# COMPARISON OF STATIC AND DYNAMIC APPROACHES FOR URBAN TRAFFIC NOISE ASSESSMENT: A CASE OF STUDY IN LYON (FRANCE)

L Leclercq A Can J Lelong Université de Lyon, INRETS, ENTPE, Laboratoire Ingénierie Circulation Transport Ghent University, Department of Information Technology, Acoustics Group

INRETS, Laboratoire Transports et Environnement

#### 1 INTRODUCTION

Traffic management can be a very efficient tool to fight against urban traffic noise. Observations have allowed for a first quantification of traffic management impact on noise. To improve such initiatives, it is crucial to take profit of a precise noise estimation model, able to assess precisely the environmental impact of traffic strategies. A-weighted sound pressure level LAeq or noise distributions estimations can be statistically derived from acoustical and traffic measurements [1]. This method can be helpful to describe urban noise, but it forbids traffic management investigations, as it is limited to specific traffic situations. Models based on a coupling between traffic data estimation (flows, speeds, etc.), noise emission laws and a sound propagation calculation, are more efficient as they can predict the changes in traffic characteristics induced by the planed traffic strategies. Their accuracy is closely linked to the traffic representation. Classical models are usually based on a static representation of traffic, which is considered as a steady flow [2]; noise levels are then estimated from flow rates and mean flow speeds. Such models are relevant to assess noise levels in inter-urban conditions. However this traffic representation is less accurate for urban traffic noise estimation, especially close to traffic signals, where traffic conditions vary a lot. Some classical models have been refined to account for traffic flow specificities in urban area. Corrections for interrupted traffic flow [3] and intersections [4] have for example been introduced, deduced from queue lengths determination. Noise estimation close to intersections can be refined once more by considering the mean vehicle trajectories [5]. However, those models are limited to energetic indicators estimation, like LAeq or Lden. Those indicators are not always sufficient to precisely describe urban traffic noise [6], which is characterized by a strong dynamic notably set by traffic signals [7]. Specific indicators have been proposed in [6] to capture this dynamics.

The breakthrough in traffic noise estimation comes with dynamic models, which output LAeq, but also instantaneous sound pressure levels [8-10]. Those models are based on a dynamic representation of traffic that gives at each time step (usually 1s) position, speed and acceleration of each vehicle on the network [8;11-12]. LAeq,1s evolution is then estimated from those data, from which classical but also specific indicators can be calculated.

This paper focuses on the relevance of traffic representations for noise assessment in urban area. Two traffic representations are compared: (i) a static one and (ii) a dynamic one provided by a traffic simulation tool (Symuvia / Symubruit). These two methodologies are applied on a real urban corridor (Cours Lafayette in Lyon) and compared to on-field noise levels. We will demonstrate how the dynamic approach outperforms the static one even for mean noise levels calculation. Moreover, the dynamic approach makes it possible to estimate more refined indicators like LAeq,1s distributions. Finally, we will show that the dynamic approach is more precise for assessing noise frequency spectrum as the static approach underestimates dramatically low frequencies, notably near traffic lights.

# 2 METHOD

## 2.1 Experimentation

The experimentation consists in traffic and acoustic measurements from 15.30 to 17.30 on a weekday. The site is a major arterial (Cours Lafayette, Lyon, France). This is a one way three-lane road (the shoulder lane is shared by buses and passenger cars) crossed by 5 intersections. The street is U-shaped with 5-floor buildings. It is quite busy, with about 1400 vehicles per hour during

the experiment. The perpendicular arterial Cours Saxe is also busy, with about 1000 veh/h, including 200 veh/h that turn right into the Cours Lafayette. Flow rates on other perpendicular streets are about 250 veh/h. The signal cycle duration is equal to 90s and traffic signals are coordinated through a green wave. The durations of green and red phases are given in Fig 1.

