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Modeling of sound propagation over ballast surfaces

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ABSTRACT

For receiver positions not too far away from a railway line the ballast bed may represent a significant portion of the ground area where reflection occurs. Extensive measurements have shown that ground reflection over ballast has to be understood as an extended-reaction process. With the application of a simple geometrical model for the ballast structure good agreement between measurement and calculation could be achieved. The model was validated for a large set of loudspeaker measurements conducted in line with the development of sonRAIL, the new Swiss railway noise calculation model [1],[2].

1. INTRODUCTION

Usually railway lines run on beds of ballast. The ballast layer has a typical thickness between 30 and 60 cm. For sound sources close to the ground there may be a significant portion of ground reflection over these ballast areas. For example such a case is given for a nearby receiver with a train running on the distant track of a double-track railway line. This configuration is especially typical for measurements of railway source characteristics. As will be shown later sound fields above such surfaces have to consider the ballast material as an extended reaction medium.

2. EXTENDED REACTION REFLECTION

A medium with extended reaction allows for propagation in the material itself. Thus the complete characterization consists of a description of impedance Z and propagation constant Γ or - equivalent - the complex wave number k. For a given material it is not obvious how to find the frequency dependencies of Z and k. In section 3 a geometrical model will be presented that describes a ballast structure as an analogy of an electrical transmission line where Z and k are well known.

Once impedance and wave number are found the interaction of a spherical sound wave with a material of given thickness d can be calculated with an empirical extension of the classical Weyl-Van der Pool formula, as proposed by Li et al. [3]. For that purpose an effective admittance β_e is introduced. This effective admittance depends on the acoustical material properties Z and k, the thickness d and the angle θ of sound incidence (measured relative to the normal direction of the surface). The extension to non flat ground can be achieved by a Fresnel zone approach as described in [4].

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3. TRANSMISSION LINE AS ELECTRICAL ANALOG OF BALLAST

From an acoustical point of view the essential characteristics of ballast can be described as a grid of connected tubes [5] complemented by cavities at the intersections as shown in Figure 1.

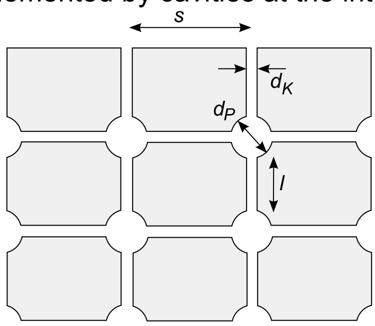

Figure 1: Geometrical model of ballast material with spherical cavities of diameter d_P connected by tubes of diameter d_K and length l (vertical) and s-d_P (horizontal) respectively.

The geometrical ballast model from Fig. 1 can be translated into an analog electrical *RLC* network (Fig. 2) [6], consisting of resistances, inductances and capacitances. In the electrical analogy voltage corresponds to sound pressure and current represents sound energy flux, that is to say the product of sound velocity and a reference area.

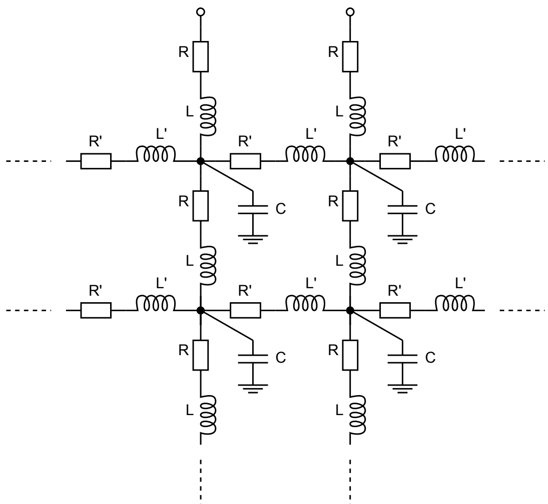

Figure 2: Electrical network (resistances R, inductances L and capacitances C) with analog behavior as the geometrical ballast model.

The impedance Z and the wave number k can be discussed for a plane wave of normal incidence (propagating from top to bottom in Fig. 1). As all cavities in a horizontal plane experience identical pressures there is no horizontal exchange of sound energy flux. For that reason the discussion of one vertical cavity/tube cascade is sufficient. This leads directly to the well known electrical network of a transmission line (Fig. 3).

