

A BASIC STUDY ON ACOUSTIC PROPERTIES OF DOUBLE-LEAF MEMBRANES

K Sakagami (1), M Kiyama (2), M Tanigawa (1) & M Morimoto (1)

(1) Environmental Acoustics Lab, Faculty of Engineering, Kobe University, Japan, (2) Graduate School of Science & Technology, Kobe University, Japan

1. INTRODUCTION

Membrane-type absorbers are normally formed by suspending a single membrane in parallel with a massive back wall. This structure forms an air cavity in the gap. Because the membrane itself has little absorptivity [1], absorption power in the cavity plays an important role in these absorbers [2-4]. A back wall is necessary to form the cavity that makes these membranes absorptive. It is generally assumed that this wall must be as massive as a concrete wall. However, it is not clear how much the back wall can be lightened and still achieve conventional mass-spring resonant absorption. It should be possible to obtain equivalent absorption characteristics with a light-weight double-leaf membrane by suitably adjusting the parameters of the structure. Furthermore, a double-membrane design could be used to improve the acoustics in buildings which use a membrane structure, e.g. in domed roofs, etc. However, no studies have yet been made on the acoustic properties of double-leaf membranes used in membrane-structure buildings. The acoustic performance of double-leaf membranes should, therefore, be clarified. In this paper, the acoustic properties of double-leaf membranes are analysed. A theory is proposed and validated experimentally. A parametric study is carried out to clarify the acoustic behaviour of double-leaf membranes.

2. ANALYSIS

Consider the double-leaf membrane of infinite extent in Fig 1. It consists of two leaves with an air cavity of depth d between them. The first leaf (leaf 1) is characterised by its surface density m_1 , tension T_1 and

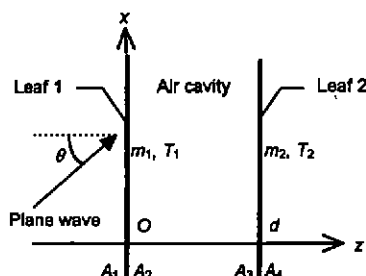


Fig 1 Geometry of a double-leaf membrane of infinite extent: surface density and tension of each membrane are m_1 and T_1 . $A_1 \dots A_4$ are specific acoustic admittance of surfaces.

surface-specific acoustic admittances A_1 (front) and A_2 (back). For the second leaf (leaf 2) those parameters are labelled m_2 , T_2 , A_3 (front) and A_4 (back). Leaf 1 is struck by a plane wave of unit pressure amplitude with an angle θ to its normal.

Solving the coupled motion of the sound field and membrane vibrations gives the displacement of each leaf, w_1 for leaf 1 and w_2 for leaf 2, which are substituted in to the Helmholtz integral to obtain the reflected and transmitted waves, p_r and p_t :

$$p_r(x, z) = \left[\frac{\cos \theta - A_1}{\cos \theta + A_1} + \frac{i \rho_0 \omega^2 \cos \theta \Gamma_1(k_0 \sin \theta)}{\cos \theta + A_1} \right] \exp[ik_0(x \sin \theta - z \cos \theta)], \quad (1)$$

$$p_t(x, z) = -i \rho_0 \omega^2 \Gamma_2(k_0 \sin \theta) \exp[ik_0(x \sin \theta + z \cos \theta)], \quad (2)$$

where the acoustic wavenumber $k_0 = \omega/c_0$ (ω is angular frequency, c_0 is sound speed), ρ_0 is air density, and

$$\Gamma_1(k) = -2iLU_1(k)/\rho_0\omega^2\Psi, \quad \Gamma_2(k) = -2iLU_1(k)/\rho_0\omega^2\Psi(B+A_4)\cos\theta U_2(k);$$

$$D_1(k) = (k_0 L/2\pi\rho_0\omega^2)(B+A_1) - [L - (B+A_1)M]U_1(k),$$

$$D_2(k) = (k_0 L/2\pi\rho_0\omega^2)(B+A_4) - [L - (B+A_4)M]U_2(k),$$

$$L = -\frac{1}{2}[(\cos \theta - A_2)(\cos \theta - A_3) \exp[ik_0 d \cos \theta] - (\cos \theta + A_2)(\cos \theta + A_3) \exp[ik_0 d \cos \theta]],$$

$$M = -\frac{1}{2}[(\cos \theta - A_3) \exp[ik_0 d \cos \theta] + (\cos \theta + A_3) \exp[ik_0 d \cos \theta]],$$

$$N = \frac{1}{2}[(\cos \theta - A_2) \exp[ik_0 d \cos \theta] + (\cos \theta + A_2) \exp[ik_0 d \cos \theta]],$$

$$\Psi = D_1(k)D_2(k) - (B+A_1)(B+A_4)\cos^2\theta U_1(k)U_2(k), \quad B = \sqrt{1 - (k/k_0)^2},$$

with $U_{1,2}(k) = [2\pi(T_1^2 - m_1^2)]^{-1}$ the transformed unit responses of the membranes.

