A BASIC STUDY ON ACOUSTIC PROPERTIES OF DOUBLE-LEAF MEMBRANES

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1. INTRODUCTION

Membrane-type absorbers are normally formed by suspending a single membrane in parallel with a massive back wall. This structure forms an air cavity in the gap. Because the membrane itself has little absorptivity [1], absorption power in the cavity plays an important role in these absorbers [2-4]. A back wall is necessary to form the cavity that makes these membranes absorptive. It is generally assumed that this wall must be as massive as a concrete wall. However, it is not clear how much the back wall can be lightened and still achieve conventional mass-spring resonant absorption. It should be possible to obtain equivalent absorption characteristics with a light-weight double-leaf membrane by suitably adjusting the parameters of the structure. Furthermore, a double-membrane design could be used to improve the acoustics in buildings which use a membrane structure, e.g. in domed roofs, etc. However, no studies have yet been made on the acoustic properties of double-leaf membranes used in membrane-structure buildings. The acoustic performance of double-leaf membranes should, therefore, be clarified. In this paper, the acoustic properties of double-leaf membranes are analysed. A theory is proposed and validated experimentally. A parametric study is carried out to clarify the acoustic behaviour of double-leaf membranes.

2. ANALYSIS

Consider the double-leaf membrane of infinite extent in Fig 1. It consists of two leaves with an air cavity of depth \( d \) between them. The first leaf (leaf 1) is characterised by its surface density \( m_1 \), tension \( T_1 \) and
surface-specific acoustic admittances $A_1$ (front) and $A_2$ (back). For the second leaf (leaf 2) these parameters are labelled $m_2$, $T_2$, $A_3$ (front) and $A_4$ (back). Leaf 1 is struck by a plane wave of unit pressure amplitude with an angle $\theta$ to its normal.

Solving the coupled motion of the sound field and membrane vibrations gives the displacement of each leaf, $w_1$ for leaf 1 and $w_2$ for leaf 2, which are substituted in to the Helmholtz integral to obtain the reflected and transmitted waves, $p_1$ and $p_2$:

\[
p_1(x,z) = \frac{\cos \theta - A_1}{\cos \theta + A_1} + \frac{i \rho_0 \omega^2 \cos \theta_1 (k_0 \sin \theta)}{\cos \theta + A_1} \exp[i k_0 (x \sin \theta - z \cos \theta)],
\]

\[
p_2(x,z) = -i \rho_0 \omega^2 \Gamma_1 (k_0 \sin \theta) \exp[i k_0 (x \sin \theta + z \cos \theta)],
\]

where the acoustic wavenumber $k_0 = \omega / c_0$ ($\omega$ is angular frequency, $c_0$ is sound speed), $\rho_0$ is air density, and

\[
\Gamma_1 (k) = -\left[2 i L U_1 (k) / \rho_0 \omega^2 \right] \Delta_1 (k), \quad \Gamma_2 (k) = -\left[2 i L U_1 (k) / \rho_0 \omega^2 \right] (B + A_4) \cos \theta U_2 (k);
\]

\[
\Delta_1 (k) = \left( k_0 L / 2 m \rho_0 \omega^2 \right) (B + A_1) - \left[ L - (B + A_4) M \right] U_1 (k),
\]

\[
\Delta_2 (k) = \left( k_0 L / 2 m \rho_0 \omega^2 \right) (B + A_4) - \left[ L - (B + A_4) M \right] U_2 (k),
\]

\[
L = -\frac{1}{2} \left[ (\cos \theta - A_2) \exp[i k_0 d \cos \theta] - (\cos \theta + A_2) \exp[i k_0 d \cos \theta] \right],
\]

\[
M = -\frac{1}{2} \left[ (\cos \theta - A_3) \exp[i k_0 d \cos \theta] + (\cos \theta + A_3) \exp[i k_0 d \cos \theta] \right],
\]

\[
N = \frac{1}{2} \left[ (\cos \theta - A_4) \exp[i k_0 d \cos \theta] + (\cos \theta + A_4) \exp[i k_0 d \cos \theta] \right],
\]

\[
\Psi = D_1 (k) D_2 (k) - (B + A_4) (B + A_4) \cos^2 \theta U_1 (k) U_2 (k), \quad B = \sqrt{1 - \left( k / k_0 \right)^2},
\]

with $U_{1,2}(k) = \left[ 2 \eta T_{1,2} / [\rho_0 \omega^2] \right]$ the transformed unit responses of the membranes.

The absorption and transmission coefficients are obtained as $1 - |p_1|^2$ and $|p_2|^2$, respectively. Field-incidence averaged coefficients (averaged over $0^\circ \leq \theta \leq 78^\circ$), typical of those in a diffuse sound field, are used in the following examples.

### 3. NUMERICAL EXAMPLES AND DISCUSSION

**Experimental validation**

To validate the theory, four types of double-leaf membranes (described in Table 1) which were made up from typical building materials. Their absorption and transmission coefficients, $\alpha$ and $\tau$, were measured. The double-leaf membrane was installed in an aperture which connects two reverberation chambers to simulate the conditions in the model analysis; the model assumes a double-leaf membrane of infinite extent which divides all space into two equivalent half spaces, therefore the experimental sound field must also be evenly divided.

