A SIMPLE MODEL OF CABINET EDGE DIFFRACTION

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1 INTRODUCTION

A knowledge of the sound field radiated by a source mounted on, or close to a surface of finite extent is of interest in a variety of situations, for example, the estimation of the output of a loudspeaker driver mounted in a cabinet. Particular details of the source itself apart, the problem reduces to one of diffraction from the edges of the mounting surface or baffle. Traditionally, diffraction problems are dealt with using the Geometric Theory of Diffraction (GTD) [1], which gives a reliable estimate of the diffraction from a sharp wedge of infinite length in the limit of high frequencies, or large distances from the wedge. The GTD approach is unsuitable however, for mid- to low-frequency problems, such as may be encountered in loudspeaker baffles, where the diffracting edges may be a few wavelengths long or smaller. More recently, powerful, but computationally-expensive, numerical methods such as the Boundary Element Method (BEM) have been applied successfully to diffraction problems of this type [2].

A study of the literature on the subject of edge diffraction [2] reveals that the complexity of the analysis overshadows the apparent physical simplicity of the problem, and that a simpler approach may be more relevant for many practical diffraction problems (indeed it is shown in [2] that this very complexity has given rise to a deal of unnecessary confusion over the interpretation of the analysis). Attempts to simplify the use of the GTD, by dividing the edges into finite-length elements [3] have gone some way toward a simpler solution, but this method is still not really suitable for low-frequency problems due to its reliance on the GTD. An alternative formulation, proposed by Svensson [4], is based on the Biot - Tolstoy exact formulation [5] and, as with [3], permits the division of the edges into finite-length elements along the lines of Medwin [6]. This method does not suffer the high-frequency limitations of the GTD-based methods and probably represents the current state-of-the-art in edge diffraction prediction models; it has more recently been successfully applied to room acoustics simulations [7]. Although very general and accurate, the Svensson method is still very complex and hides, to some extent, the apparent physical simplicity of the problem, particularly for the finite-baffled source problem described above.

This paper introduces a simple method for tackling diffraction problems when the source is mounted on a baffle of finite extent. It begins by considering the simple case of a compact source mounted on a small baffle radiating very low frequencies. A number of facts, based on observations about this system, are then used to build the more general model. The resulting model can be used to yield good estimates of the diffracted sound field at low to high frequencies for sources on baffles of arbitrary shape and size.

2 THE STRENGTH OF THE TOTAL DIFFRACTED WAVE FIELD

Consider first, a point monopole source mounted on one side of an infinite, plane baffle. The sound waves emitted by the source at position $\boldsymbol{i}$ propagate in a hemispherical manner and the sound pressure at any field point $\boldsymbol{x}$ away from the source is given by

$$p_b(\boldsymbol{x}, k) = G_h(\boldsymbol{x} \mid \boldsymbol{i}, k)\; q \;, \tag{1}$$

where G_h is the half-space free-field Green function given by

$$G_h(\boldsymbol{x} \mid \boldsymbol{i}, k) = \frac{-j\rho_0 c_0 k}{2\pi} \frac{e^{-jk|\boldsymbol{x} - \boldsymbol{i}|}}{|\boldsymbol{x} - \boldsymbol{i}|} \;, \tag{2}$$

where ρ_0 is the density of air, c_0 is the sound speed, q is the volume velocity of the source and $k = 2\pi f / c_0$ is the wavenumber where f is the frequency. If the same monopole is placed on a baffle of finite extent, any change in the sound field may be attributed entirely to the presence of the diffraction caused by the edges of the finite baffle, thus:

$$p(\boldsymbol{x}, k) = p_b(\boldsymbol{x}, k) + p_{diff}(\boldsymbol{x}, k) \;, \tag{3}$$

where p is the sound field radiated by the source on the finite baffle and p_{diff} is the diffracted wave field. In the limit of low frequencies where the wavelength is large compared to the dimensions of the finite baffle, the sound field is the same as that of a monopole in free-space:

