TOWARD ABERRATIONS CORRECTION IN AUDIO SYSTEMS WITH ACOUSTIC METAMATERIALS

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1 ABSTRACT

Immersive audio is crucial for immersive experiences, but the tools aiming to deliver it to the listeners are still limited to headphones and bulky loudspeaker arrays. In this paper, we discuss the possibility of using a single loudspeaker, whose emission is augmented coupling acoustic metamaterial lenses with it, to obtain sound field control and thus to deliver localized sound cues in a 2D environment around a listener. Together with the sound field control, the reduction of the form-factor of the system is obtained. In fact, the realization of personal sound zones typically require at least 13 loudspeakers, placed around the listener but metamaterials have already proven that they can contribute to obtain more compact and lighter audio systems, allowing an effective directivity control and cancelling emissions in unwanted directions and thus, contributing to deliver the intended acoustic field in the region of interest. Just like an optical objective made of only two lenses, however, it was recently found that such a system suffers from the acoustic equivalent of spherical aberration. In this work, inspired by optics, we use numerical methods to design an additional metasurface to passively correct this aberration. Perspectives of combining systems of multiple lenses to correct audio systems will be discussed.

2 INTRODUCTION

Focusing sound in a defined small area of a listening space is a well-known challenge. Today, this can be achieved placing digitally controlled wavefront shape loudspeakers around the listening area. Most of the nowadays digital signal processing (DSP) approaches aim on the reproduction of a desired sound field in a target region, while attenuating acoustic energy in a second target region. Sound zone filters and highly directional filters can be applied to create so called, bright and dark zones to shape the wavefront following a desired sound field [17]. Due to the limitation in the acoustical properties provided by naturally available materials, however, wavefront engineering is still challenging and generally has to rely on using a bulky array of individually controlled active elements, such as the piezoelectric transducers for producing high-intensity focused sound. In addition, control of acoustic waves of long wavelength with a compact device would be highly desirable in a great variety of applications such as noise cancellation, but is extremely difficult for existing technologies since classic acoustics theory usually requires the device size and the wavelength to be comparable for giving rise to effective wave-structure interaction.

Therefore, current solutions tend to result effective at the cost of a large number of sound sources, e.g. surrounding the listener or employing linear arrays of loudspeakers, that usually creates virtual sound sources to focus sound in proximity to the listener. This type of installation is found, per example, in the home audio systems, where it is assumed that the listener is positioned in a precise location of the space. However, one of the main issues of these methods based on DPS is the ability to accurately reproduce the bright zone, without failing on attenuating the second region. Consequently, the precision with which sound can be delivered is limited and it also varies with the reproduced frequency [17].

Acoustic metamaterials have already proven that they can contribute to obtain more compact and lighter sound systems, allowing an effective directivity control and cancelling emissions in unwanted directions and thus, contributing to deliver the intended acoustic field in the region of interest [8,11]. When coupled at the outlet of a sound source, acoustic metamaterials can create acoustic lenses [7], superlenses [11] and meta-filters, passively embedding part of the required signal processing, contributing to create sound homogeneity or providing special effects for sound such as, the creation of a target zone in a listening area were sound pressure is maximized (or minimized), enhancing the contrast with respect to the rest of the environment. In this work, we aim to focus sound by combining two acoustic lenses with a sound source and we will show the effects of trying to limit the spherical aberration due to an acoustic superlens by means of a corrective acoustic metasurface, capable of locally manipulate phase and intensity of a sound wave in a small footprint, by changing its geometrical parameters.

In previous work was found that acoustic lenses suffer from aberrations, just as it happens in the equivalent optical device. In optics, real lenses behaves differently from how they are modelled using the thin lens equation thus, producing aberrations that imply a distortion in the image created. There are different types of aberrations due to the lens size, its material, and position of the object. One common type is chromatic aberration, which is caused by the fact that the index of refraction of the lens depends on colour (or wavelength) of the light passing through. As a consequence, light rays are bended differently and images are produced at multiple places and with different magnifications depending on the colour. Quite often in imaging systems the object is off-centre. Therefore, different parts of a lens or mirror do not refract or reflect the image to the same point. This type of aberration is called coma and as a consequence, the image appears pear-shaped. Another common aberration, which is related to this work, is the spherical aberration, where rays converging from outer edges of a lens converge to a focus closer to the lens (positive) and rays closer to the axis focus further (negative). It is found in optical systems that are composed by elements characterized by spherical surfaces and this deviation reduces the quality of images produced by optical systems. This effect can be observed in Figure 1 and was first identified by Ibn al-Haytham who discussed it in Kitāb al-manāẓir.