The recorded traffic data are the number of vehicles at each intersection for each movement and at each traffic cycle, and the precise bus trajectories (including stopping time at bus stations). Acoustic recordings are the  $L_{Aeq,1s}$  evolution for the selected points. The eight selected points for acoustical measurements are typical of urban situations:

- in front of a bus station downstream to a traffic signal (P<sub>1</sub>),
- between two consecutive traffic signals (P<sub>2</sub>),
- close to a traffic signal: in front of (P<sub>3</sub> and P<sub>6</sub>) and downstream (P<sub>4</sub>, P<sub>5</sub> and P<sub>7</sub>),
- Set back from the major street (P<sub>B</sub>) (flow rate on the perpendicular street is 250 veh/h).

Note that spectrum measurements are only performed at points P<sub>1</sub>, P<sub>6</sub>, P<sub>7</sub> and P<sub>8</sub>. Measurement points are 2m high. Their exact location is given in Fig 1Fig 1.

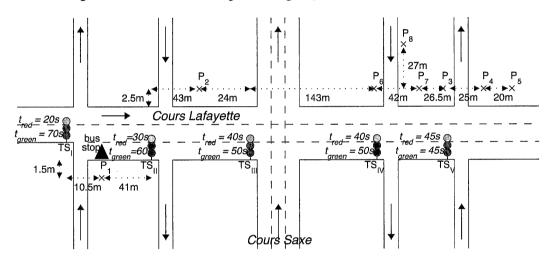


Fig 1: experimental site. Position of traffic signals TS and their green time  $t_{green}$  and red time  $t_{red}$  durations.

## 2.2 Noise estimation

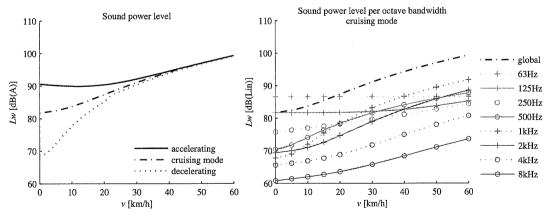


Fig 2: Noise emission laws for light vehicles: accelerating (a=0.8m/s<sup>2</sup>), cruising (a=0m/s<sup>2</sup>) or decelerating (a=3m/s<sup>2</sup>).

Each acoustic value  $L_x$  consists in an 8 elements vector, which correspond to the values of the octave bands with center frequencies from 63 Hz to 8 kHz:  $\{L_{x,63};...; L_{x,8k}\}$ . Emission and propagation are computed for each octave. Each lane of the traffic network is divided into noise cells i, which are characterized by their sound power level  $L_{w,i}$ . Cell lengths are between 9 and 18

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m.  $L_{w,i}$  is calculated by gathering the emissions of all vehicles present inside the cell. The Harmonoise model is used to predict the sound power level  $L_{w,k}$  of one given vehicle k, in terms of its speed  $v_k$  and its acceleration  $a_k$  [13], see Figure 2. The way those variables are obtained depends on the traffic flow representation included into the noise prediction model (see section 2.3). The contribution  $L_{p,i}$  of each cell for a receiver P is then determined thanks to the propagation model NMPB96 implemented in Mithra [14], which gives the sound attenuation from i to P. Finally, the sound pressure level  $L_p$  at P (which is also a 8 elements vector) is the sum of the contributions of each cell:

$$L_p = 10\log\left(\sum_{i \in cell} 10^{\frac{L_{p,i}}{10}}\right) \tag{1}$$

# 2.3 Traffic flow representations

The static and the dynamic models differ on their approach and resolution. The dynamic model allows for  $L_{p,1s}$  evolution estimation as it describes the traffic variables evolution in time. The static model only allows for the  $L_{p,2h}$  estimation, as it considers global values for traffic variables over the 2h simulation period. Hence, only the former allows for specific indicators calculation, based on  $L_{p,1s}$  values.