The absorption and the transmission coefficients are obtained as $1 - |p_r|^2$ and $|p_t|^2$, respectively. Field-incidence averaged coefficients (averaged over $0^\circ \leq \theta \leq 75^\circ$), typical of those in a diffuse sound field, are used in the following examples.

3. NUMERICAL EXAMPLES AND DISCUSSION

Experimental validation

To validate the theory, four types of double-leaf membranes (described in Table 1) which were made up from typical building materials. Their absorption and transmission coefficients, α and τ , were measured. The double-leaf membrane was installed in an aperture which connects two reverberation chambers to simulate the conditions in the model analysis; the model assumes a double-leaf membrane of infinite extent which divides all space into two equivalent half spaces, therefore the experimental sound field must also be evenly divided.

Table 1 The four types of double-leaf membrane used in the experiment. All types are 2.41×2.91 [m²].

| No. | Surface density [kg/m ²] | | Cavity depth [m] |
|-----|--------------------------------------|--------|------------------|
| | Leaf 1 | Leaf 2 | |
| 1 | 0.495 | 2.100 | 0.05 |
| 2 | 0.995 | 3.300 | 0.05 |
| 3 | 0.495 | 2.100 | 0.05 |
| 4 | 0.995 | 3.300 | 0.05 |

Figure 2 shows a comparison of the measured results with field-incidence averaged theoretical predictions. The measured results are in fairly good agreement with the theoretical values: Both experimental and theoretical results show the significant peaks in α and $\alpha-\tau$ that are characteristic of single-leaf membrane-type absorbers with rigid back walls. This implies that double-leaf membranes can be used as sound absorbing elements.

Parametric study

The primary question in design practice is how massive must the second leaf (leaf 2) be in order to give the double-leaf membrane enough absorptivity at the mass-spring resonance peak. Figure 3 shows the effect of m_2 on $\alpha-\tau$: the peak value increases and the peak frequency decreases as m_2 increases, though no peak appears if m_2 is too small, e.g. 0.25 kg/m^2 , which considered alone might suggest that a massive membrane must be used for the second leaf. But significant peaks appear if m_2 approaches a certain value, in this example, 4 kg/m^2 , as seen in Fig 4 which more clearly indicates the peak value and frequency plateaux as m_2 increases. This indicates that m_2 should have some minimum value, but above that extra mass does not significantly raise absorptivity.

Figure 5 shows the effect on $\alpha-\tau$ of m_1 , which is also of critical interest in design practice. Increasing m_1 makes leaf 1 more difficult to vibrate, resulting in low absorptivity. But the relationship between m_1 and the peak value is not simple; it cannot be said that the more m_1 increases, the more $\alpha-\tau$ decreases. Instead, a maximum peak value is found implying the optimum value for m_1 exists, in this case $m_1 = 0.5 \text{ kg/m}^2$. This phenomenon was also identified in a membrane-type absorber with a rigid back wall in [4], and is explained by the behaviour of system impedance at the resonance frequency. Since m_1 is a major

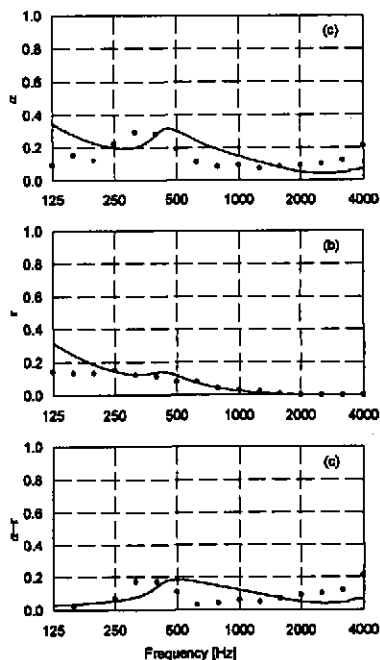


Fig 2 Comparison of the theoretical (dots) and the experimental (solid lines) results for the double-leaf membrane No 1 in Table 1: (a) α , (b) τ , and (c) $\alpha-\tau$.

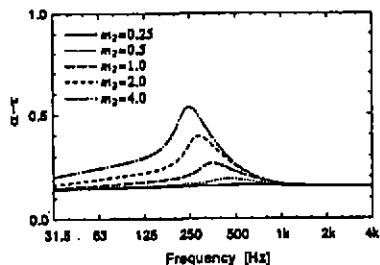


Fig 3 Effect of the surface density of leaf 2, m_2 , on $\alpha-\tau$ of double-leaf membranes. $A_1 \dots A_4 = 0.028$, $m_1 = 2.0 \text{ kg/m}^2$, $d = 0.05 \text{ m}$.