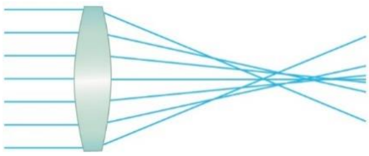

Figure 1 - Spherical Aberration

A spherical lens has an aplanatic point (i.e., no spherical aberration) only at a radius that is equal to the ratio between the radius and the index of refraction of the lens material. Considering a typical value of refractive index for crown glass equals to 1.5, it indicates that only about 43% of the area (67% of the diameter) of a spherical lens is useful. This effect tends to be pronounced at short focal ratios.

Previous work [12] demonstrates the validity of the thin lens equation also for the design of a superlens made of metamaterials, which is able to create a focus beyond the limit of diffraction, at audible frequencies. Moreover, [12] highlighted also that the designed acoustic superlens suffers from aberration, in fact the focal point spread to become a focal spot having dimensions related to both the lens dimensions and the wavelength of the sound passing through the metasurface.

2.1 How to correct aberrations

In optics, to correct aberrations, lenses may have specially shaped surfaces, as opposed to the simple spherical shape that is relatively easy to fabricate. Spherical aberration can be eliminated by making lenses with an aspheric surface. Descartes showed that lenses whose surfaces are well-chosen Cartesian ovals, revolved around the central symmetry axis, can perfectly image light from a point on the axis or from infinity in the direction of the axis. Such a design yields completely aberration-free focusing of light from a distant source [1]. In a 1949, Wasserman and Wolf formulated the problem how to design a lens without spherical aberration in an analytical way, and it has since been known as the 'Wasserman-Wolf problem' [16]. In 2018, Rafael G. González-Acuña and Héctor A. Chaparro-Romo, finally found a closed formula for a lens surface that eliminates spherical aberration [2]. Their equation can be applied to specify a shape for one surface of a lens, where the other surface has any given shape. They proposed to use two aspheric adjacent surfaces to correct spherical and coma aberrations, with a solution consisting of two first-order simultaneous differential equations, which are solved numerically according to Malacara-Hernández et *al.* [3].

Further advances in material science have resulted in lenses with a range of refractive indices or graded index lenses (GRIN). For example, in a design consisting of a single lens with spherical surfaces and a given object distance o, image distance q, and refractive index RI, spherical aberration can be minimized by adjusting the radii of curvature R_1 and R_2 on the front and back surfaces of the lens such that:

$\frac{R_1+R_2}{R_1-R_2} = \frac{2(RI-1)}{RI+2}\left(\frac{q+o}{q-o}\right)$. The sign of the radii follows the Cartesian sign convention.

In this work, we explore the possibility of using acoustic metasurfaces to reduce the spherical aberration due to an acoustic superlens. Acoustic metamaterials here employed are labyrinthine structures where sound can pass through and acquire a phase delay imposed by the geometrical characteristics of each so-called bricks, as described by Memoli et *al.* [15], composing the metastructure. In particular, the aim of this work is to employ hill descending algorithm as design method of a corrective device required to be coupled with a given superlens to focus sound, reducing the spherical aberration the superlens exhibits. This corrective device may be a second metasurface, combined at the outlet of the superlens or, the determined phase profile may be encoded in the superlens itself, varying the labyrinthine path of each component brick and thus its geometric parameters.

The superlens here presented, as well as the corrective device, are both two dimensional metasurfaces. The first one is built by juxtaposing the centre row metamaterials of the superlens presented in [12]. This superlens is designed to focus sound at a central design frequency equal to 2k [Hz] and having a focal length equal to 150 mm. The corrective metasurface will be characterized by the same number of bricks as the converging lens, and the phase profile aimed to reduce the spherical aberration, will be determined following a procedure recently presented in [13] and which will be detailed in the method section below.

3 METHOD

Previous literature demonstrates that sound fields with high spatial resolution can be created from a discrete set of 16 phase-delay metamaterial labyrinthine structures [14], with a constant amplitude. The superlens and the corrective device here described have 12 labyrinthine structures each. The converging lens (Figure 2) was designed for a central frequency of operation $f_d = 2k\ [Hz]$ and for a focal length $f = 150\ [mm]$. If the focal point is defined as the point with maximal intensity, considering sound propagation in air at a temperature of $20°C$ (speed of sound $c = 343\ [m/s]$), to confirm it is a superlens we therefore expect to find it at a distance less than one wavelength ($\lambda_d = 17.1\ [cm]$).