# 2.3.1 Static representation

Two classes c of vehicles are considered: the light vehicles lv and the buses bus, defined by their flow rates  $Q^{lv}$  and  $Q^{bus}$  respectively. Sound power level  $L_W^c$  of a given cell is deduced from the mean speed  $v^c$ , acceleration  $a^c$  and flow rate  $Q^c$  on the cell:

$$L_W^c = L_W(v^c, a^c) + 10\log(Q^c/v^c)$$
 with  $c = \{lv, bus\}$  (2)

No acceleration or deceleration zones are considered in usual static models: vehicles are supposed to pass through intersections without stopping. Hence,  $a^c=0$  whatever the cell is. Finally, global noise emission  $L_W^{lv}$  of the cell is the acoustical sum  $\oplus$  of the emissions  $L_W^{lv}$  and  $L_W^{bus}$  of the cell:

$$L_W = L_W^{l_V} \oplus L_W^{bus} \tag{3}$$

# 2.3.2 Dynamic representation

Dynamic traffic models aim at predicting how key traffic variables evolve along the network. The model used in this study is SYMUVIA (jointly developed by INRETS and ENTPE), which is based upon a detailed and individualized vehicle representation. SYMUVIA is involved in the simulation package SYMUBRUIT that dynamically estimates noise in urban area [10;15]. It gives position  $x_k(t)$ , speed  $v_k(t)$  and acceleration  $a_k(t)$  of each vehicle k on the network at each time step (usually about 1s). Motion of vehicles on the network is governed by three parameters: the maximal speed u reached when traffic is free, the wave speed w at which a starting wave spills back on the network (a starting wave can be easily observed when successive vehicles start driving at a green light), and the minimum spacing  $s_m$  between two vehicles, observed when vehicles are stopped for example at a traffic signal. Position of a vehicle k at the next time step  $x_k(t+\Delta t)$  is the minimum between the position it is willing to reach when traffic is free and the position it cannot overpass when traffic is congested. The time-step is fixed to  $\Delta t = s_m / w$  to avoid numerical viscosity [15]. Then:

$$x_{k}(t + \Delta t) = \min \left( \underbrace{x_{k}(t) + u\Delta t}_{\text{position when traffic is free position when traffic is congested}}, \underbrace{x_{k-1}(t) - s_{m}}_{\text{position when traffic is free position when traffic is congested}} \right)$$
 (4)

Speed  $v_k(t)$  and acceleration  $a_k(t)$  are then deduced from positions  $x_k(t)$  and  $x_k(t+\Delta t)$ . Noise power level  $L_{W,k}(t)$  of a vehicle is then calculated from its speed and acceleration at t. Finally, the noise power level  $L_{W,k}(t)$  of a cell i is the acoustical sum of noise emissions of vehicles on the cell at t:

$$L_{W,i}(t) = 10 \log \left( \frac{\sum_{k \in i} 10^{\frac{L_{W,k}(t)}{10}}}{l_i} \right)$$
 (4)

where  $l_i$  is the length of the cell. The model has been refined to take into account the bounded acceleration of vehicles [17], the influence of slow motion of buses [18], the lane-changing phenomena [19], and conflicts at junctions [20-21].

#### 2.3.3 Calibration

The static models have been calibrated to fit the on field observations. Vehicle kinematic parameters are: an average deceleration rate  $d=-3\text{m/s}^2$  and an average acceleration rate  $a=0.8\text{m/s}^2$ . Note that the low acceleration value could be due to the traffic signal settings, which incite vehicles to accelerate slowly to benefit from the green wave. The other vehicle properties are its wave speed w=-3.33m/s, minimum spacing  $s_m=5\text{m}$ , and maximal speed u. The maximal speed of light vehicles depends on the location on the network:  $u_1=17\text{m/s}$  at the beginning of the Cours Lafayette (up to the second intersection),  $u_2=15\text{m/s}$  at the end of the Cours Lafayette (after the second intersection), and  $u_3=10\text{m/s}$  on the crossing roads. The maximal speed of buses is  $u_{\text{bus}}=10\text{m/s}$ . Finally, a constant 51dB(A) noise was added to take account of the background noise.

