NON-MINIMUM PHASE BEHAVIOUR OF LOUDSPEAKERS AT LOW FREQUENCIES

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1 INTRODUCTION

Between the years 1998 and 2015, the first author carried out objective reviews of monitor loudspeakers for *Studio Sound* and *Resolution* magazines. A total of 85 loudspeakers were tested, with each mounted in the same position in the large anechoic chamber at ISVR, and measured using the same equipment. These tests have yielded a valuable resource for investigating the differences or similarities between different loudspeakers, and the purpose of this paper is to look closely at one aspect of the performance of these loudspeakers, specifically, the phase of the on-axis frequency response at low frequencies. In a paper presented at Reproduced Sound 19 in 2003 [1], it was shown that the phase response of many of the loudspeakers tested showed varying degrees of non-minimum-phase behaviour, which affected the response of the loudspeakers to transient signals. The current paper looks at these responses in detail, and attempts to explain the phase differences using computer simulations.

2 MEASUREMENT ENVIRONMENT AND EQUIPMENT

The measurements of the on-axis frequency responses of the loudspeakers were carried out in the large anechoic chamber at ISVR. The chamber has overall dimensions of 9.15m x 9.15m x 7.32m and is fully lined with 91cm-long glass-fibre wedges, giving free-field conditions at all frequencies above 80Hz [2]. The loudspeakers were mounted on a pole, 1.75m from a free-field measurement microphone which was aligned with the principle listening axis of the loudspeaker. The pink noise input and microphone output signals were generated and recorded using a digital acquisition system for subsequent off-line computer post-processing. The 16384-line complex frequency response functions were estimated using Welch's method [3], using 175 process averages with 50% overlap and raised-cosine windowing.

2.1 Monopole Calibration at Low Frequencies

As stated above, the ISVR anechoic chamber can be considered free-field for frequencies above about 80Hz. At lower frequencies, even 91cm wedges do not absorb all of the incident sound and some minor reflection from the rigid walls of the outer chamber does exist. The effect of these reflections on the measurements can be identified as peaks and troughs at low frequencies in the example frequency response shown in Figure 1a. All of the measurements described in Section 1 were carried out with the loudspeakers and microphone in the same place within the chamber, so it is may be reasonable to assume that the effects of the room will be the same for all the loudspeakers. A calibration file was created on the assumption that a compact loudspeaker behaves as a monopole at low frequencies. The theoretical pressure on the surface of a circular piston on a compact sealed cabinet at low frequencies is given by [4]

$$p_s \approx \rho c u \left\{\frac{k^2a^2}{4} + j0.6ka\right\} \qquad (1)$$

where ρ is the density of air, c is the speed of sound, u is the surface velocity of the piston, k is the acoustic wavenumber [4] and a is the radius of the piston. The pressure at a far-field position a distance r from the same piston is given by [4]

$$p_0 = j\rho cku\pi a^2 \exp\{-jkr\}/4\pi r \quad (2)$$

A measurement was made of the transfer function T between the pressure next to the surface of a compact loudspeaker diaphragm, and that at the measurement position, with the loudspeaker and microphone in the same positions in the chamber as for all of the loudspeaker measurements. The monopole calibration for that pair of loudspeaker and microphone positions in the chamber is then given by the ratio of the measured and theoretical transfer functions,

$$Cal = T\frac{p_0}{p_c}. \quad (3)$$

Figure 1b shows the response of the loudspeaker in Figure 1a after application of the calibration curve; a smooth roll-off, consistent with the expected behaviour of a loudspeaker [4] can be seen.

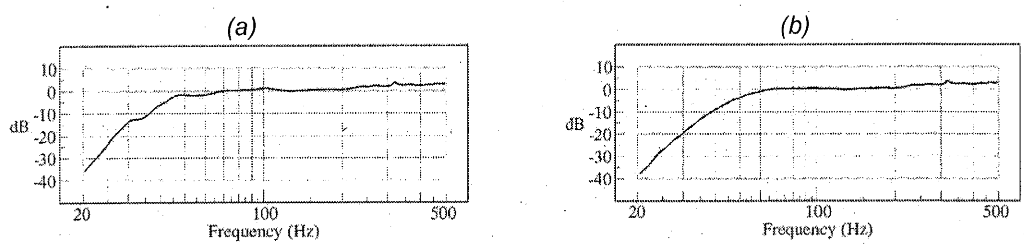

Figure 1 Typical frequency response measured in the anechoic chamber: (a) without calibration, (b) with monopole calibration.