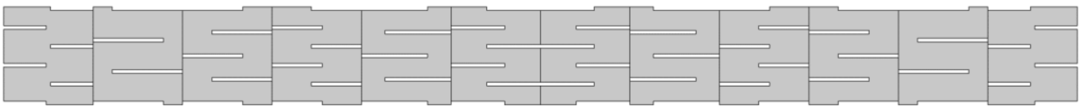

Figure 2 – 2D superlens 2k [Hz]

For the sake of simplicity, the design method of the metasurface here presented to correct the acoustic lens exploits the symmetry with respect to its central axis. Therefore, the problem variables are the phases of the 6 acoustic metamaterials. Each brick can take 8 possible phase values, resulting in a 6D space search problem. Phase delays are associated with indices, to which correspond values ranging from 0 to $7/4\pi$ [rad], in steps of $\pi/4$. Assuming that only a symmetrical distribution of delays is possible, and thus limiting the search of a desired pressure distribution with a corrected focus on-axis with respect to the sound source, the number of possible combinations is reduced to 8^6.

The input space is retrieved by finite-element modelling, using COMSOL Multiphysics software package. The 2D numerical model concerns the superlens, placed at the centre of the right boundary of a reflection-free listening area $(50 \times 50\ [cm])$, where a desired pressure distribution is aimed to be obtained searching among all the possible bricks combinations by hill descending algorithm. The corrective metasurface is modelled by imposing a parametric phase to the sound passing through each of the 12 metamaterial bricks, aiming to limit the focus width (x-axis) and therefore, the spherical aberration.

In this work, the sound system is not modelled, and the metamaterial geometry is explicitly represented here for the bricks composing the superlens only. In fact, previous literature [9] allows to determine the geometrical parameters the corrective metamaterial device needs to have, once the desired wavefront amplitude and phase at its outlet are known. Previous work demonstrates how a superlens can be designed by following the thin lens equation [12].

In the 2D listening area, the sound wavefront propagation in air is modelled as a planewave, emitted by the metasurfaces. Therefore, following the thin lens equation and simulating the sound source placed at an infinite distance with respect to the superlens thus, the focus is expected to be centred at $150\ mm$. Together, the metasurfaces determine the total wavefront emitted. The acoustic corrective metamaterial device $(2.5\ \times 30\ [cm])$ consists of sound transmission bricks, represented with 12 air squares $(brick_x = brick_y = 2.5\ [cm])$, each of which imposes a precise phase delay on the emitted wavefront. Pressure Acoustic, Frequency Domain interface computes the pressure variations for the propagation in air of acoustic waves at quiescent background conditions. Perfectly Matched Layers (PMLs) 10 cm thick are added to the external boundaries, to absorb all outgoing wave energy. Sound Hard Boundary is assigned to all the external boundaries of the metastructure components and the internal metamaterial bars thus, here the normal component of the acceleration results zero. A Background Pressure Field is assigned to each metamaterial brick composing the meta-corrective device with initial nil phase.

The calculation is performed by considering a minimum element size mesh equal to 1/5th of the shortest wavelength studied. The study is run three times, analysing the results obtained around the design frequency of the superlens $(f_{1,2,3} = 1980,\ 2000,\ 2020\ [Hz])$ and it provides, the 6D search space needed for the algorithm to work. Since the lens appears to perform better at a slightly lower frequency rather than the design one ($(f_1 = 1980\ Hz$), we will show the results for this frequency.

Data are then exported from COMSOL to MATLAB in a single spreadsheet, containing bidimensional space coordinates and the root mean square value of total acoustic pressure (p_{rms}), evaluated on 2601 points (10 mm step) regular grid (N) in the listening area or region of interest (ROI), for all the 8^6 configurations, having excluded both PMLs and metasurfaces form the sampled region.