#### 2.3.4 Indicators

The indicators considered in this study are the energetic indicator  $L_{Aeq}$ , statistical indicators ( $L_1$ ,  $L_{10}$ ,  $L_{50}$ ,  $L_{90}$ ) and  $L_{Aeq,1s}$  distributions. The noise level spectrum is also proposed and compared to noise rating curves. Those curves have been developed by the International organization of Standardization to rate noisiness. Each x dB NR<sub>x</sub> curve is built as follows: the value NR<sub>x,Bi</sub> allocated to the octave bandwidth Bi is the sound level that a sound at the frequency Bi should have to be as noisy as a sound of x dB(Lin) at 1kHz (thus NR<sub>x,1kHz</sub> = x).

#### 3 RESULTS

# 3.1 L<sub>Aeq</sub> estimation

 $L_{\rm Aeq}$  estimations and measurements are depicted in Tab 1. The coarse static calculation seems insufficient fore  $L_{\rm Aeq}$  estimation even if a 3dB(A) error is accepted. The model overestimates noise levels since it considers that all vehicles go at their free speed u all along the network. The estimation is correct in the entrance of the network (1.5dB(A) error at P<sub>1</sub>), where vehicle speeds are high indeed. Yet, error is larger farther, where vehicles are actually slowed down by green wave and stops at intersections.

·	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>7</sub>	P <sub>8</sub>	mean
measurement	73.8	73.5	70.3	71.2	71.2	71.3	71.2	64.2	error
static representation	75.1	76.5	74.4	74.3	74.4	74.8	74.6	73.2	3.8
dynamic representation	75.8	71.9	70.6	70.3	70.7	70.2	70.2	61.0	1.3

Tab 1 : LAeq estimation at the 8 points of experimentation; in clear green: error exceeds 1dB(A); in dark grey: error exceeds 2dB(A); in black: error exceeds 3dB(A)

The dynamic model guarantees a precise estimation of  $L_{\text{Aeq}}$  (error falls below 2dB(A) for all the points except the point set back from the major street), since the specificities of the traffic flow in urban area (queue formation and discharge at each traffic signal, platoons of vehicles behind buses, etc.) are represented. Estimation seems particularly precise in front of the traffic signal: error falls below 1dB(A) for  $P_3$  to  $P_7$ , what can be linked to a precise localization of accelerating zones.

Moreover, the noise decrease at those five points is underlined by the dynamic model while it is not by static models: it is due to vehicles that arrive from the Cours Saxe and turn right into the Cours Lafayette. Those vehicles slow down traffic flow, what cannot be seen by the static models. Finally, the slight underestimation of noise levels with the dynamic model is due to peaks of noise (such as klaxons), which increase  $L_{\text{Aeq}}$  and are not taken into account by the model.

# 3.2 Specific indicators estimation with Symubruit

Tab	measurement									
	P1	P2	P3	P4	P5	P6	P7	Р8		
$L_{Aeq}$	73.8	73.5	70.3	71.2	71.2	71.3	71.2	64.2		
L <sub>1</sub>	81.8	80.6	78.3	79.7	79.2	79.2	79.3	71.0		
L <sub>10</sub>	<b>7</b> 6.0	74.9	<b>7</b> 3.6	74.3	74.6	73.8	74.3	64.6		
L <sub>50</sub>	71.4	69.7	67.1	67.8	67.5	68.5	68.9	61.0		
L <sub>90</sub>	64.4	63.4	61.1	60.1	58.1	63.6	60.8	57.2		

Dynamic noise prediction model										
P1	P2	Р3	P4	P5	P6	P7	P8			
75.3	71.9	70.6	70.3	70.7	70.2	70.2	61.0			
86.3		78.0	78.1	78.2	77.5	77.4	67.3			
77.7	75.7	74.0	74.1	74.9	73.7	73.5	64.2			
71.8	70.1	68.0	66.8	67.4	68.2	68.0	59.9			
r si	61.6	62.6	59.4	57.2	63.3	60.2				

Tab 2: specific indicators estimation with dynamic model at the 8 points of experimentation; in white: error is under 1dB(A); clear grey: error is between 1dB(A) and 2dB(A); in dark grey: error is between 2dB(A) and 3dB(A); in black: error exceeds 3dB(A).