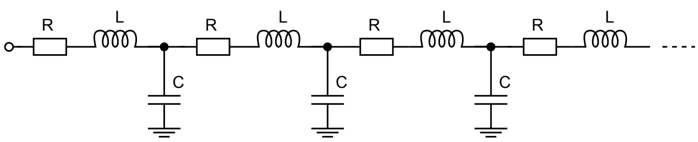

Figure 3: Transmission line as analog electrical network for the ballast material.

The electrical transmission line shown in Fig. 3 has an impedance Z' at the left-side terminals given by Eq. 1

$$Z' = \sqrt{\frac{L}{C} - j\frac{R}{\omega C}} \qquad (1)$$

with angular frequency $\omega = 2\pi f$. The propagation constant Γ' per unit length is given by Eq. 2.

$$\Gamma' = \sqrt{-\omega^2 LC + j\omega RC} \qquad (2)$$

The values for R, L and C are found as

$$R = \frac{8l\eta}{\pi r^4}G \qquad L = \frac{\rho l}{A}G \qquad C = \frac{V}{\rho c^2}\frac{1}{G} \qquad (3)$$

with l: tube length, r: tube radius ($r = d_k/2$), η: viscosity coefficient for air = 1.82x10^{-5} Nsm^{-2}, ρ: density of air, A: tube cross sectional area ($A = \pi d_k^2/4$), V: cavity volume ($V = \pi d_P^3/6$), c: speed of sound. The factor G = 1 [Vm^5/NAs] is necessary for the conversion of units in the analogy.

The impedance Z' according to Eq. (1) describes the ratio of sound pressure to sound energy flux. By scaling with the tube cross sectional area A the familiar impedance definition (ratio of sound pressure to sound velocity) is found. This impedance is valid at the entrance of the tube. It is assumed that the rest of the surface of the material is acoustically hard which implies vanishing sound velocity. The average impedance Z at the surface is thus found by weighting with the ratio of the areas (Eq. 4).

$$Z = \sqrt{\frac{L}{C} - j\frac{R}{\omega C}} \cdot A \cdot \frac{s^2}{A} = s^2\sqrt{\frac{L}{C} - j\frac{R}{\omega C}} \qquad (4)$$

Z in Eq. (4) assumes an $e^{j\omega t}$-time dependency. For the calculation of the reflection as described by Li et al. the $e^{-j\omega t}$ convention is used. Therefore the sign of the imaginary part of Z has to be changed.

The propagation constant Γ' in Eq. (2) has to be understood per unit length. The absolute value of Γ is thus given by division of the physical length of an RLC element (Eq. 5).

$$\Gamma = \frac{1}{d_P + l}\sqrt{-\omega^2 LC + j\omega RC} \qquad (5)$$

Finally from Γ follows the wave number k according to Eq. (6).

$$\mathrm{Re}[k] = \mathrm{Im}[\Gamma]$$
$$\mathrm{Im}[k] = \mathrm{Re}[\Gamma] \qquad (6)$$

4. PARAMETER ADJUSTMENT

The geometrical parameters of the ballast model such as tube diameter d_K, tube length l, cavity diameter d_P and distance between the rows s were determined by variation 'by hand' until a best fit with a reference propagation measurement was reached. For that purpose the frequency dependent attenuation between a loudspeaker and a microphone above a ballast bed of approximately 0.35 m thickness was evaluated. The loudspeaker with uniform directivity in the vertical plane was located 0.5 m above ground, the microphone height was 1.2 m, distance between loudspeaker and microphone was 7.5 m. By comparison with a measurement under free field conditions the influence of the ground, that is to say the *ground effect* could be determined.
With the best fit parameter settings (d_K = 0.00127 m, l = 0.00132 m, d_P = 0.0056 m, s = 0.006 m) and a thickness of the ballast bed of 0.33 m a very good agreement between measurement and calculation could be achieved up to frequencies of about 1.5 kHz (Fig. 4). At higher frequencies the calculation shows larger deviations from the measurement. This is probably due to scattering losses at the rough surface.