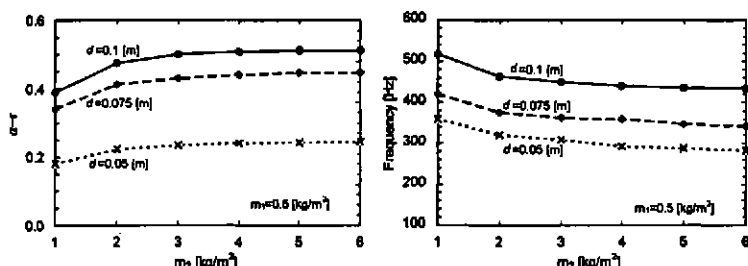


Fig 4 Variation of value (left) and frequency (right) of the peak in $\alpha\tau$ due to changes in the surface density of leaf 2 of double-leaf membranes of various cavity depth.

determinant of the frequency at which the peak appears and the peak value depends on the frequency, the maximum peak value will be determined by the optimum m_1 once d has been set. Thus, m_1 should be chosen considering desired peak frequency and value.

A similar rising and falling tendency is seen when d is varied, as is shown in Fig 6. The peak value increases as d increases up to 0.025m, but decreases as d increases further.

The acoustic admittances of each surface of both leaves have different effects. As A_2 and A_3 , admittances of the surfaces facing the air cavity, increase, $\alpha\tau$ also increases (at the frequencies around the peak.) As admittance A_1 , that of the front surface of leaf 1 increases, $\alpha\tau$ also increases (at all frequencies). The admittance of the back surface of leaf 2, A_4 , has little effect on $\alpha\tau$.

Acknowledgements

The authors wish to thank Dr Daiji Takahashi at Fukui University for his valuable comments. Thanks are also due to Nobuhiro Ohira for his assistance in performing the experiment.

References

- [1] K Sakagami, M Morimoto & D Takahashi, *Acustica* 80, 569 (1994).
- [2] K Sakagami, D Takahashi, H Gen & M Morimoto, *Acustica* 78, 288 (1993).
- [3] K Sakagami, H Gen, M Morimoto & D Takahashi, *Acustica/Acta Acustica* 82, 45 (1996).
- [4] K Sakagami, M Kiyama, M Morimoto & D Takahashi, *Appl. Acoust.* (in press).

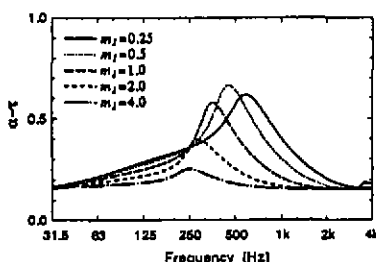


Fig 5 Effect of the surface density of leaf 1, m_1 , on $\alpha\tau$ of double-leaf membranes. $A_1 \dots A_4 = 0.026$, $m_2 = 2.0 \text{ kg/m}^2$, $d = 0.05 \text{ m}$.

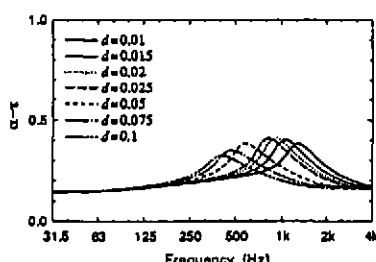


Fig 6 Effect of the cavity depth, d , on $\alpha\tau$ of double-leaf membranes. $A_1 \dots A_4 = 0.026$, $m_1 = 2.0 \text{ kg/m}^2$, $d = 0.05 \text{ m}$.

MATHEMATICAL MODEL OF THE ACOUSTICAL ABSORPTION BEHAVIOR OF THE OPEN CELL POLYURETHANE FOAMS AND COMPARISON WITH EXPERIMENTAL RESULTS

M L Szary (1) & O O Nwankwo (2)

(1) College of Engineering, Southern Illinois University, Carbondale, Illinois 62901, USA, (2)
United Technologies Automotive, 1641 Porter St, Detroit, Michigan, 48216-1984, USA

This study concentrates on mathematical model of the sound absorption behavior of the open cell polyurethane foams which are commonly used by different branches of industry as a sound attenuating materials.

LIST OF SYMBOLS

| | |
|-------------|--|
| A | area normal to the sound wave in m^2 |
| a | mean equivalent cell size in m |
| $C(\omega)$ | frequency dependent compressibility of the air in the foam |
| c_o | velocity of the sound in the free air at the normal conditions |
| d | average thickness of solid rib in m |
| e | depth of the material (thickness) in m |
| f | frequency in Hz |
| H | porosity of the foam = cell volume fraction |
| i | $(-1)^{0.5}$ |
| $K(\omega)$ | frequency dependent dynamic bulk modulus of the air |
| k_s | structure factor |
| N | total number of cells |
| N_s | number of cells in thickness of the foam e |
| N_{Pr} | Prandtl number for the air = 0.71 |
| R_s | radius of equivalent cells cylindrical part without ribs = $0.5a$ |
| R_d | radius of equivalent cells cylindrical part with ribs = $0.5(a-d)$ |
| U | air velocity averaged over a unit of cross-sectional area of the foam (at the surface but outside the medium) in ms^{-1} |
| v | complex average macroscopic fluid velocity perpendicular to the surface A inside the foam in ms^{-1} |
| Z | acoustic impedance |