$$p(\boldsymbol{x}, k) \approx G(\boldsymbol{x} \mid \boldsymbol{i}, k)\; q \;, \tag{4}$$

where G is the full-space free-field Green function given by

$$G(\boldsymbol{x} \mid \boldsymbol{i}, k) = \frac{-j\rho_0 c_0 k}{4\pi} \frac{e^{-jk|\boldsymbol{x} - \boldsymbol{i}|}}{|\boldsymbol{x} - \boldsymbol{i}|} \;. \tag{5}$$

Comparing Equations (1) and (4), the pressure field radiated by a source on a small baffle is half of the pressure field radiated by the same source when mounted on the infinite baffle. Thus it follows from Equation (3) that, at low frequencies at least, the total diffracted wave field, which is equal to the integral of the contribution to the diffracted wave field along the entire length of the edge of the finite baffle, is equal to minus one half of the field radiated by the source on the infinite baffle, thus:

$$\int_0^{2\pi} p_d(\boldsymbol{x}, k)\, d\theta = \frac{-p_b(\boldsymbol{x}, k)}{2} \;, \tag{6}$$

where p_d is the contribution of the diffracted wave field per unit angle subtended from the source in the plane of the baffle.

At low frequencies, the relationships in Equations (1), (3) and (6) are independent of the physical size or shape of the finite baffle so p_d can therefore be assumed to be independent of the position along the baffle edge and can be taken out of the integral in equation (6). The value of p_d can then be evaluated:

$$p_d(\boldsymbol{x}, k) = \frac{-p_b(\boldsymbol{x}, k)}{4\pi} \text{ per unit angle.} \tag{7}$$

3 THE FREQUENCY DEPENDENCE OF THE DIFFRACTED WAVE FIELD

Consider a plane wave incident upon an infinite-length, infinitely-sharp edge. The strength of the diffracted wave field must be independent of the wavelength of the incident plane wave, as the shape of the edge remains unchanged with scale. This being the case, the above argument for low frequencies may be extended to higher frequencies as any frequency dependence of the total diffracted wave (and indeed, the shadow region in high-frequency ray acoustics) must be entirely due to geometry, that is, the interference between the diffracted wave contributions from the different parts of the edge. In the absence of the diffracted field, the relative phase of p_b at a position on the edge depends upon the distance from the source to that point. It is reasonable therefore to assume that the phase of p_d at a point on the baffle edge is also dependent upon the distance from the source as the wave originates from the source and propagates to the edge from where it is diffracted. Defining p_d per unit angle in Equation (7) rather than per unit length of edge, takes care of the reduction in wave amplitude due to spherical spreading. The diffracted wave contribution from any arbitrary shape of baffle and for any frequency may therefore be found by dividing the baffle edge into a number of finite-length edge elements, the values of p_d for which may be determined by considering the distance from the source to the element and the angle subtended by the element at the source (see Figure 1).

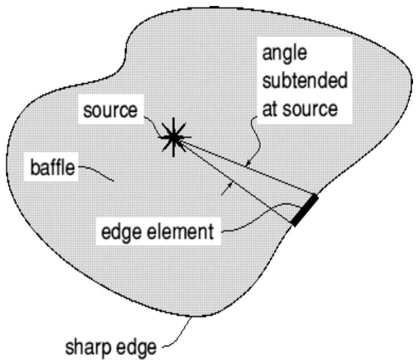

Figure 1 An element of the baffle edge subtends an angle at the source. This angle determines the strength of the contribution of that element to the diffracted wave field, and the distance from the source to the element determines the phase

4 THE POLARITY OF THE DIFFRACTED WAVE FIELD

Some confusion is apparent in the literature about the relative polarity (phase) of the direct and diffracted wave components. The confusion arises from considering the sound field as divided into two regions: the direct region, where the receiver point can be 'seen' by the source, and the shadow region, where it cannot. The direct field is then assumed not to exist in the shadow region, but both the direct and diffracted fields interfere in the direct region. The confusion stems from the fact that the diffracted wave needs to have the same polarity as the direct field in the shadow region but needs to have reverse polarity in the direct region to destructively interfere with the direct field. This situation leads to a discontinuity in the diffracted wave field at the plane joining the two regions which is taken care of by the total absence of a direct field component on the shadow side of the plane.