The hill descending algorithm used here is a local search method, often employed when the gradient of the objective function is unknown. It is an iterative algorithm which starts with an arbitrary solution, then attempts to find a better one by incrementally changing (i.e., mutating) it. If the change produces a better result, another incremental change is made to the new solution, and so on until no further improvements can be found. Thus, two objective functions (or Scores) to be minimised are defined (Eq. (1) and Eq.(2)), for every brick configuration. In particular, the second objective function aims to enhance the contrast between the focus and the rest of the listening area.

$$OBJ1 = \sum_{i=1}^{N} \frac{\left| p_{rmsROI} - (\max(p_{rmsFocus}) * mean(p_{rmsOuter})) \right|_i}{N} \ (1).$$

$$OBJ2 = -(\max(p_{rms}) * \ mean(p_{rms}))\ (2).$$

Here $\max(p_{rmsFocus})$ is a matrix having $\max(p_{rms})$ in the first target zone, where the sound field is expected to be maximised, while limiting the spherical aberration (i.e. the focus), and ones in the second target zone (i.e. elsewhere). $\mathrm{mean}\ (p_{rmsOuter})$ instead, is a matrix characterized by $\mathrm{mean}\ (p_{rms})$, in the outer region points and ones in the focus area.

The two target zones are obtained selecting data by using a rectangular mask for both the objective functions and imposing its dimensions equal to the desired reduced focus width ($w_{desired} = 10\ mm,$) and height ($h_{desired} = 250\ mm$). In this study, we also changed the rectangular mask dimensions in order to be able to reduce spherical aberration.

A mutation function is performed by the algorithm, starting with a random metamaterial brick, selecting a random phase value for the selected brick and then replacing the current value with a newly generated one. The algorithm was run for a maximum number of 500 iterations which was sufficient for convergence for the objective function used here and an exit clause is chosen to stop searching if the global minimum is reached.

4 RESULTS

Below the effectiveness of the two objective functions is compared. In Figure 3a, c the score over time is shown for the analysed frequency, whose behaviour indicates the error always decreases and reaches convergence. As described above, here a rectangular mask has been applied to define the focus, whose dimensions are imposed to be equal to de desired ones. 500 are the maximum iterations for the algorithm to find a solution which minimise the error, but it not necessarily is the global optimal one. Figure 3b, d represents the change of the brick configurations over time, obtained by hill descending algorithm. Here, y-axis indicates the index associated to each brick, and colours are associated to a phase value between the 8 possible ones.

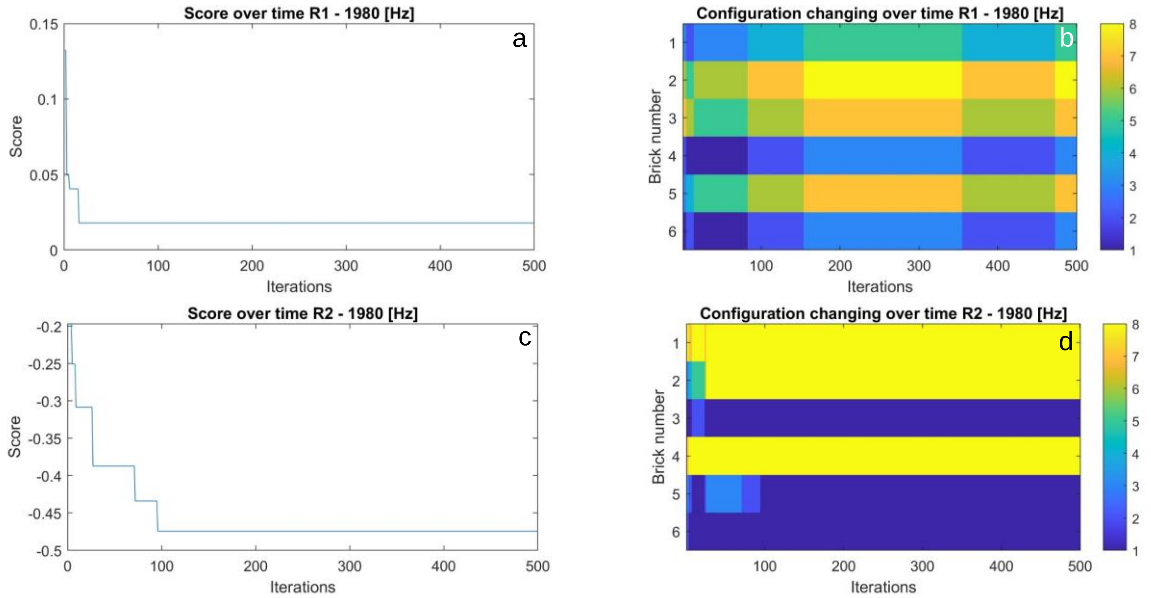

Figure 3 – 1980 Hz Two objective functions comparison: (a, c) Score and (b, d) Configuration over time using the rectangular mask.

