A POWER MODE TECHNIQUE FOR ESTIMATING POWER TRANSMISSION TO A FLEXIBLE RECEIVER

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1. INTRODUCTION

An understanding of vibration power transmission in complex, built-up systems is essential in the attempt to solve many vibration problems of practical concern, such as noise radiation from submarines, fatigue of aircraft structures and mechanical equipment noise in buildings. For the case of a multipoint-coupled source/receiver system, the frequency response function (FRF) based sub-structuring technique is found to be attractive [1-2]. This utilizes the individual uncoupled component FRFs to construct the total system response. However, this method may be inconvenient since it inherently requires the evaluation of many terms (e.g. the full mobility matrices of the source and of the receiver) before an exact description for the vibration power is obtained [3]. Furthermore the exact predictions require exact knowledge of these FRFs, and this is rarely available in practice, especially at higher frequencies, e.g. the audio-frequency range.

In this paper an alternative technique, the power-mode method [4-5], is described. A set of force/velocity sources is weighted by a set of orthogonal functions and hence transformed into a new set of power modal forces/velocities. As a result the vibration power transmitted by N forces can be considered as the power transmitted by N independent power modes. The transformation involves the eigenfunctions of the receiver mobility matrix. The approach was first suggested in [6]. The "multipole" approach of [7-8] is somewhat similar, except that the transformation matrices are preselected sums and differences of the applied forces, so that they can be regarded as monopole, dipole, quadrupole terms etc.

In this paper power mode theory is briefly reviewed and then applied to the power transmission from a stiff source to a flexible receiver through discrete points. Approximations are then found for the upper and lower bounds of the transmitted power, together with an approximation for the frequency average value [5], which depends only on the diagonal elements of the mobility matrices of the source and the receiver. Finally, a numerical example of a four-point-coupled beam-plate system is considered and the approximate expressions are compared to the results of the FRF-based sub-structuring method. The expressions and results presented here help to shed some light on simplifications for the prediction and measurement of power transmission.

2. POWER MODE THEORY

The power mode method is in effect a transformation from physical to modal forces and responses, analogous to the "multipole" approach [7-8], but of a more general form so that the transformed

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power can be calculated by

$$P = \frac{1}{2}\mathbf{F}^{H}\mathrm{Re}[\mathbf{M}]\mathbf{F} \quad (1)$$

where $\mathbf{F}$ is the force excitation vector, $\mathbf{M}$ the complex mobility matrix of the receiver structure, and and the superscript H denotes the conjugate transpose. By matrix theories [9], $\mathrm{Re}[\mathbf{M}]$ can be decomposed into the form

$$\mathrm{Re}[\mathbf{M}] = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^{T} \quad (2)$$

where $\mathbf{\Lambda}$ is a real and non-negative diagonal matrix of the eigenvalues of $\mathrm{Re}[\mathbf{M}]$, $\mathbf{\Psi}$ the normalized orthogonal matrix composed of the corresponding eigenvectors (in columns), and the superscript T denotes the transpose. Therefore $\mathbf{\Psi}\mathbf{\Psi} = \mathbf{I}$. Let the force vector $\mathbf{F}$ be weighted by $\mathbf{\Psi}$ so as to give a new set of power modal forces defined by

$$\mathbf{Q} = \mathbf{\Psi}^{T}\mathbf{F} \quad (3)$$

It follows that

$$\sum_{n=1}^{N}|Q_n|^2 = \sum_{n=1}^{N}|F_n|^2 \quad (4)$$

Combining equations (1), (2) and (3), the transmitted power to the receiver can be written as

$$P = \frac{1}{2}\sum_{n=1}^{N}|Q_n|^2 \lambda_{nn} \quad (5)$$

Equation (5) shows that the vibration power input to the receiver by N forces can be regarded as the power input by N independent power modes by transforming the forces $\mathbf{F}$ into power modal forces $\mathbf{Q}$ and describing the receiver by a set of power modal mobilities λ_{nn}. It is seen then that the power modes of a structure are analogous to the “radiation modes” sometimes used to describe the power radiated by a vibrating surface into a surrounding acoustic medium [10]. Instead of there being N^2 terms contributing to the power P in the “multipole” approach [7-8], there are only N terms, one for each power mode, while fewer terms are involved.

Conceptually power modes are very helpful but they have no practical advantages over the conventional mobility matrix method since full knowledge of $\mathrm{Re}[\mathbf{M}]$ is required to determine its eigen-properties. However, advantages do occur because simple approximations can be developed for the upper and lower bounds of P. From equations (4) and (5), the upper and lower bounds are

(6)

$$P_{low} = \frac{1}{2}\left(\sum_{n=1}^{N}\left|F_n\right|^2\right)\lambda_{min} \qquad (7)$$

where λ_{max} and λ_{min} are the maximum and minimum eigenvalues of $\mathrm{Re}[\mathbf{M}]$, respectively. It is seen then that the bounds on power are independent of the force distribution and the relative phases of the forces. Equations (6) and (7) were also given in [6].