The assumptions inherent in the subsequent use of this calibration to adjust the measured response of a loudspeaker are: 1) that the loudspeaker is compact compared to a wavelength, 2) that the temperature - and hence speed of sound - are the same for the calibration and the measurement, and 3) that the loudspeaker radiates as a monopole. The first assumption holds true for all the loudspeakers tested at the low frequencies of interest (sub 80Hz). The anechoic chamber is very temperature-stable, so the second assumption is reasonable. The third assumption, however, appears to rule out ported loudspeakers as these do not radiate as monopoles at very low frequencies, and at frequencies below the port-tuning frequency are actually closer to dipole radiators. In practice though, the errors are small, and confined to frequencies where the response of the loudspeaker has rolled-off considerably. However, it is possible that application of this calibration to the measurement of ported loudspeakers may be responsible for some, or all, of the observed phase response differences. This question is explored in Section 4.4.

3 PHASE, MINIMUM PHASE AND EXCESS PHASE

A complex frequency response function (frf) is a set of complex numbers, each of which represents the change in amplitude and phase of a single-frequency signal as it passes through the system of interest. Linear-systems theory [5] shows that, for a linear, time-invariant system, knowledge of the frf at all of the frequencies of interest is sufficient to describe the response of the system to any and all signals. The time-domain counterpart of the frf is the impulse response, and transforming from one to the other is lossless via the Fourier Transform.

For each frequency, the frf can be written

$$\hat{H}(f) = \frac{\hat{y}(f)}{\hat{x}(f)} = |H| \exp\{\, j\emptyset(f)\,\} \quad (4)$$

Where $x(f)$ is the input signal, $y(f)$ is the resultant output signal $|H|$ is the amplitude of the frf and ϕ is the phase. A symbol ^ over a quantity indicates that it is complex. The phase part of the frf can be further separated into minimum- and excess-phase components, ϕ_m and ϕ_e, respectively

$$\hat{H}(f) = |H| \exp\{\emptyset_m\} \exp\{\emptyset_e\} \tag{5}$$

the frf can be then be expressed as the product of two responses; a minimum-phase response, having the amplitude of the frf and minimum phase, and an all-pass response having unit amplitude and the excess phase

$$\hat{H}(f) = \hat{H}_m(f)\hat{A}(f) \tag{6}$$

The amplitude of an frf and its minimum phase have a unique relationship in that they are linked via the Hilbert transform [6]. Using this transform, the minimum phase corresponding to the amplitude of an frf can be exactly (re)created from a knowledge of the amplitude only.

This unique relationship between the amplitude of an frf and its corresponding minimum phase has profound implications when considering the equalization of systems. Most conventional filters are minimum phase, and the product of two minimum-phase responses is also minimum phase. It therefore follows that if a system is minimum phase, equalization of the amplitude of its frf to achieve a flat frequency-response will 'automatically' also equalize the phase - as the minimum phase of a system with flat amplitude-response is zero. However, if the frf of the system contains excess phase, then that part of the phase will remain when the system is equalized, and a perfect response will not be achieved. In general, it is difficult to equalize systems having excess phase in real time, as the required filters may need to introduce a significant delay into the signal path in order to reverse the excess phase. It has been proposed in [1, 4, 7] that the presence of excess phase in monitor loudspeaker responses may have a detrimental effect on the quality of music mixes carried out using those monitors. It was particularly noticed that those loudspeakers with bass-reflex ports and high-pass electrical protection-filters showed most evidence of excess phase in their responses.

4 PORTED LOUDSPEAKER RESPONSE SIMULATION

4.1 Conventional Lumped-Parameter (Spring) Model

Lumped-parameter modelling is the most commonplace practice for modelling the low-frequency behaviour of loudspeakers. Indeed, the famous Thiele-Small Parameters [8, 9, 10] are themselves lumped parameters. Figure 2 shows a graphical representation of a lumped-parameter model of a ported loudspeaker. In this model, both the driver suspension and the air within the cabinet are modelled as mechanical springs, characterised entirely by their stiffnesses, and the driver and port moving masses entirely by their mass. The lever mechanism represents the area difference between the driver and the port.