This representation of two discontinuous sound field components comes from the high-frequency approximation that sound waves behave as rays, and cannot, therefore, turn corners; the diffracted component therefore becomes necessary to ensure continuity of the total sound field. A more intuitive description of the sound field follows from the low-frequency consideration above. In this case, the direct field always propagates around an edge but in doing so encounters an increase in solid angle into which it is propagating. This gives rise to a local reduction in pressure which propagates away from the edge with negative polarity — the diffracted wave. This diffracted wave is exactly in phase opposition to the direct wave at the edge and thus, everywhere in the shadow region it partially cancels the direct wave (which also now emanates from the edge), and everywhere in the direct region it interferes with the direct wave (which emanates from the source) as shown in figure 2.

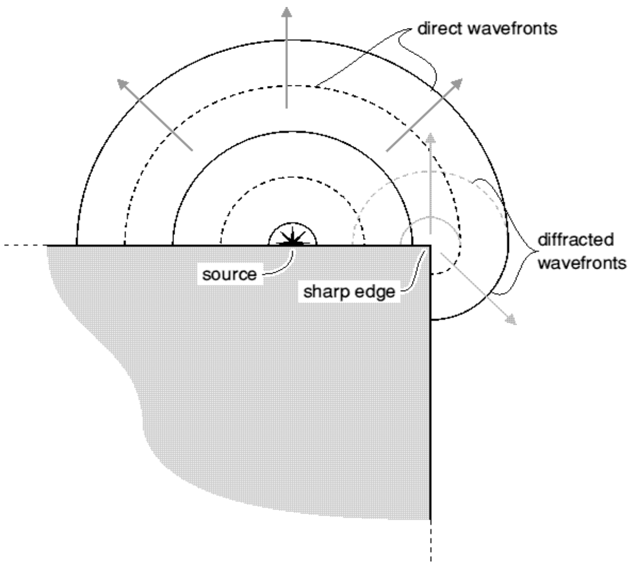

Figure 2 The negative polarity diffracted wave emanates from the same position as the direct field in the 'shadow zone', but spatially-separated from the direct field in the 'direct zone'.

In the direct region, there will exist path-length differences between the direct wave and the diffracted wave giving a complex interference field, but within the shadow region no such path-length differences exist and the two waves will be in exact phase opposition everywhere. As both the direct and diffracted waves exist continuously everywhere, there is no need for any sudden transition into the shadow region; instead the edge of the shadow region is marked only by the beginning of exact phase opposition between the two wave components.

5 IMPLEMENTATION OF THE SIMPLE DIFFRACTION MODEL

The simple diffraction model may be used to simulate the diffraction due to any arbitrary-shaped, finite-sized, plane baffle with a sharp right-angled edge. The edge of the baffle is first divided into elements having lengths that are small compared to the shortest wavelength of interest. For each element, the angle subtended from the source is calculated from the positions of the two ends of the elements along with the distance from the source to the centre of the element. Using Equation (7), for each frequency of interest the total diffracted sound field radiated to a field point $\boldsymbol{x}$ is estimated by summing the contributions of all of the elements at that point, thus:

$$p_{diff}(\boldsymbol{x}, k) = \frac{-q}{4\pi} \sum_{e} \theta_e G(\boldsymbol{x} \mid \boldsymbol{e} \mid \boldsymbol{i}, k) \quad , \qquad (8)$$