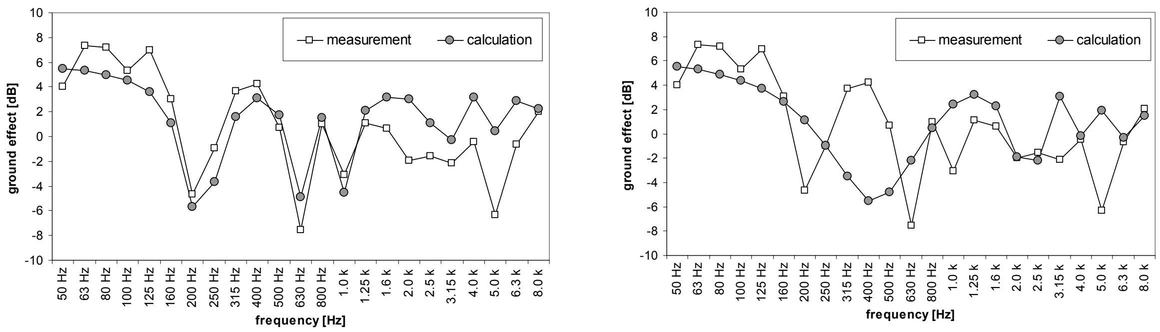

Figure 4: Measured and calculated ground effect with optimally adjusted model parameters (left) and for comparison a calculation assuming local reaction with a flow resistivity of 50 kPa s m^{-2}.

The calculation model with the optimal parameter setting was further validated at the occasion of loudspeaker measurements at eight different sites. As the thickness of ballast beds may vary from location to location, this parameter has to be adjusted individually. By doing so very good agreement was found with unchanged geometry parameters d_K, l, d_P, and s [7].

5. RELEVANCE OF THE EXTENDED REACTION APPROACH IN CONNECTION WITH EMISSION MEASUREMENTS

Measurements to determine the sound power of train vehicles are usually done in a standard distance of 7.5 m from the center of the track at a height of 1.2 m above track level. Based on measured sound pressure, sound power is found by inverse calculation of the propagation effects such as geometrical spreading and interference with the ground reflection. For that reason an accurate model to estimate the ground effect is very important. Here a comparison is given for ground effect calculations assuming local and extended reaction respectively. The source is located above a bed of 0.40 m ballast, adjacent to grass land with a Delany Bazley [8] flow resistivity of 200 kPa s m^{-2}. For the local reaction calculation the flow resistivity of the ballast surface was set to 50 kPa s m^{-2} [9]. Fig. 5 shows the corresponding geometry. It is assumed that microphone M1 captures train passages on the distant track no. 1 and M2 measures events on the nearby track no. 2. The geometry parameters were set according to Table 1. Figure 6 demonstrates the calculated ground effect for an omnidirectional point source, integrated over a passage within an angle of +/-45°. For the lowest source position (0.0 m) even the passage on the nearby track leads to considerable differences in the spectrum of the ground effect. For sources higher up on the close-by track these differences vanish. Ground effect calculations for the distant track show tremendous deviations between the local and extended reaction assumption, also for higher source positions.

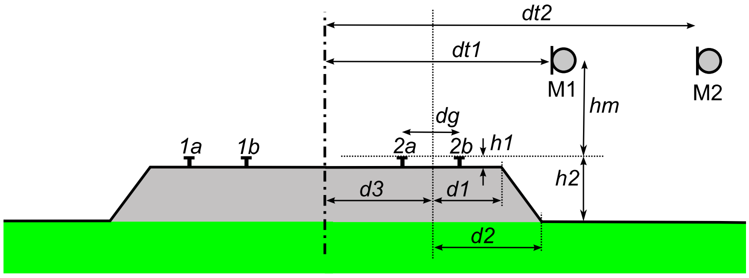

Figure 5: Geometry used to investigate the ground effect in emission situations. A 0.40 m thick bed of ballast lies on grass land. The source is located above rail 1b for microphone position M1 (distant track) and above rail 2b for microphone position M2 (nearby track).

Table 1: Geometry parameters used in the calculations.

d1	*d2*	*d3*	*dg*	*h1*	*h2*	*dt1*	*dt2*	*hm*
1.90 m	2.22 m	2.00 m	1.50 m	0.17 m	0.57 m	5.50 m	9.50 m	1.20 m

