctions. Because in the high frequency range, the transmission of vibrational energy from one subsystem to another subsystem is unquestionably dependent upon the local structural details of the junction, it is advised to apply finite elements in order to model the junction in an accurate way.

The so-called "finite element wave approach" is developed, implemented and worked out in the case of beam-beam junctions. The technique makes it possible to take into account whatever geometrical complexity within the beam-beam junction.

A typical example manifesting the significance of the technique concerns the incorporation of round-offs at the junction. The theory developed in here is shown to work properly for this example. Other examples, highlighting the importance of the finite element wave approach, are dealing with the application of solid elements and the influence of a reinforcing beam at the junction on the transmission coefficient, see DE LANGHE et al. [2-3].

The ultimate goal of the technique is to optimize a junction (with respect to energy transfer) through the appropriate application of finite element models and relevant optimization tools.

5. REFERENCES

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[3] DE LANGHE, K.; SAS, P.; VANDEPITTE, D.; *Proceedings ISMA19 Volume 1, pp. 471-480; Leuven, Belgium; 1993*