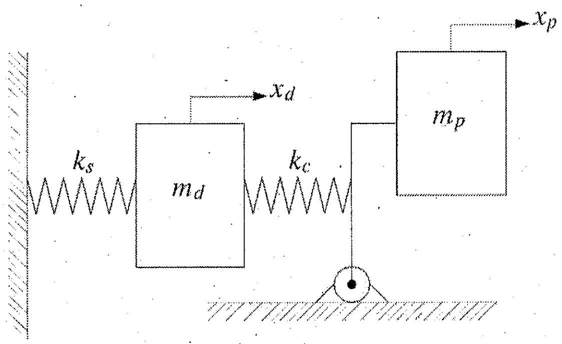

Figure 2 Lumped-parameter model of a ported loudspeaker: k_s and k_c are stiffnesses of driver and cabinet springs, m_d and m_p are the moving-masses of the driver and the air in the port respectively and x_d and x_p are their displacements.

The combined volume velocity of the driver and the port is given by

$$\hat{q} = j\omega(\hat{x}_d S_d + \hat{x}_p S_p) \quad (7)$$

where S_d and S_p are the radiating areas of the driver and port respectively. The displacements x_d and x_p are found by solving the coupled simultaneous equations of motion of the two masses. This volume velocity is then used to calculate the pressure at a distance r from the loudspeaker, using an expression similar to Equation (2). Note that the use of Equation (7) is equivalent to assuming the same path-length to the observation point from the driver as from the port.

Figure 3 shows the output of a computer simulation of a lumped-parameter model of a ported loudspeaker. Figure 3a is the amplitude of the frf, 3b is the phase and 3c is the excess phase. It is shown that the lumped-parameter model predicts an entirely minimum-phase response for a ported loudspeaker, as the excess phase is zero at all frequencies.

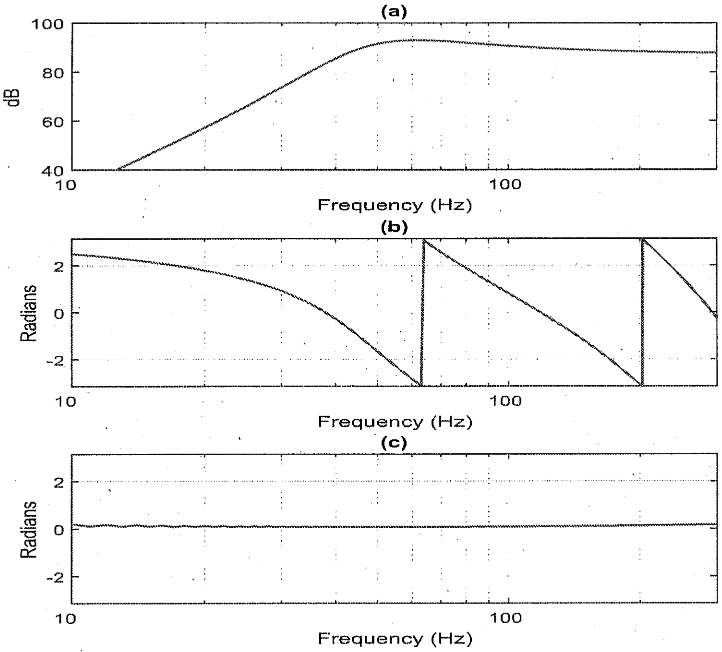

Figure 3 Output of a computer simulation of a lumped-parameter model of a ported loudspeaker: a) amplitude, b) phase, c) excess phase.

4.2 Wave Propagation Model

The lumped-parameter model using a spring to represent the air in the cabinet, whilst adequate for most purposes, does not take into account the physical distance between the driver and the port, and the resultant finite time for an acoustic wave to propagate from the driver to the port (at the speed of sound). It is conceivable that the delay between the output of the driver and that of the port – not considered in the spring model – may explain the observed measured phase differences. To investigate this, a more realistic wave-propagation model is simulated. In this model, the spring representing the air in the cabinet is replaced by an air-filled pipe, with a piston at each end representing the driver and the port, which are both still modelled as lumped masses. The sound field within the pipe is represented by the superposition of forward (from driver to port) and backward propagating waves. Any delay due to the propagation distance between the driver and the port is therefore included. Figure 4 shows a graphical representation of the wave-propagation model, and Figure 5 shows a comparison between the response of the ported loudspeaker using the spring model and that using the wave model with a typical 0.3 m between the driver and the port. Equation (7) is used to calculate the combined volume velocity, which is again equivalent to assuming the same path-length to the observation point from the driver and from the port. The two responses agree to within an acceptably small margin, giving confidence in both models.