**Table 1** The four types of double-leaf membrane used in the experiment. All types are $2.41 \times 2.91 \text{m}^2$.

<table>
<thead>
<tr>
<th>No.</th>
<th>Leaf 1</th>
<th>Leaf 2</th>
<th>Cavity depth [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.495</td>
<td>2.100</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.495</td>
<td>3.300</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>0.495</td>
<td>2.100</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>0.995</td>
<td>3.300</td>
<td>0.05</td>
</tr>
</tbody>
</table>

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Figure 2 shows a comparison of the measured results with field-incidence averaged theoretical predictions. The measured results are in fairly good agreement with the theoretical values: Both experimental and theoretical results show the significant peaks in $\alpha$ and $\alpha - \tau$ that are characteristic of single-leaf membrane-type absorbers with rigid back walls. This implies that double-leaf membranes can be used as sound absorbing elements.

**Parametric study**

The primary question in design practice is how massive must the second leaf (leaf 2) be in order to give the double-leaf membrane enough absorptivity at the mass-spring resonance peak. Figure 3 shows the effect of $m_2$ on $\alpha - \tau$, the peak value increases and the peak frequency decreases as $m_2$ increases, though no peak appears if $m_2$ is too small, e.g. 0.25 kg/m$^2$, which considered alone might suggest that a massive membrane must be used for the second leaf. But significant peaks appear if $m_2$ approaches a certain value, in this example, 4 kg/m$^2$, as seen in Fig 4 which more clearly indicates the peak value and frequency plateaux as $m_2$ increases. This indicates that $m_2$ should have some minimum value, but above that extra mass does not significantly raise absorptivity.

Figure 5 shows the effect on $\alpha - \tau$ of $m_1$, which is also of critical interest in design practice. Increasing $m_1$ makes leaf 1 more difficult to vibrate, resulting in low absorptivity. But the relationship between $m_1$ and the peak value is not simple; it cannot be said that the more $m_1$ increases, the more $\alpha - \tau$ decreases. Instead, a maximum peak value is found implying the optimum value for $m_1$ exists, in this case $m_1=0.5$ kg/m$^2$. This phenomenon was also identified in a membrane-type absorber with a rigid back wall in [4], and is explained by the behaviour of system impedance at the resonance frequency. Since $m_1$ is a major

![Fig 2 Comparison of the theoretical (dots) and the experimental (solid lines) results for the double-leaf membrane No 1 in Table 1: (a) $\alpha$, (b) $\tau$, and (c) $\alpha - \tau$.](image)

![Fig 3 Effect of the surface density of leaf 2, $m_2$, on $\alpha - \tau$ of double-leaf membranes. $A_1=A_2=0.026$, $m_1=2.0$ kg/m$^2$, $d=0.05$ m.](image)
The determinant of the frequency at which the peak appears and the peak value depends on the frequency, the maximum peak value will be determined by the optimum \( m_1 \) once \( d \) has been set. Thus, \( m_1 \) should be chosen considering desired peak frequency and value.

A similar rising and falling tendency is seen when \( d \) is varied, as is shown in Fig 6. The peak value increases as \( d \) increases up to 0.025m, but decreases as \( d \) increases further.

The acoustic admittances of each surface of both leaves have different effects. As \( A_2 \) and \( A_3 \), admittances of the surfaces facing the air cavity, increase, \( \alpha-\tau \) also increases (at the frequencies around the peak.) As admittance \( A_1 \), that of the front surface of leaf 1 increases, \( \alpha-\tau \) also increases (at all frequencies). The admittance of the back surface of leaf 2, \( A_2 \), has little effect on \( \alpha-\tau \).

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References
This study concentrates on mathematical model of the sound absorption behavior of the open cell polyurethane foams which are commonly used by different branches of industry as a sound attenuating materials.

LIST OF SYMBOLS

- \( A \): area normal to the sound wave in m
- \( a \): mean equivalent cell size in m
- \( C(\omega) \): frequency dependent compressibility of the air in the foam
- \( c_0 \): velocity of the sound in the free air at the normal conditions
- \( d \): average thickness of solid rib in m
- \( e \): depth of the material (thickness) in m
- \( f \): frequency in Hz
- \( H \): porosity of the foam = cell volume fraction
- \( i \): \((-1)^{25}
- \( K(\omega) \): frequency dependent dynamic bulk modulus of the air
- \( k_s \): structure factor
- \( N \): total number of cells
- \( N_c \): number of cells in thickness of the foam e
- \( N_{eq} \): Prandtl number for the air = 0.71
- \( R_a \): radius of equivalent cells cylindrical part without ribs = 0.5a
- \( R_d \): radius of equivalent cells cylindrical part with ribs = 0.5(a -d)
- \( U \): air velocity averaged over a unit of cross-sectional area of the foam (at the surface but outside the medium) in m/s
- \( v \): complex average macroscopic fluid velocity perpendicular to the surface A inside the foam in m/s
- \( Z \): acoustic impedance