For structures with low mode-count (e.g. low and mid-frequency vibration), however, λ_{max} tends to be much bigger than λ_{min}, especially at the resonant frequencies where $\lambda_{max} \to \sum_{n=1}^{N}\mathrm{Re}\{M_{nn}\}$ and $\lambda_{min} \to 0$ [4]. As a result the power band will be too broad to be of practical value. For these cases, it is more appropriate to replace the strict lower bounds by the approximate first order power mode [4].

Moreover, the mean value of the power transmitted to the receiver, when averaged over all the power modes, can then be approximated in terms of the mean square force and the mean point mobility as

$$\bar{P} = \frac{1}{2}\left(\sum_{n=1}^{N}\left|F_n\right|^2\right)\mathrm{Re}\left(\sum_{n=1}^{N}M_{nn}\Big/N\right) \qquad (8)$$

Equation (8) is in agreement with the result of [3].

3. APPROXIMATION OF THE POWER TRANSMISSION FROM A STIFF SOURCE TO A FLEXIBLE RECEIVER

Based on the power mode theory described in the previous section, the power transmitted to a flexible receiver from a stiff source structure through discrete couplings can be predicted rather simply and accurately by approximating its upper and lower bounds as well as its frequency average value, which depends only on the diagonal elements of the mobility matrices of the source and the receiver. The main steps are described below with full details given in [5].

For a multipoint-coupled source/receiver system, the power transmitted to the receiver substructure is given by [5]

$$P = \frac{1}{2}\mathrm{Re}\left\{\mathbf{V}_{sf}^{\;H}\left[\left(\mathbf{M}_S + \mathbf{M}_R\right)^{-1}\right]^H \mathbf{M}_R\left(\mathbf{M}_S + \mathbf{M}_R\right)^{-1}\mathbf{V}_{sf}\right\} \qquad (9)$$

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free velocity has been used to define the source strength, which has the advantage of allowing simple comparisons between different sources [3]. Physically an approximation for the maximum power transmission is the power when the source substructure is rigid so that the receiver structure is in effect excited by a set of free velocity sources directly. Therefore a convenient approximation to the upper bound of the transmitted power is

$$P_{up} = \frac{1}{2}\mathrm{Re}\left\{\mathbf{V_{sf}}^H \mathbf{M_R}^{-1} \mathbf{V_{sf}}\right\} \qquad (10)$$

The interface force acting on the receiver is given in [5] as

$$\mathbf{F_I} = \left(\mathbf{M_R} + \mathbf{M_S}\right)^{-1} \mathbf{V_{sf}}$$

(11)

It follows that

$$\sum_{n=1}^{N} \left|F_{I,n}\right|^2 = \mathbf{V_{sf}}^H \left[\left(\mathbf{M_R} + \mathbf{M_S}\right)^{-1}\right]^H \left(\mathbf{M_R} + \mathbf{M_S}\right)^{-1} \mathbf{V_{sf}} \qquad (12)$$

Equation (12) has a positive definite quadratic form, and hence its strict lower bound can be found using steps similar to those described in Section 2. It is such that

$$\sum_{n=1}^{N} \left|F_n\right|^2 \geq \left(\sum_{n=1}^{N} \left|V_{sf,n}\right|^2\right) \frac{1}{\lambda_{\max}^{RS}} \qquad (13)$$

where $\lambda_{\max}^{RS}$ is the maximum magnitude of the eigenvalues of $\left(\mathbf{M_R} + \mathbf{M_S}\right)\left(\mathbf{M_R} + \mathbf{M_S}\right)^H$. From [11], it follows that

$$\sqrt{\lambda_{\max}^{RS}} \leq \max_n \left|M_{R,nn} + M_{S,nn}\right| + \sqrt{\sum_{n \neq m}\sum \left|M_{R,nm} + M_{S,nm}\right|^2} \qquad (14)$$

where $M_{R,nn}$ and $M_{S,nn}$ are the (n,n) diagonal terms of matrices $\mathbf{M_R}$ and $\mathbf{M_S}$, and $M_{R,nm}$ and $M_{S,nm}$ the (n,m) off-diagonal terms, respectively. Combining equations (7), (9), (13) and (14), a strict lower bound of power can be determined as

$$P_{low} = \frac{\frac{1}{2}\left(\sum_{n=1}^{N}\left|V_{sf,n}\right|^2\right)\lambda_{\min}^{R}}{\left(\max\left|M_{\cdot} + M_{\cdot\cdot}\right| + \sqrt{\sum\sum\left|M_{\cdot} + M_{\cdot\cdot}\right|^2}\right)^2}$$

where $\lambda_{\min}^{R}$ is the smallest eigenvalue of the mobility matrix $\mathrm{Re}\left[\mathbf{M_R}\right]$.