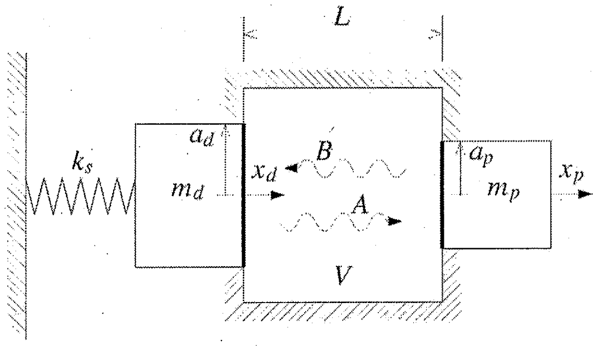

Figure 4 Graphical representation of the wave-propagation model of a ported loudspeaker: k_s is the stiffness of driver spring, m_d and m_p are the moving-masses of the driver and the air in the port respectively, x_d and x_p are their displacements, L is the distance between the driver and the port, V is the volume of the cabinet, a_d and a_p are the effective radii of the driver and the port respectively, and A and B are the (complex) amplitudes of the forward and backward propagating waves, respectively.

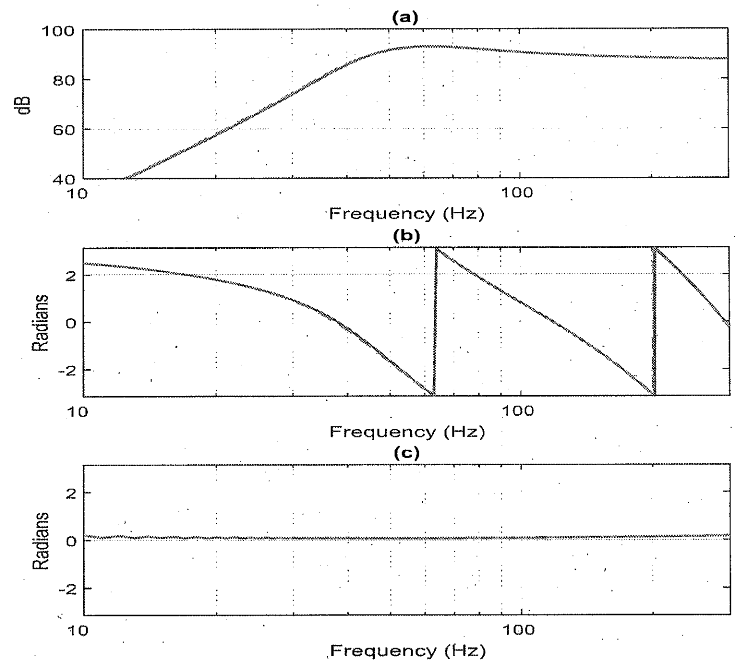

Figure 5 Comparison between the response of the ported loudspeaker using the spring model (solid) and the wave model with 0.3 m between the driver and the port (dashed): a) amplitude, b) phase, c) excess phase.

Figure 6 shows the response of the same ported loudspeaker, but with an excessive 0.75 m between the driver and the port (the volume of the cabinet is the same). The excessive delay has changed the amplitude and phase of the response, but the response remains minimum phase. Note that the resonant response at around 230Hz is the first standing wave resonance of the pipe representing the cabinet; this would occur in a real cabinet of this length, but would (hopefully) be damped through the use of absorbent stuffing etc.

