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EXPERIMENTAL STATISTICAL ENERGY ANALYSIS: INTERNAL AND COUPLING LOSS FACTOR MATRIX VALIDATION

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1. INTRODUCTION

Over the last few years, experimental Statistical Energy Analysis (ESEA), the counterpart of analytical SEA, has been shown to be an effective predictive tool for complicated vibro-acoustic structures like trains, cars, engines, ..., exhibiting a high modal density and overlap in the medium and high frequency range. Based on partitioning the test structure into subsystems and conducting 'in-situ' measurements, the internal and coupling loss factor matrix can be derived. This paper gives a brief overview of some approaches to calculate the loss factors and describes how the quality of the experimentally derived loss factors can be assessed.

2. DERIVING THE LOSS FACTORS

Balancing the time-averaged power input to each subsystem with the power dissipated in the subsystem and the net power flows to coupled subsystems gives rise to the SEA power balance equations. According to the Power Injection Method (PIM) [1], a response energy matrix can be composed whereby each element $<\overline{E}_{ij}>$ represents the subsystem response energy for subsystem i due to injected power $<\overline{P}_j>$ in subsystem j. The brackets < > and the bar $^{-}$ denote respectively the space and time averaging. This gives the following equation :

$$\begin{bmatrix} \sum_{j=1}^{n}\eta_{1j} & -\eta_{21} & \cdots & -\eta_{n1} \\ -\eta_{12} & \sum_{j=1}^{n}\eta_{2j} & \cdots & -\eta_{n2} \\ \vdots & & & \\ -\eta_{1n} & & & \sum_{j=1}^{n}\eta_{nj} \end{bmatrix} \begin{bmatrix} \langle\overline{E_{11}}\rangle & \langle\overline{E_{12}}\rangle & \cdots & \langle\overline{E_{1n}}\rangle \\ \langle\overline{E_{21}}\rangle & & & \vdots \\ \vdots & & & \\ \langle\overline{E_{n1}}\rangle & & & \langle\overline{E_{nn}}\rangle \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} \langle\overline{P_1}\rangle & 0 & \cdots & 0 \\ 0 & & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & & & \langle\overline{P_n}\rangle \end{bmatrix} \text{ or } [L][E] = \frac{1}{\omega}[P] \quad (1)$$

where n is the number of subsystems, ω the center band frequency, η_{ii} the internal loss factor of subsystem i and η_{ij} the coupling loss factor between subsystems i and j.

Obviously, the most straightforward way to derive the loss factors is to invert the energy matrix of the left-hand side of equation (2) which is well conditioned in case of weak coupling as large terms occur on the leading diagonal. This can be written as follows :

$$[L]=\frac{1}{\omega}[P][E]^{-1} \tag{2}$$

Figure 1 depicts the condition of the energy matrix for a section of a railway carriage consisting of 20 subsystems. With values lower than 15, it is clear that the condition of the matrix is reasonably low. This does not necessarily imply a good quality of the loss factors as in this case, inverting the energy matrix turns out to yield some negative loss factors. Several studies have shown that this might be partially due to small measurement errors. Another reason is the susceptibility to omitting energies of non-adjacent subsystems which would be very tempting from the point of view of measurement effort reduction. Numerical simulations have pointed out that neglecting an energy $<\bar{E}_{ij}>$ in equation (2) yields a negative coupling loss factor η_{ji} [2]. This problem can be partly solved by using an alternative technique based on matrix inversion which explicitly takes into account physically unconnected subsystems by forcing the corresponding coupling loss factors to zero [3].

Another approach is based on approximate equations as found below [3].

$$\eta_{ii}=\frac{1}{\omega}\frac{\langle\overline{P_i}\rangle}{\langle\overline{E_{ii}}\rangle} \text{ and } \eta_{ij}=\frac{1}{\omega}\frac{\langle\overline{E_{ji}}\rangle}{\langle\overline{E_{ii}}\rangle}\frac{\langle\overline{P_j}\rangle}{\langle\overline{E_{jj}}\rangle} \tag{3}$$

These are derived on the basis that the energy in non-driven subsystems is significantly lower than in the directly driven subsystem (weak coupling) and offer the advantage that they cannot produce negative loss factors.

3. TECHNIQUES TO VALIDATE THE DERIVED LOSS FACTORS

Checking the sign of the loss factors is evidently needed. Evaluation of the confidence limits for the calculated loss factors might justify whether negative values can be adjusted to zero. A second validation is that all terms of the inverted total loss factor matrix [L] should be positive, which can be easily understood from equation (2). Additionally, from a physically acceptable point of view, if power is only inserted into one subsystem, the net flows transmitted to all coupled subsystems should be positive.