For a very flexible and/or heavily damped receiver, the point mobility tends to be much larger than the transfer mobility so that the points can be taken as "uncoupled", i.e.,

$$\mathbf{\Lambda_R} \approx \mathbf{M_R},\ \mathrm{Re}\left[\mathbf{\Lambda_R}\right] \approx \mathrm{Re}\left[\mathbf{M_R}\right] \tag{16}$$

The upper- and lower-bounds of the transmitted power can then be approximated by

$$P_{up} \approx \frac{1}{2}\sum_{n=1}^{N}\left|V_{sf,n}\right|^2 \mathrm{Re}\left\{\frac{1}{M_{R,nn}}\right\} \tag{17}$$

$$P_{low} \approx \frac{1}{2}\left(\sum_{n=1}^{N}\left|V_{sf,n}\right|^2\right)\frac{\min_n\left\{\mathrm{Re}\left[M_{R,nn}\right]\right\}}{\left(\max_n\left|M_{R,nn}\right| + \left(1+\sqrt{N(N-1)}\right)\max_n\left|M_{S,nn}\right|\right)^2} \tag{18}$$

It is seen then that these bounds are simply determined by only the diagonal elements of the mobility matrices, and that the stiffer the source and/or the more flexible the receiver, the closer are the values of the upper and lower bounds, i.e., the narrower the range between these limits. Therefore both the upper and the lower bounds of the transmitted power can be quite close to the "exact" value of the transmitted power when the mobility mismatch between the source and the receiver is big enough. However, since the approximation for the upper bound is found by assuming that the source only generates free velocities at the interfaces whereas the lower bound is given by assuming a general linear source [12], the lower bound is more accurate than the upper bound. It is known the average point mobility of a finite structure equals that of the equivalent infinite structure [13]. Therefore it is reasonable to expect that the frequency average of the power transmitted from a stiff source to a finite flexible receiver can be simply and accurately approximated by

$$P \approx \frac{1}{2}\left(\sum_{n=1}^{N}\left|V_{sf,n}\right|^2\right)\frac{\mathrm{Re}\left\{M_{R,nn}^{\infty}\right\}}{\left(\left|M_{R,nn}^{\infty}\right| + \left(1+\sqrt{N(N-1)}\right)\max_n\left|M_{S,nn}\right|\right)^2} \tag{19}$$

where $M_{R,nn}^{\infty}$ corresponds to the point mobility of the equivalent infinite receiver. The bigger the mobility (or stiffness) mismatch between the source and the receiver, the more accurate is this approximate expression. Finally, if the source structure is much stiffer than the receiver structure so that

$$\max_{n}\left|M_{R,nn}\right| >> \left(1+\sqrt{N(N-1)}\right)\max_{n}\left|M_{S,nn}\right| \tag{20}$$

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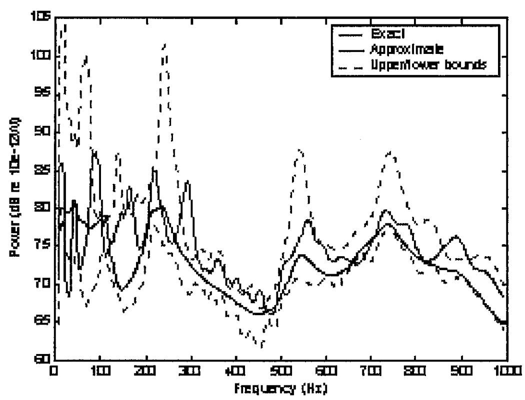
In the above sections, only the power transmitted by forces has been considered, this usually being the most significant [3]. However, the power transmitted by other components of motion, particularly moments, can also be included. Under such circumstances, the power transmission due to moment excitations can be approximated by analogy with equations (17), (18) and (19), in which the free velocity vector is composed of a set of rotational "velocities" and the mobility corresponds to the frequency response function between the moment excitation and the rotational velocity.

4 NUMERICAL EXAMPLE



In the previous section various expressions for the transmitted power were developed. These included strict bounds for the power together with approximations for these bounds which are much less conservative. Three approximate expressions have been derived to predict the power transmitted from a stiff source to a flexible receiver. These are given by equations (17), (18) and (19).

A numerical example is considered in this section. Figure 1 shows a point-coupled beam-plate system connected at four evenly spaced points. The beam has a length of 2m in the x-direction, a width of 0.059m in the y-direction, and a height of 0.068m. A unit harmonic force acts at a distance $\xi = 0.73$ m to one end of the beam. The plate has the same length as the beam in the x-direction with a width of 0.9m in the y-direction. Three different plate thicknesses of 0.010m, 0.005m and 0.002m are used to investigate how stiffness mismatching between the source and the receiver affects the accuracy of the approximations. The material of the system is chosen to be perspex with a Young's modulus of 4.4×10^9 N/m^2, a density of 1152kg/m^3, a material loss factor of 0.05 and a Poisson's ratio of 0.38.