Figure 4 shows the pressure distribution comparison obtained with the two objective functions employing the rectangular mask, enabling the evaluation of both spherical aberration and resulting contrast. In particular, the pressure distribution without the use of a corrective device (wo) (Fig.4a, d), with the last combination of bricks composing the corrective metasurface found by the optimization (Fig.4b,e) and the difference of pressure distribution between the last configuration and the one obtained without the use of a corrective device can be observed (Fig.4c,f).

Observing the pressure distribution obtained in the listening area without the use of a corrective device in Figures 4a and 4d, it should be noted that five small areas on axis where the sound is maximised are created. This is due to the presence of minima in the total acoustic pressure which results in maxima when the RMS of the pressure is showed. The sound field outside these areas is attenuated with the exception of the zone 20 mm away from the superlens. Therefore, without a corrective device the superlens exhibits a main focus which is found by the algorithm to be cantered in (33, 26), which correspond to 180 mm distance from the superlens and means 30 mm away from the design focal length. This suggests that the focus moves slightly away with respect to what is expected, possibly due to the limited dimensions of the superlens with respect to the case where the whole superlens is modelled [12].

Combining a corrective device with the superlens and applying the optimisation algorithm shifts the focus closer in both analysed sound fields (Fig 4b, e), at a distance which is less than one wavelength of the sound passing through. This can be due to the fact that the effective path length for the sound to pass through is increased with respect to the wo corrective device case.

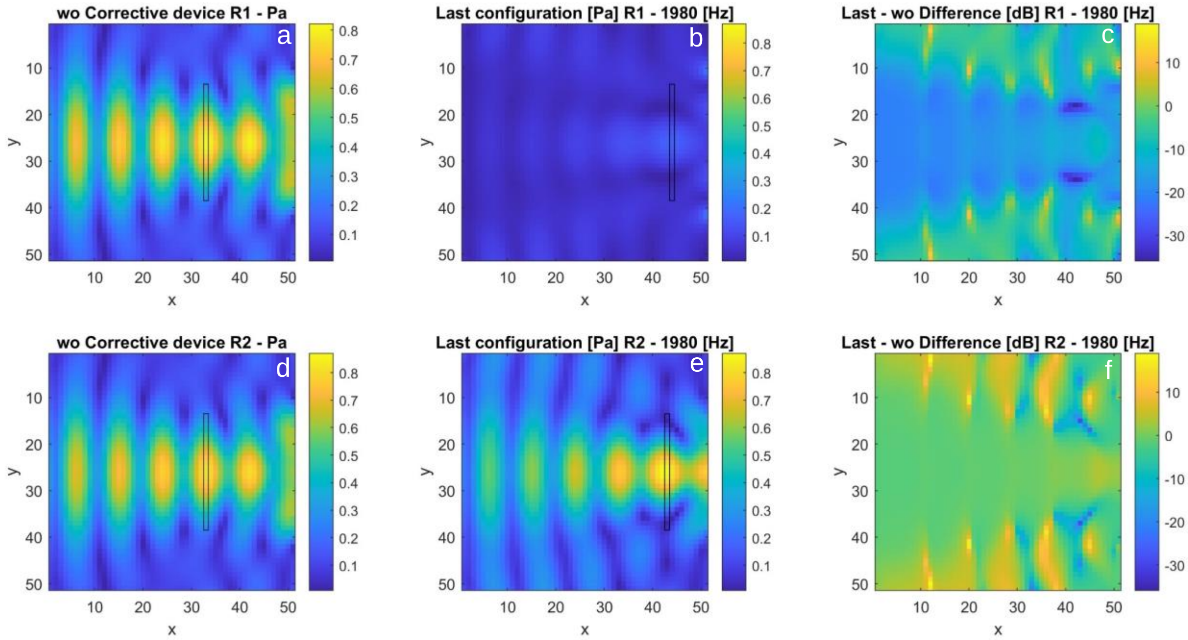

Figure 4 – 1980 Hz Two objective functions comparison: Pressure distributions: (a, d) without corrective device, (b, e) with last combination of metamaterials found by the algorithm and (c, f) SPL difference between the last and without corrective device.