where θ_e is the angle subtended at the source at $\boldsymbol{i}$ by edge element e at position $\boldsymbol{e}$ and

$$G(\boldsymbol{x} \mid \boldsymbol{e} \mid \boldsymbol{i}, k) = \frac{-j\rho_0 c_0 k}{2\pi} \frac{e^{-jk(|\boldsymbol{e}-\boldsymbol{i}| + |\boldsymbol{x}-\boldsymbol{e}|)}}{(|\boldsymbol{e}-\boldsymbol{i}| + 2|\boldsymbol{x}-\boldsymbol{e}|)} ,$$

is evaluated over the distance from $\boldsymbol{i}$ to the field point $\boldsymbol{x}$ via the element at $\boldsymbol{e}$. It should be noted that, as the edge of the baffle surrounds the source,

$$\sum_i \theta_i = 2\pi \tag{9}$$

so that in the low-frequency limit, $p_{diff}(\boldsymbol{x}, k) = -p_b(\boldsymbol{x}, k)/2$. The total pressure when $\boldsymbol{x}$ is in the direct zone is therefore given by Equation (3) above.

When $\boldsymbol{x}$ is in the shadow zone, the sides of the baffle shield some of the edge elements from $\boldsymbol{x}$, and secondary or tertiary edge diffraction would need to be taken into account to estimate the entire field. However, in most cases, these contributions are likely to be small, in which case, remembering that the direct and diffracted waves are in phase opposition everywhere in this region, the total pressure may be approximated as

$$p(\boldsymbol{x}, k) \approx -\tilde{p}_{diff}(\boldsymbol{x}, k) \;, \tag{10}$$

where $\tilde{p}_{diff}$ refers to only those edge elements that are within 'line-of-sight' of $\boldsymbol{x}$.

6 DISTRIBUTED AND PISTON SOURCES

The simple diffraction model is based on a single point monopole source mounted on an infinite baffle with a correction to the radiated pressure field due to the edges of the finite baffle. Under infinite plane baffle conditions, the radiation from complex, distributed sources can be modelled using a discrete form of the Rayleigh integral

$$p_b(\boldsymbol{x}, k) = \sum_i q_i\, G_h(\boldsymbol{x} \mid \boldsymbol{i}, k) \;, \tag{11}$$

where the source region is divided into a number of small elemental sources i having volume velocities q_i, and the individual contributions from each source element are summed at the field point of interest $\boldsymbol{x}$. For sources mounted on finite baffles the Green functions G are not known in general so Equation (11) cannot easily be applied. However, the diffracted wave fields due to each source element may also be summed and added to Equation (11) to yield the total sound field

$$p(\boldsymbol{x}, k) = \sum_i q_i \left(G_h(\boldsymbol{x} \mid \boldsymbol{i}, k) + \frac{1}{4\pi} \sum_e \theta_e\, G(\boldsymbol{x} \mid \boldsymbol{e} \mid \boldsymbol{i}, k) \right) . \tag{12}$$

For source regions with regular geometry it may be possible to simplify Equation (12) by exploiting analytical expressions for the directivity of the source. For example, the far-field radiation of a rigid, circular piston in an otherwise infinite baffle (read simple loudspeaker model) can be written

$$p_{ff}(\boldsymbol{x}, k) = q\, G_h(\boldsymbol{x} \mid \boldsymbol{i}, k)\, D(\phi, k, a) \;, \tag{13}$$

where D is the circular piston directivity function given by

$$D(\phi, k, a) = \frac{2\, \mathbf{J}_1(ka \sin(\phi))}{ka \sin(\phi)} \;, \tag{14}$$

where $\mathbf{J}_1$ is a Bessel function, a is the radius of the piston and θ is the angle of point $\boldsymbol{x}$ from the piston axis. Evaluating Equation (14) for $\theta = \pi/2$ gives an approximate scaling factor for the sound field radiated along the baffle towards the edge and hence for the entire diffracted field. If the baffle edge and point $\boldsymbol{x}$ are sufficiently far from the piston, the diffracted field may be approximated as that for a single monopole source at the piston centre but scaled by the piston directivity function at 90°,