Noise characterization can be refined with a dynamic traffic representation, which allows specific indicators estimation; see Tab 2. The  $L_{Aeq,1s}$  distributions are also estimated with a strong accuracy; see Fig 3. The two modes of the distributions, which are due to green and red phases of the traffic signal and are typical of one-lane roads noise dynamics [6], are well reproduced by the model.

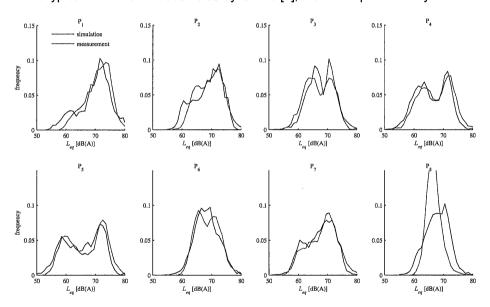


Fig 3:  $L_{Aeq,1s}$  distributions from measurements and from the dynamic noise prediction model.

Statistical indicators are also precisely estimated. However,  $L_1$  is overestimated close to the bus station at  $P_1$ , which may be due to an overestimation of the noise emitted by the bus when staying at and leaving the bus station. This may be due to the inadequacy of buses acceleration when constructing buses noise emission laws compared to their acceleration in real traffic conditions. Other statistical indicators are estimated with an error under 3dB(A) for all points (except for the point set back from the major street), and with an error under 1dB(A) for most of the points.

# 3.3 Spectrum analysis

Spectrum measurements are only available for points  $P_1$ ,  $P_6$ ,  $P_7$  and  $P_8$ . The sound spectra estimated at these four points thanks to the static and the dynamic models are compared to

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measurements in Fig 4. Sound spectra of the four points present similarities. For instance the sound levels in dB(Lin) tend to decrease with the frequency. The decrease between the 63 Hz and the 8 kHz sound levels reaches at least 20 dB for the four points. This can be explained by the road traffic noise spectrum, which contains more low than high frequencies (see Fig 2). This results in a low Spectrum Gravity Centers (SGC) for the four points, between 266 Hz and 375 Hz (see Tab 3).

Moreover, spectra show a peak at 1 kHz, which is caused by vehicles cruising at their free speed and may correspond to tire-road contact noise. It can effectively be seen on Fig 2 that this frequency is predominant for speeds above 40 km/h. This peak causes high NR values (except for the point  $P_8$  that stands in retreat of the main road), since this indicator reflects the most loud octave bandwidth. Note that the 1 kHz peak is less pronounced at  $P_6$  that is in front of the traffic signal, because many vehicles have to stop at this point, and pass accelerating at low speeds, for which the 1kHz octave bandwidth is less energetic.

Finally, the sound levels at  $P_8$  decrease faster with the frequency, mainly because of its specific location. This point is indeed less noisy than the others since it is set back from the street; but it still contains lots of low frequencies probably due to urban background noise. Those low frequencies explain why the SGC is lower at this point than for the other points.

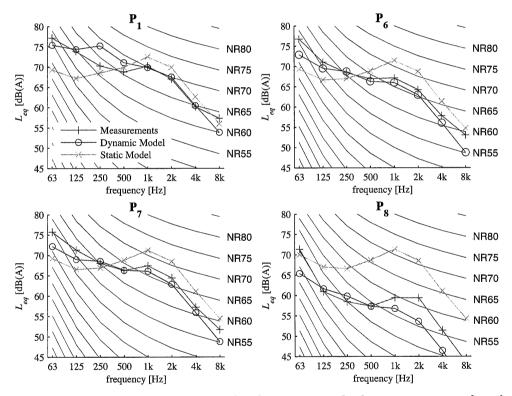


Fig 4: Equivalent sound pressure level spectrum at the four measurement locations from measurements and both static and dynamic models estimates

	NR				SGC			
	P <sub>1</sub>	$P_6$	$P_7$	P <sub>8</sub>	P <sub>1</sub>	P <sub>6</sub>	P <sub>7</sub>	P <sub>8</sub>
Measurements	70	67	67	62	376	288	315	267
Dynamic Model	70	66	66	57	383	353	379	288
Static Model	72	71	71	71	921	875	866	837