The first objective function realises a single focus (Fig. 4b) which is ~70 mm away from the metasurface, whose intensity is strongly reduced with respect to the case without a corrective device. The width is also not reduced, while the height seems to be. Moreover, comparing Figure 4a and 4b it can be observed that the sound field is strongly attenuated throughout the outer target zone with the exception of the top and lower boundaries where the corrective device acts like a diverging lens.

By applying the second objective function instead, the multiple foci are maintained, and the resulted pressure distribution shows the main focus (Fig. 4e) ~80 mm away from the metasurface, whose intensity is slightly increased with respect to the case without having coupled a corrective device to the superlens. Its width is increased but as in the previously analysed case, the height is reduced.

This suggests the best objective function to be used is the second one (Eq. (2)). By changing the rectangular mask dimensions and also by weighting the maximum value inside the focus and the outer mean differently, it is possible to optimise the sound field. Figure 5 shows the pressure distributions obtained without the use of a corrective device, with the last combination of bricks found by the algorithm and the sound pressure level difference between the last configuration and the one without the use of a corrective device in two different cases where the rectangular dimensions changes. The width of the rectangular mask has been increased to $w_{desired} = 30\,mm$ first, having maintained the desired height as in the previously analysed cases ($h_{desired} = 250\,mm$). Then, the height of the rectangular mask has been increased to $h_{desired} = 410\,mm$, having restored its desired width to $10\,mm$. These results are obtained by multiplying the maximum value by 0.75 and the outer mean by 0.25.

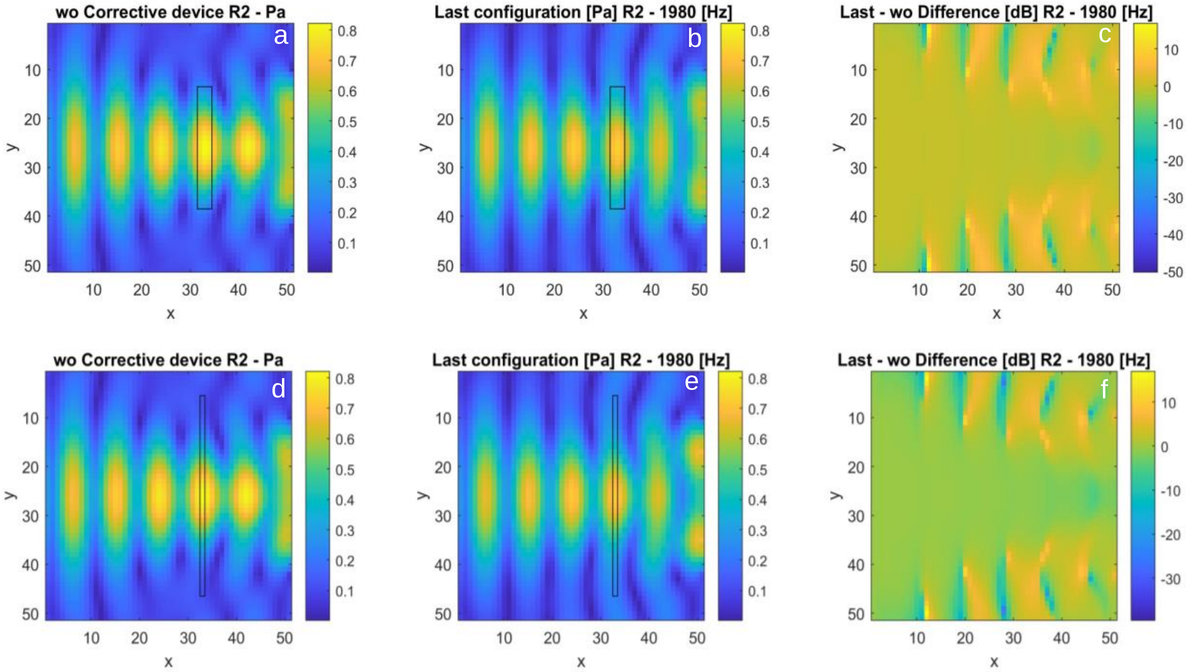

Figure 5 –1980 Hz OBJ2, comparison between different rectangular masks: Pressure distributions: (a, d) without corrective device, (b, e) with last combination of metamaterials found by the algorithm and (c, f) SPL difference between the last and without corrective device.