$$p_{ff}(\boldsymbol{x}, k) \approx q \left(G_h(\boldsymbol{x} \mid \boldsymbol{i}, k)\, D(\phi, k, a) + \frac{D(\pi/2, k, a)}{4\pi} \sum_e \theta_e\, G(\boldsymbol{x} \mid \boldsymbol{e} \mid \boldsymbol{i}, k) \right) . \tag{15}$$

7 RADIATED SOUND POWER ESTIMATES

In addition to estimations of the radiated sound field, the simple diffraction model can be used to estimate the radiation impedance and hence the power radiated by a source on a finite-sized baffle. For example; measurements of the vibrational velocity of parts of a finite-sized surface could be combined with estimates of radiation impedance from the diffraction model to yield the contribution of the individual vibrating parts to the total radiated sound power.

This may be achieved by positioning the field point $\boldsymbol{x}$ at the source point $\boldsymbol{i}$ and using the model to estimate the diffracted pressure field alone. The power output of the source on the finite baffle in terms of the power output of the same source on an infinite baffle is then given by

$$\frac{W_{fb}}{W_b} = \frac{\Re e\left\{Z_r + p_{diff}(\boldsymbol{i}, k) S/q\right\}}{\Re e\left\{Z_r\right\}} , \qquad (16)$$

where Z_r is the radiation impedance of the source on an infinite baffle, which could be that of a piston for example, S is the surface area of that source, $p_{diff}(\boldsymbol{i}, k)$ is the diffracted pressure field evaluated at the source position using Equation (8), and $\Re e\{\}$ refers to the real part. If the wavelength is assumed to be large compared to the source dimensions, the radiation impedance is independent of the source geometry and Equation (16) may be reduced to

$$\frac{W_{fb}}{W_b} \approx 1 + \frac{2\pi \Re e\left\{p_{diff}(\boldsymbol{i}, k)/q\right\}}{\rho_0 c_0 k^2} . \qquad (17)$$

8 DISCUSSION

This paper discusses the concepts behind a simple model of baffle edge diffraction and describes a way of implementing that model. As the model is based on a low-frequency approximation, it should prove particularly useful for assessing the effects of diffraction on the radiation of sound from loudspeakers. At the time of writing, no experimental verification of the model predictions has been carried out, although qualitative comparisons with measurements and predictions found in the literature (not reported here) have shown promise. A thorough set of verification measurements are planned for the near future.

9 REFERENCES

[1] A D Pearce, *Acoustics – An Introduction to its Physical Principles and Applications*, Acoustical Society of America, 1991, ISBN: 0-99318-612-8

[2] J R Wright, “Fundamentals of Diffraction”, *Journal of the Audio Engineering Society*, **45**[5], 347-356, 1997.

[3] J Vanderkooy, “A Simple Theory of Cabinet Edge Diffraction”, *Journal of the Audio Engineering Society*, **39**[12], 923-933, 1991.

[4] U P Svensson, R I Fred and J Vanderkooy, “An Analytic Secondary Source Model of Edge Diffraction Impulse Responses”, *Journal of the Acoustical Society of America*, **106**[5], 2331-2344, 1999.

[5] M A Biot and I Tolstoy, “Formulation of Wave Propagation in Infinite Media by Normal Coordinates with an Application to Diffraction”, *Journal of the Acoustical Society of America*, **29**[3], 381-391, 1957.

[6] H Medwin, “Shadowing by Finite Noise Barriers”, *Journal of the Acoustical Society of America*, **69**[4], 1060-1064, 1981.

[7] U P Svensson and P T Calamia, “The Use of Edge Diffraction in Computational Room Acoustics”, *Journal of the Acoustical Society of America*, **120**, 2006.