Tab 3: Noise Ratings values and Sound Gravity Spectrum from Measurements and both static and dynamic models estimates

#### 3.3.1 Static Model

The spectra assessed by mean of the static model tend to have the same envelope whatever the traffic situation is. It corresponds to the noise emitted by a flow of vehicles moving at their free speed. Since it cannot capture the specificities of urban traffic flow (stops of vehicles at traffic signals, speed variations along the network due to traffic, etc.), this model fails in reproducing the real spectra envelope that correspond to each traffic situation. In particular, the low frequencies, which are mainly due to stops and slow vehicles, are underestimated by the static model. This results in a poor estimation of SGC, which is moved forward too high frequencies (errors exceed 100%), due to the speed overestimation. Finally, the NR estimation is also biased by the speed overestimation: sound levels around 1kHz are overestimated, which fix NR into a too high value.

#### 3.3.2 Dynamic Model

The dynamic model improves the estimation of spectrum envelopes, since it takes speed variations set by the traffic flow into account. The low frequencies emitted at slow speeds and the 1 kHz frequencies mainly emitted at free flow speed can thus be reproduced by the model. This improvement in the estimation of vehicles kinematics also results in an improvement of the SGC estimation: errors fall between 2 to 18%. Nevertheless, the estimation of the SGC could be improved at the points  $P_6$  and  $P_7$ , where the 63Hz bandwidth sound level is underestimated. This error may be the result of an underestimation of the background noise at those points. Finally, the right consideration of the proportion of vehicles that move at free speed improves the 1 kHz bandwidth sound level estimation. It results in a precise estimation of the NR, which is "fixed" by this frequency, as it is most often the noisiest one. NR is indeed estimated with errors under 1 dB(A) for the points located on the main road ( $P_1$ ,  $P_6$  and  $P_3$ ). NR estimation is not as good for the point  $P_8$ , mainly because high frequencies are underestimated. This underestimation might be related to specific propagation phenomena (the site here is large with trees close to  $P_8$ ) or to an underestimation of vehicles speed on this secondary road.

# 4 CONCLUSION

Two traffic representations have been tested in this paper. Comparison was achieved by confronting estimates to on-field data, collected at 8 points corresponding to different traffic situations: close to a bus station, in front of a traffic signal, down a traffic signal, and set back from the corridor.

The coarse static representation often gives errors in  $L_{\text{Aeq}}$  estimation that exceed 3 dB(A). This representation overestimates noise levels, as it is based on a coarse estimation of vehicle mean speeds. Moreover, the static model fails in reproducing the spectra envelopes along the corridor. In particular, low frequencies sound levels are systematically underestimated, as they are mainly emitted by vehicle at slow speeds or accelerating vehicles, which are not reproduced by the static model. This could be problematic when achieving noise impact studies, especially in the presence of noise reducers, which often offer better results for high frequencies than for low frequencies. Results of the static model could be improved by using speed distributions instead of a mean speed when assessing noise emissions. This could be obtained by elaborating noise emission laws that correspond to real traffic situations.

The dynamic representation overcomes the static one. Firstly, it guarantees the estimation of the classical noise indicator  $L_{\text{Aeq}}$  with errors under 1 dB(A). Secondly, refined acoustic indicators, like the  $L_{\text{Aeq},1s}$  distribution, may then be calculated with a good accuracy. Thirdly, this representation also improves the estimation of the spectra envelopes, because it is able to capture vehicles moving slowly as well as vehicles moving at their free speed. This guarantees a precise estimation of indicators that describe the sound spectra, such as the Spectrum Gravity Center, which informs on the pitch of the sound, and the Noise Rating value, which is deduced from the value of the noisiest octave bandwidth.

The accurate estimation of urban traffic noise characteristics that enables the dynamic noise prediction model offers new perspectives in the investigation of the environmental effects of traffic strategies: (i) by closely taking traffic characteristics into account, it is more in accordance with the

precise noise emission and noise propagation that already exist, and it offers a wide variety of traffic management that can be tested, (ii) by allowing the determination of refined noise indicators, it will offer at midterm a tool able to assess physical but also perceptive impact of traffic strategies.