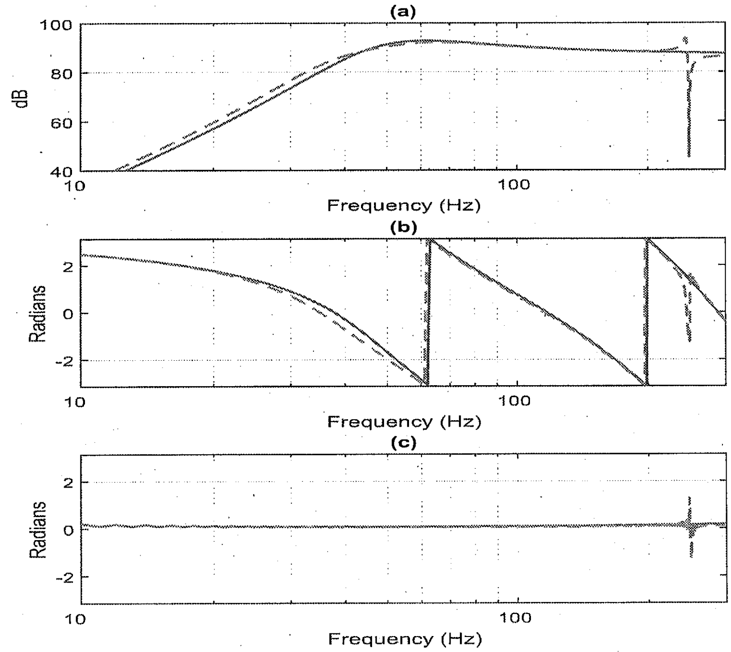

Figure 6 Comparison between the response of the ported loudspeaker using the spring model (solid) and the wave model with 0.75 m between the driver and the port (dashed): a) amplitude, b) phase, c) excess phase.

4.3 Improved Spatial Summing Model

The assumption of equal distance between driver and port to the observation point may not be realistic in some circumstances; for example, the port may exit at the rear of the cabinet instead of the front, or the observation point may be above or below the plane of equidistance. To investigate this, the wave-propagation model is extended by replacing Equations (2 & 7) with one equation which takes account of the physical distances between the driver, port and observation point:

$$\hat{p}_0 = -\rho\omega^2 \left\{\frac{\hat{x}_d S_d exp(-jkr_d)}{4\pi r_d} + \frac{\hat{x}_p S_p exp(-jkr_p)}{4\pi r_p}\right\}. \quad (8)$$

Figure 7 shows the response of the ported loudspeaker with 0.3 m vertical distance between driver and port, but with the observation point raised to an angle of 45 degrees relative to the plane of equidistance; thus, the observation point is closer to the driver than to the port. The minimum-phase behaviour is unaffected by this shift but, interestingly, the amplitude and phase responses are different; this change in response due to a change in the physical positions of the driver and port could not be predicted with either of the first two models.

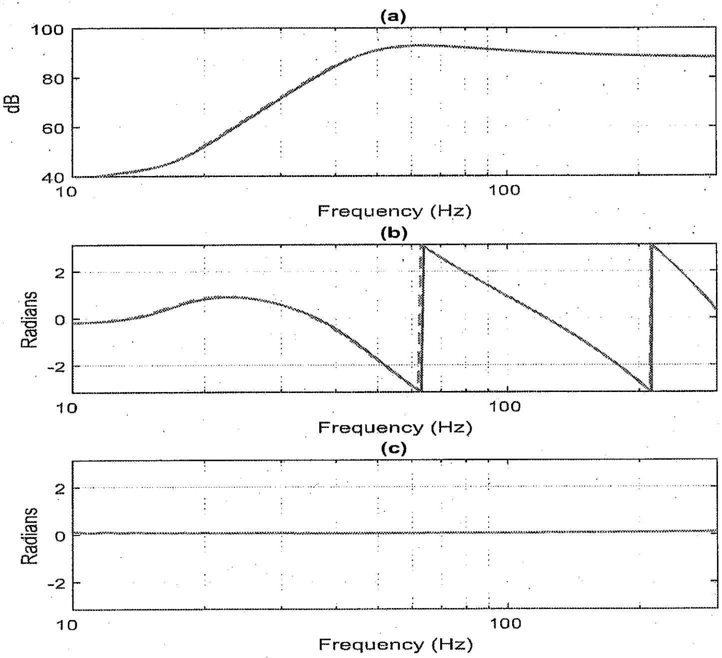

Figure 7 Comparison between the response of the ported loudspeaker using the spring model (solid) and the wave model with 0.3 m between the driver and the port (dashed), and the observation point raised to an angle of 45 degrees above the plane of equidistance between the driver and the port: a) amplitude, b) phase, c) excess phase.

Figure 8 is as Figure 7, but with the observation point lowered by an angle of 45 degrees, bringing the observation point closer to the port than to the driver. Under these conditions, with the sound from the port reaching the observation point before that from the driver, the response shows significant excess phase. This result suggests that the physical position of the port does affect the phase response of a loudspeaker and that this observation may explain the presence of excess phase in some of the measured responses.