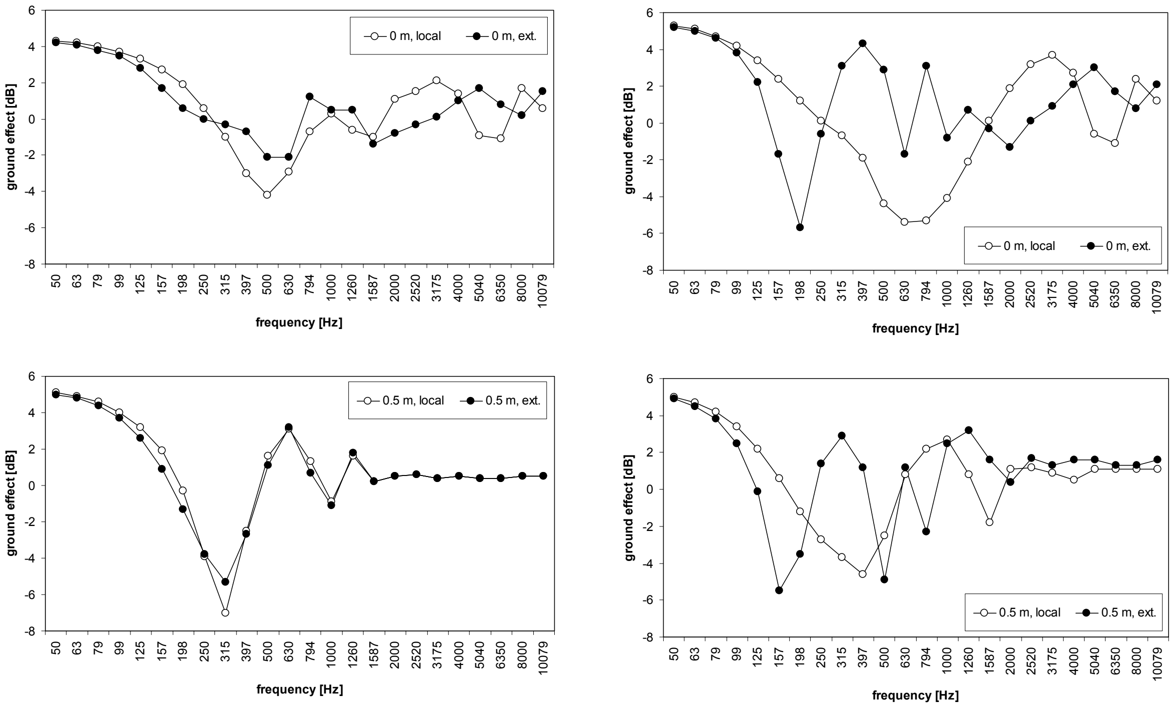

Figure 6: Calculated ground effect with local and extended reaction assumption for the geometry in Table 1 and source heights of 0.0 and 0.5 m (from top to bottom), for the nearby (left column) and distant (right column) track.

6. CONCLUSIONS

It has been shown that reflected sound over a ballast bed has to be calculated with an extended-reaction approach whereas the local reaction assumption leads to significant errors. Good agreement with measurements was achieved by application of a model by Li [3] and frequency responses of impedance and wave number according to a simple cavity/tube model. The tuning of the ballast model yielded values for the geometrical parameters with a plausible order of magnitude.

The calculations confirm a pronounced sensitivity of the ground effect relative to the thickness of the ballast bed. In situations with significant reflection over ballast, this parameter needs to be known. If no information is available the thickness should be measured, either directly or by an indirect method. As an indirect method propagation measurements with a loudspeaker have proven to deliver useful results. After adjustment of this parameter accurate ground effect calculations are possible for all source heights of interest.

ACKNOWLEDGMENTS

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REFERENCES

1. D. Sehu, Railway noise: a problem? A contribution to the solution: sonRAIL, Euronoise 2009.

2. J. M. Wunderli, The sound propagation model of sonRAIL, Euronoise 2009.

3. K. M. Li, T. Waters-Fuller and K. Attenborough, “Sound propagation from a point source over extended-reaction ground”, *J. Acoust. Soc. Am* **104**, pp. 679-685, (1998).

4. B. Plovsing and J. Kragh, “Nord2000. Comprehensive Outdoor Sound Propagation Model”, DELTA Acoustics & Vibration Report AV 1849/00, 2000.

5. J. Hofmann and K. Heutschi, “Simulation of outdoor sound propagation with a transmission line matrix method”, *Applied Acoustics* **68**, pp. 158-172, (2007).

6. L. L. Beranek, Acoustics, published for the Acoustical Society of America, 1986.

7. K. Heutschi, Sound propagation over ballast surfaces, submitted to Acta Acustica united with Acustica.

8. M. E. Delany, E. N. Bazley, Acoustical Properties of Fibrous Absorbent Materials, Applied Acoustics, **3**, pp. 105-116, (1970).

9. P. van der Stap, E. Verheijen et. al. Practical data collection for the Harmonoise source model: measurement guidelines and analysis, Harmonoise deliverable: HAR12TR-031203-AEA12.doc (2004).