In addition to these three simple validation checks, the quality of the loss factors can be evaluated by synthesizing the energy matrix for the derived loss factor matrix and evaluating the difference between the predicted energies and the energies obtained from the PIM measurements. The synthesized energy matrix is given by

$$[Esynth] = \frac{1}{\omega}[L]^{-1}[P] \qquad (4)$$

Of course, if all energies are measured and the straight inversion based on equation (2) is then applied for the loss factor calculation, both will perfectly match. However, this validation technique is extremely helpful to evaluate whether it is justified to put negative loss factors to zero. For a box structure consisting of 6 plates, figure 2 shows a comparison between the approximate method based on equation (3) and the matrix inverse method according to equation (2) without including couplings between non-adjacent subsystems. Negative coupling loss factors obtained by neglecting the unconnected energies were set to zero. The figure illustrates clearly that on the one hand the approximate method is less accurate when synthesizing the energies of driven and non-driven subsystems and on the other hand adjusting the negative values introduced by the inverse type of solution to zero seems to be justified.

Although the loss factors are derived without explicit knowledge of the modal densities, the SEA consistency relationships can be used as well to derive an interesting validation tool. For any closed loop of three subsystems i,j,k, the following relationships can be written down :

$$\eta_{ij} n_i = \eta_{ji} n_j \text{ and } \eta_{jk} n_j = \eta_{kj} n_k \text{ and } \eta_{ik} n_i = \eta_{ki} n_k \qquad (5)$$

where n_i denotes the modal density of subsystem i and η_{ij} the coupling loss factor between subsystems i and j. Eliminating the modal densities in equation (5) leads to the so-called triple product rule [3] :

$$\eta_{ij}\eta_{jk}\eta_{kl} = \eta_{ji}\eta_{kj}\eta_{ik} \qquad (6)$$

By calculating the ratio of the left-hand to the right-hand side of equation (6) for each possible combination of three interconnected coupling loss factors and by averaging these ratios for each involved coupling loss factor, a matrix can be constructed for which the non-zero off-diagonal elements should be ideally equal to unity. Such a matrix corresponding to the center band frequency of 800Hz is shown in figure 3 for the section of a railway carriage, which enables the quality of each coupling loss factor to be verified and compared. The closer the value to unity, the better the quality of the coupling loss factor. Figure 4 illustrates the triple product ratios as function of frequency for the approximate and the matrix inverse method with regard to a particular coupling loss factor between two plates of the box structure. It shows clearly that the difference in quality is small for both methods.

4. CONCLUSION

The quality of the loss factors calculated by approximate methods or by matrix inverse methods where energies of unconnected subsystems are omitted can be successfully assessed by synthesizing the energies and comparing these with the measured energies obtained from the PIM tests.

By calculating the averaged ratios of the triple products for each closed loop of three interconnected subsystems, a matrix is built which enables the loss factors which do not fully comply with the SEA consistency relationships to be pinpointed.

5. REFERENCES

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[3] N. Lalor, Practical Considerations for the measurements of internal and coupling loss factors on complex structures, ISVR Technical Report No. 182, June 1990.

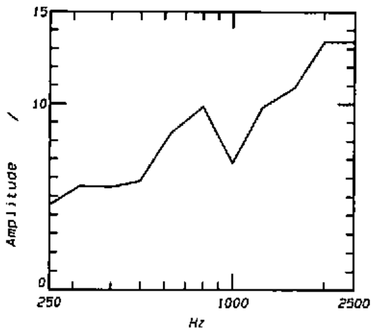

Figure 1 : Condition number of energy matrix. Section of railway carriage

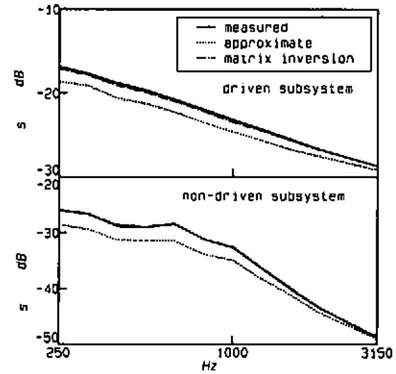

Figure 2 : Comparison between measured and synthesized energies normalized to unit input power for box structure

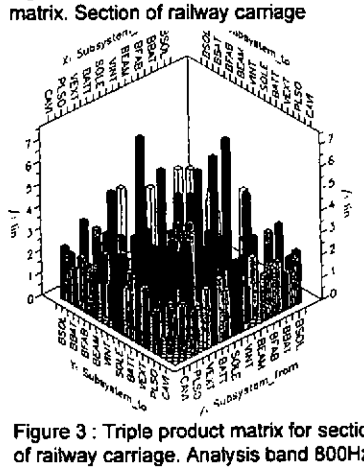

Figure 3 : Triple product matrix for section of railway carriage. Analysis band 800Hz

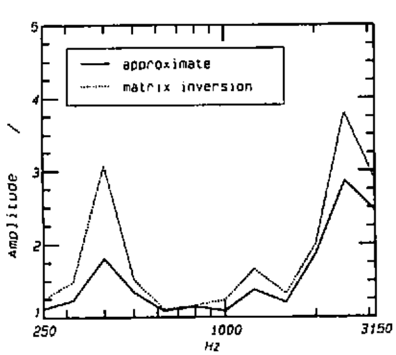

Figure 4 : Triple product comparison for coupling loss factor between two plates of box structure