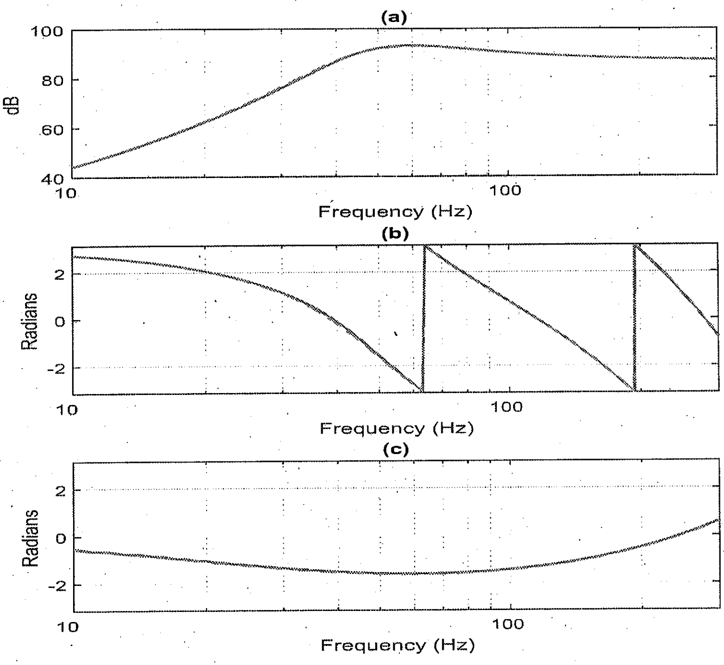

Figure 8 Comparison between the response of the ported loudspeaker using the spring model (solid) and the wave model with 0.3 m between the driver and the port (dashed,) and the observation point lowered to an angle of 45 degrees below the plane of equidistance between the driver and the port: a) amplitude, b) phase, c) excess phase.

4.4 Imperfect Anechoic Chamber at Low Frequencies

As mentioned in Section 2, a monopole calibration was applied to all of the loudspeaker measurements discussed here, to account for the imperfect free-field conditions in the anechoic chamber at low frequencies. This calibration is based on the assumption of a monopole source and, as stated in Section 2.1, this assumption is not necessarily valid for ported loudspeakers. To investigate this, the ported loudspeaker model was extended to include a floor reflection. In this simulation, the floor is 1 m below the port, which is 0.3 m below the driver. In a worst-case scenario, the wedge-covered floor is assumed to be perfectly-reflecting, which is unlikely, but the simulation results are insensitive to the amplitude of the reflection or the distance to the floor. In the simulation, the loudspeaker is replaced by a monopole, and the responses at the observation point with and without the floor reflection are subsequently used to calibrate the response of the ported loudspeaker model. Figure 9 shows the response of the ported loudspeaker, including the floor reflection, but without calibration, and Figure 10 is the same with the monopole calibration applied. Some excess phase is present, probably due to the reflection of the port being closer to the observation point than the reflection of the driver (see Section 4.3); however, the excess phase is unaffected by the application of the monopole calibration. It can be concluded that the application of monopole calibration to a non-monopole source, in the presence of significant floor reflections, does not introduce excess phase.

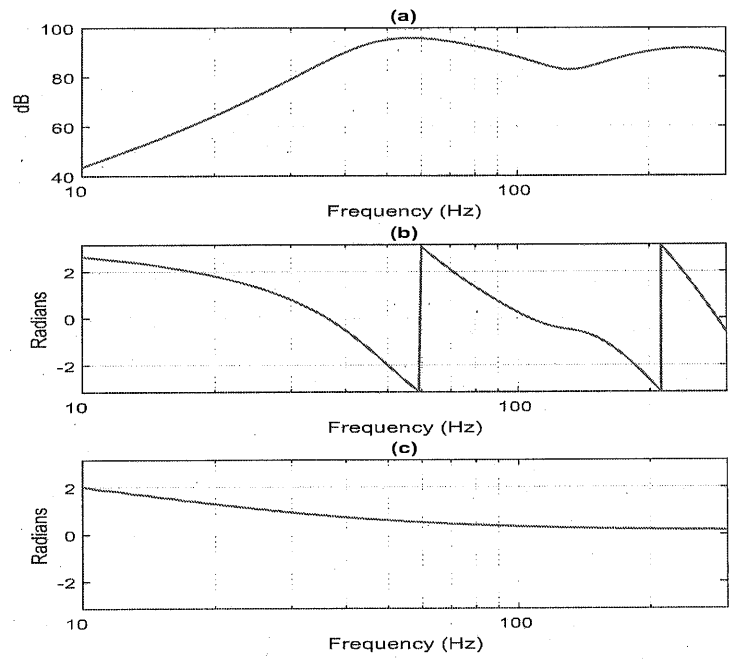

Figure 9 Output of a computer simulation model of a ported loudspeaker with a floor reflection: floor 1 m below the loudspeaker, no calibration: a) amplitude, b) phase, c) excess phase.