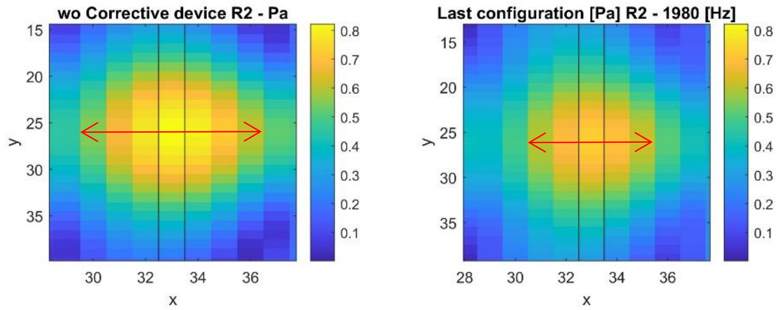

Figure 6 –1980 Hz OBJ2: Enlargement on the focus of Figures 5d, e

Observing the sound fields showed (Fig. 5b, e) and also the enlargement on focuses in Figure 6, it can be concluded that the width of the focus has been reduced with the proposed method, at the cost of having lowered the energy inside the focus with respect to the case without the corrective device coupled with the superlens. Moreover, it can be seen that the additional metasurface behaves like a diverging lens, increasing pressure in the outer target region. Further research is needed to enhance this result, but this study highlights the importance of the choice of the objective function in the proposed method.

5 CONCLUSION

In this work, we use hill climbing algorithm to retrieve the metamaterial bricks required to build up an acoustic metasurface, aiming to correct the spherical aberration due to a superlens, i.e., reducing the width of its focus. The combination of the two lenses is also aimed to enhance the contrast between two sound target zones: the focus area and the outer region. The focus area has been defined by searching for the maximum in the sound pressure distribution and having employed a rectangular mask for choosing the two target zones. The proposed method is tested comparing results obtained with two different objection functions and it is applied to a simplified case, concerning a bidimensional reflection-free listening area, where a system of two metasurfaces are coupled to a single sound source at a single frequency. The form factor is strongly reduced, considering that the effective path length is given by the labyrinthine structures of each brick. Therefore, the thickness of the combined lenses system which exhibits the best sound field in the listening area, is equal to ~1/4 of the effective path length.

The 6D input space is retrieved using finite element numerical method. The hill descending is used to search for the optimal combination of metamaterial bricks to build the corrective device. The best sound field, with a reduced width of the focus, in 8^6 possible ones due to different combinations of metamaterial bricks is searched, for a single frequency nearby the superlens design one ($f_1 = 1980\,Hz$). In conclusion, this work highlight the importance of choosing the objective function in the proposed optimisation method. The best result is obtained using the second objective function here presented, by weighting the maximum value inside the focus and the outer mean pressure differently.

This study shows that the balance between the two dimensions of the rectangular mask also play an important role in the reduction of spherical aberration of the superlens here presented. As can be observed in the best sound field found by hill descending, reducing the width of the focus is therefore quite challenging, a trade off between losing energy within the focus, and distributing pressure more uniformly across the listening area, with respect to the case without a corrective metasurface.

Therefore, by combining a system of two lenses i.e., a corrective metasurface retrieved by the proposed optimisation algorithm and a superlens, the focus width can be reduced. It should be mentioned that the smaller the focal length of the lens, the more difficult it is to reduce spherical aberration. In future work, this method will be applied to superlenses being designed to focus at a larger distance with respect to the metasurface to compare the efficiency in reducing the focus width.

The key limitation of acoustic metasurfaces as the one designed in this work is their bandwidth. Currently operating over a whole octave, thus sufficient to focus a simple melody, systems of lenses like the one described in this work can create special effects for sound and can be thought to be coupled to a single sound source to focus sound at target frequencies in applications such as recording studios monitoring and reduction of interference with obstacles in home audio systems.

This method could be applied in order to obtain more complex sound shapes such as, multiple soundless areas or controlled distance sound propagation. In future studies, different objective functions will be tested, as well as different mutation function, to approach a higher control of sound propagation. Using the presented method, we expect to design lenses, diffractors and other compact tools to create special effects for sound. This simplified procedure allows to reduce the time required to design acoustic metasurfaces targeting a desired pressure distribution in a listening area. Further modelling of the sound source and additional research are required in order to enhance solutions and to validate this method toward multi-objective optimisation.

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