# 5 REFERENCES

- 1. Bazaras J, Jablonskyte J, Jotauniene E. Interdependance of noise and traffic flow. Transport, 23(1):67–72 (2008).
- 2. CERTU. Review of French road and railway noise prediction models («Catalogue des logiciels de calcul du bruit des infrastructures routières et ferroviaires, un aperçu du marché français»). Centre d'études sur les réseaux, les transports, l'urbanisme et les constructions publiques. 47 p. [in french]. (2002).
- 3. Makarewicz R, Fujimoto M, Kokowski P. A model of interrupted road traffic noise. Applied Acoustics, 57:129–37 (1999).
- 4. De Coensel B, Botteldoren D, Vanhove F, Logghen S. Microsimulation based corrections on the road traffic noise emission near intersections. Acta Acustica United Acustica, 93:241–52 (2007).
- 5. Picaut J, Bérengier M, Rousseau E. Noise impact modelling of a roundabout. In inter-noise, Rio de Janeiro, Brésil; 2005.
- 6. Can A, Leclercq L, Lelong J, Defrance J. Capturing urban traffic noise dynamics through relevant descriptors. Applied Acoustics, 69:1270–80 (2008).
- 7. Nelson P. Transportation noise. Reference book, Butterworth; 520 p., 1987.
- 8. De Coensel B, De muer T, Yperman I, Botteldoren D. The influence of traffic flow dynamics on urban soundscape. Applied Acoustics, 66:175–94 (2005).
- 9. Leclercq L. A traffic flow model for dynamic estimation of noise. Ph D report. 317 p. [in French] (2002).
- 10. Can A, Leclercq L, Lelong J, Defrance J. Accounting for traffic dynamics improves noise assessment: experimental evidence. Applied Acoustics, 70:821-829 (2009).
- 11. Rotranomo. In: Final conference on 28.09.2005. <a href="http://www.rotranomo.com/">http://www.rotranomo.com/</a> [on line].
- 12. Oshino Y, Tsukui K, Hanabusa H, Bhaskar A, Chung E, Kuwahara M. Study on road traffic noise prediction model taking into account the citywide road network. In inter-noise, Istanbul. 8 p. (2007).
- 13. HARMONOISE, Source modelling of road vehicles, work package 1.1. Project funded by the EC under the Information Society and Technology (IST) Programme. 52 p. (2004).
- 14. Mithra, Manuel technique. Mithra 5.0, 58 p. (2002).
- 15. Chevallier, E., Can, A., Nadji, M., Leclercq, L. Improving noise assessment at intersections by modeling traffic dynamics. Transportation Research Part D, 14(2): 100-110 (2009).
- 16. Leclercq, L., Laval, J., Chevallier, E. The Lagrangian coordinates and what it means for first order traffic flow models. In: Allsop, R.E., Bell, M.G.H., Heydecker, B.G. (Eds) Proceedings of the 17th International Symposium on Transportation and Traffic Theory. London: Elsevier, 735-753 (2007).
- 17. Leclercq, L. Bounded acceleration closed to fixed and moving bottlenecks. Transportation Research Part B, 41(3): 309-319 (2007).
- 18. Leclercq, L., Chanut S. and Lesort J.-B. Moving bottlenecks in Lighthill-Whitham-Richards Model: A unified theory. Transportation Research Record: Journal of the Transportation Research Board, 1883: 3-13 (2004).
- 19. Laval, J. and Leclercq L. Microscopic modeling of the relaxation phenomenon using a macroscopic lane-changing model. Transportation Research Part B, 42(6): 512-522 (2008).
- 20. Chevallier, E., Leclercq, L. Do microscopic merging models reproduce the observed priority sharing ratio in congestion? Transportation research part C, 17(3):328-336 (2009).
- 21. Chevallier, E., Leclercq, L. A microscopic dual-regime model for single-lane roundabouts. Journal of Transportation Engineering, 135(6):386-394 (2009).