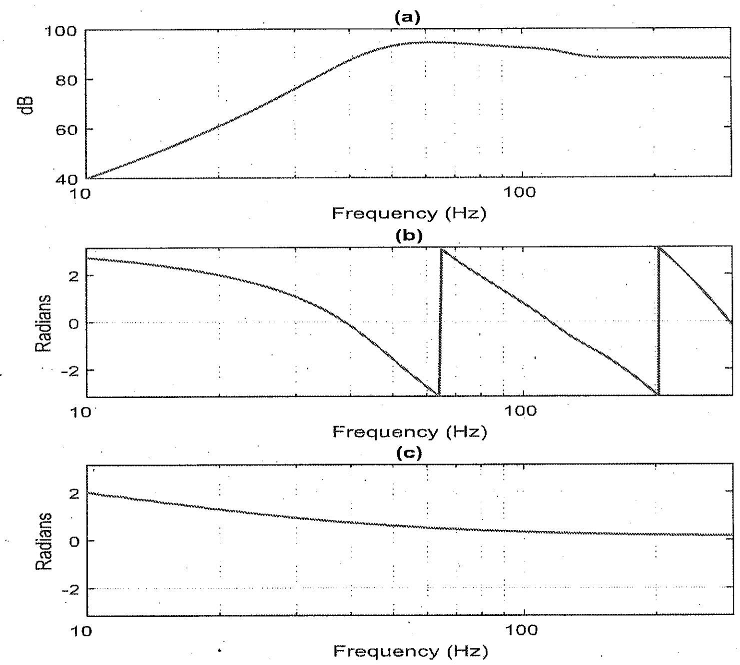

Figure 10 Output of a computer simulation model of a ported loudspeaker with a floor reflection: floor 1 m below the loudspeaker, with monopole calibration: a) amplitude, b) phase, c) excess phase

5 DISCUSSION

5.1 Electronic High-Pass Filters

Many of the loudspeakers measured are equipped with electronic high-pass protection filters, designed to limit the excursion of the low-frequency driver at very low frequencies. As discussed in [1], the use of these filters, when combined with the mechanical roll-off of the loudspeaker, can result in very steep low-frequency roll-offs; in some cases as high as 8^{th}-order. Often the same set of filters is used to 'sharpen' the knee of the roll-off, to maintain a flatter response down to the chosen roll-off frequency. The combination of sharp knee and high-order roll-off results in significant phase effects at low frequencies, even if the overall system is minimum phase. To illustrate this, Figure 11 shows the on-axis frequency response of an example of a loudspeaker measured to have an 8^{th}-order low-frequency roll-off as reported in [1]. Figure 12a shows the corresponding waterfall plot, and Figure 12b is the same but with the excess phase removed, using the technique described in Section 3 and [1]. For completeness, Figure 13 shows the waterfall of just the excess phase added to an idealised minimum phase 20Hz 2^{nd}-order high-pass filter.

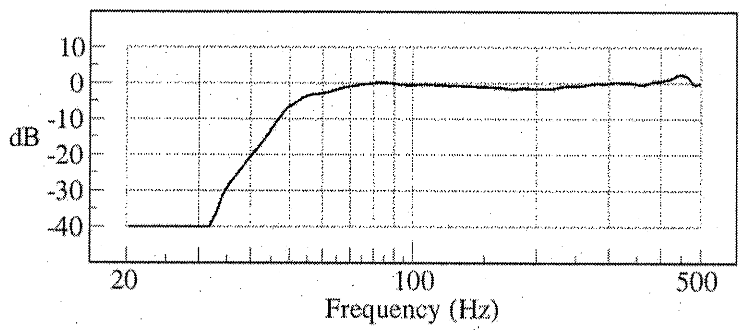

Figure 11 On-axis frequency response of an example loudspeaker with an 8^{th}-order low-frequency roll-off [1].