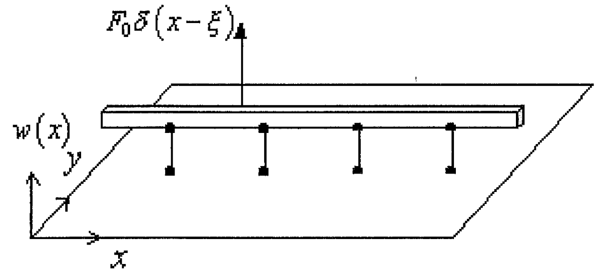


Figure1 Point coupled beam-plate system

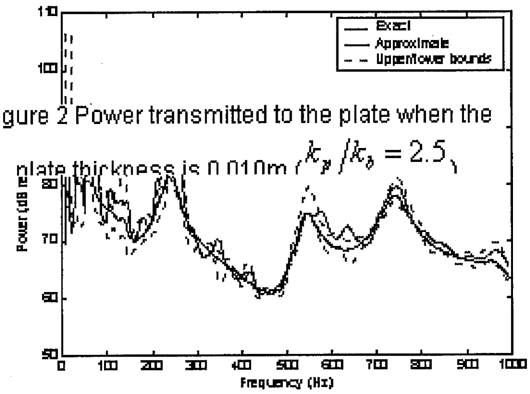


Figure 2 Power transmitted to the plate when the plate thickness is 0.010m ($k_p/k_b = 2.5$)

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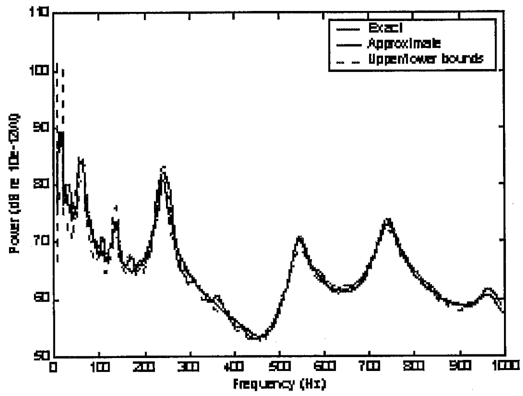
Figure 3 Power transmitted to the plate when the plate thickness is 0.005m ($k_p/k_b = 3.5$)

shown in Figures 2-4 corresponding to plate thicknesses of 0.010m, 0.005m and 0.002m, respectively. A running frequency average technique has been used in the calculations to determine the broad features of the transmitted power. The bandwidth used in the frequency averaging is 10 Hz wide so that each band consists of a few (plate) vibration

Figure 4 Power transmitted to the plate when the plate thickness is 0.002m ($k_p/k_b = 5.6$)

respectively.) It can be seen from Figures 2-4 that the range between the approximations to the upper and lower bounds becomes smaller as the plate receiver gets more flexible, as we would expect. When the receiver becomes more flexible compared to the source, or the source mobility gets smaller compared to that of the receiver, the source structure exhibits more velocity-source-like behaviour [12], i.e., the source-generated velocities at the interface DOFs become less affected by the generated interface forces. The source then behaves almost as a set of free velocity sources. As a result the transmitted power can then be simply approximated by that transmitted by a set of free velocities sources, as shown in Figure 4. In this case, the receiver is actually behaving in a "fuzzy" manner [14-15]. Figures 2-4 also show clearly that the frequency average power transmitted to the receiver can be rather accurately approximated using equation (19) provided the mobility mismatching between the source and the receiver is big enough, as shown in Figures 3 and 4 (e.g. the wavenumber of the plate is no less than three or four times of that of the beam). Otherwise the approximation tends to underestimate the frequency average value of the transmitted power, as shown in Figure 2.



5. CONCLUSIONS

This study was an attempt to simplify the estimation of the transmitted power from a stiff source to a flexible receiver by discrete point coupling. Three new approximate expressions for the power transmission were derived using the power mode method. The main results are as follows.

(1) The maximum and minimum power transmission can be simply approximated by upper and lower bounds. The usefulness of these upper and lower power bounds depends on the range of power they cover. The bigger mismatch between the source and the receiver, the more narrow is this band.

(2) The frequency average power can be approximated quite accurately provided the stiffness misr A Power Mode Technique For Estimating Power Transmission – L Ji, BR Mace, RJ Pinnington n tenc

(3) When the receiver structure is much more flexible than the source and behaves in a "fuzzy" manner, the power can be regarded as transmitted mainly by a set of free velocities.

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