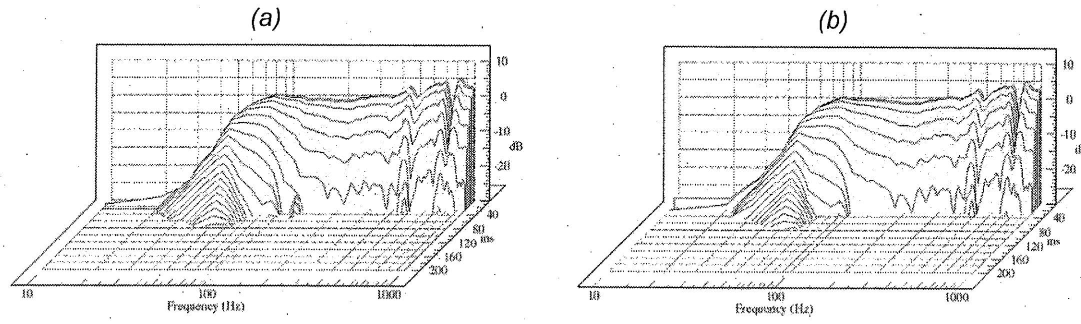

Figure 12 Waterfall plot for loudspeaker in Figure 11 (a) as measured (b) with excess phase removed

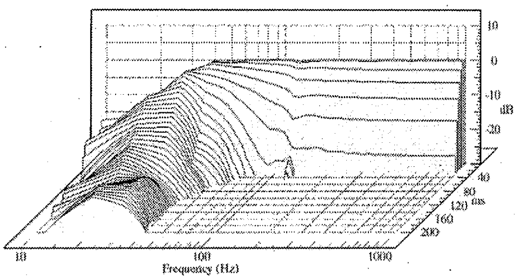

Figure 13 Waterfall plot of excess phase added to a minimum phase 20Hz 2nd-order high-pass filter (see [1])

Removing the excess phase from the measurements does improve the transient response, as is evidenced by a slightly more rapid decay at low frequencies in Figure 12, but the remaining response still suffers due to the sharp roll-off. The presence of the high-pass protection filter is detrimental to transient response even if the filter and loudspeaker combination are minimum-phase. Adding just the excess phase to an idealised minimum-phase high-pass filter is equivalent to applying a 'perfect' minimum-phase equalisation filter to achieve an ideal 20Hz high-pass response (see [1]). Clearly, under these conditions, the presence of the (unequalised) excess phase is enough to spoil the transient response of the equalised loudspeaker.

It is acknowledged that the sole purpose for loudspeakers of the type discussed here is to generate sounds that are listened to by people, and so, ultimately, any objective observations of performance must be related back to what is perceived. However, the audibility or otherwise of phase distortions in loudspeakers has been little studied to date, so it is unclear whether the differences identified in this paper have any relevance to the sound of the loudspeakers. It must be borne in mind, however, that all of the loudspeakers tested have been marketed as monitors, and as such are quality-control tools used by the music and broadcast industries, and not by consumers for entertainment. Carefully-controlled subjective listening tests would need to be carried out to establish the audibility, and hence importance, of these differences. Furthermore, with the growing use of the automated equalisation of monitors, the effect of excess phase on the decision-making *within* the automated systems needs to be carefully considered, as it could lead to attempts to equalise parts of the responses that are not amenable to correction by equalisation. Consequently, it is important to be aware of the excess-phase parts of the response of any loudspeaker system to be equalised in this manner.

The careful assessment of the subjective aspects of the excess phase also needs to be carried out in rooms with a range of decay times, to evaluate the degree to which the room effects may swamp any audible effects. Indeed, in the past, this potential for rooms to mask the audibility of the excess phase has frequently been used as justification for *not* considering it to be a significant issue, but much modern music-monitoring is carried out at close distances in rooms with decay times of 250 ms or less [11], even at frequencies as low as 60 Hz, and often after automated equalisation.

6 CONCLUSIONS

The following are conclusions drawn from this study.

- As reported in references [1, 4 & 7], excess phase is observable in measurements of the on-axis responses of a number of commercially-available loudspeakers.

- The presence of excess phase in the responses is detrimental to the transient responses of loudspeakers.

- Simulations show that:

    - The measured excess phase is not caused by the use of bass reflex ports in the loudspeakers.
    - The measured excess phase is not caused by using monopole calibration of the measurement system.
    - The measured excess phase *may* be caused by the relative positions of the driver, the port and the observation point; specifically, when the path length from the driver to the observation point is longer than that from the port to the observation point.

- Removal of the excess phase from the response of a loudspeaker having a rapid, high-order, low-frequency roll-off does improve the transient response, but only by a small amount; the transient response of the remaining minimum-phase loudspeaker is still poorer than that of loudspeakers having lower-order low-frequency roll-offs.

- The presence of the excess phase can compromise the equalisation of the response of the loudspeaker using conventional filters.

7 REFERENCES

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