AN FFT BASED METHOD FOR BROADBAND CONSTRAINED BEAMFORMING

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ABSTRACT

This paper presents a method of computing optimal weights of the time domain broadband beamformer employing a tapped delay line structure. The method is based upon narrowband beamforming concepts where signals are processed in frequency domains by multiplying the signals derived from each element by complex weights. The method offers potential for computational saving for real time signal processing in time domain processors and provides framework for comparing time domain and frequency domain processing methods under similar implementation conditions.

1. INTRODUCTION

Figure 1 shows a general set of a broadband beamforming structure consisting of L omnidirectional elements, each followed by a tapped delay line filter of length J. Assuming that the array is immersed in a homogeneous media consisting of uncorrelated directional sources, isotropic noise and white noise, the output of the array is given by

$$y(t) = \sum_{j=1}^{J} \sum_{i=1}^{L} x_i(t - T_i - (j-1)T) w_{ij} \quad (1)$$

where T_i denotes the steering delay on the ith element and is selected to steer the array in the direction of interest, $x_i(t)$ denotes the voltage induced on the ith element, T is the delay associated with each tap and w_{ij} denotes the real weight on the ith channel before jth tap. These weights are selected by solving a constrained beamforming problem.

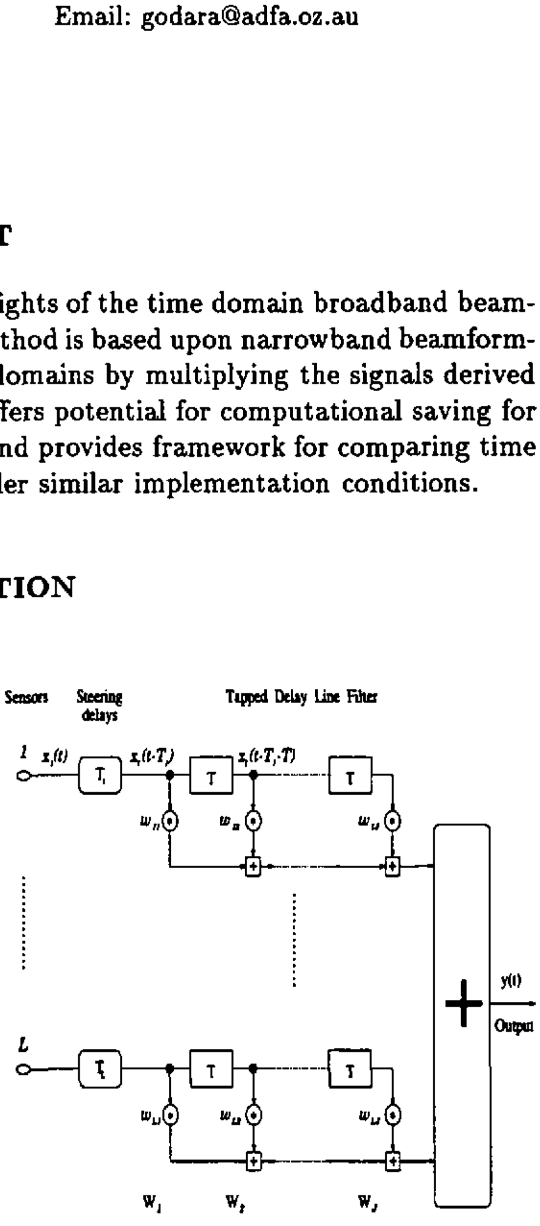


Figure 1: Broadband time domain beamformer structure.

In its simplistic form, the problem is to minimise the mean output power subject to a specified response in the direction of interest. When the desired signal is in this direction and specified response is such that the desired signal is received without distortion, the solution to this problem in the absence of errors maximises the output signal to noise ratio (SNR). Let an LJ vector $\underline{\hat{W}}$ be the solution to this problem. It is given by [1]

$$\underline{\hat{W}} = R^{-1}C(C^T R^{-1} C)^{-1}\underline{F} \tag{2}$$

where R is the $LJ \times LJ$ dimensional array correlation matrix,

$$C = \begin{bmatrix} \underline{S}_0 & 0 & \cdots & 0 \\ 0 & \underline{S}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \underline{S}_0 \end{bmatrix}, \tag{3}$$

$\underline{S}_0$ is an L dimensional vector of 1s and the J dimensional vector $\underline{F}$ specifies the frequency response in the desired direction.

2. FFT BASED METHOD

This method estimates the optimal weights of the time domain broadband constrained beamformer of Figure 1 by minimising the mean output power of each frequency bin, rather than minimising the mean output power as is done by the time domain method of the previous section. However, it maintains the same frequency response in the desired direction as is done by the time domain method. This method is similar to frequency domain schemes of narrowband beamforming where broadband time domain data are Fourier transformed and beams are formed at each frequency bin by multiplying the signals derived from each element by a complex weight and the output is produced by summing the results. The weights are then optimised by minimising the mean out power at each bin subject to a point constraint to maintain a specified response in the desired signal direction.

The main difference between this method and the frequency domain beamforming methods is that this method uses the optimised complex weights by the narrowband scheme to estimate the optimal weights of the time domain processor using a unique transformation [2]. Thus the processor is implemented in the time domain rather than in the frequency domain as is the case for narrowband schemes. Therefore, the received signal flows in the time domain structure of Figure 1 and does not encounter the delay associated with the frequency domain methods which may be important for some applications. The performance of the time domain processor using this method to estimate weights is the same as those of the frequency domain methods and this presents a framework to compare the performance of the time domain and the frequency domain methods under similar implementation conditions. Furthermore, the method offers a potential for large amounts of computational saving for real time applications due to its parallel nature as discussed later. The method uses the following algorithm to estimate the optimal weights of the time domain processor. The details of the algorithm may be found in [2].

FFT BASED METHOD

1. Estimate narrowband array correlation matrix $R_f(k)$, $k = 0, 1, \cdots, (J-1)$ using

$$(R_f(k))_{\ell,i} = \underline{e}^H(k) R_{\ell,i} \underline{e}(k) \qquad \ell, i = 1, 2, \cdots, L \tag{4}$$

where

$$e_m(k) = e^{j\frac{2\pi}{J}(m-1)k} \qquad m = 0, 1, \cdots, J-1 \tag{5}$$

and $R_{\ell,i}$ is a $J \times J$ matrix denoting the correlation between elements ℓ and i in time domain.

2. Estimate $\hat{\underline{h}}(k)$, $k = 0, 1, \cdots, (J-1)$ using

$$\hat{\underline{h}}(k) = \frac{R_f^{-1}(k) \underline{S}_0 \tilde{f}_k^*}{\underline{S}_0^H R_f^{-1}(k) \underline{S}_0} \qquad k = 0, 1, \cdots, J-1 \tag{6}$$

which are the solutions of the following narrowband beamforming problems

$$\underset{\underline{h}(k)}{Minimise} \qquad \underline{h}^H(k) R_f(k) \underline{h}(k) \tag{7}$$

$$Subject\ to \qquad \underline{h}^H(k) \underline{S}_0 = \tilde{f}_k \qquad k = 0, 1, \cdots, J-1 \tag{8}$$

where

$$\tilde{f}_k = \sum_{m=1}^{J} F_m e^{j\frac{2\pi}{J}(m-1)k} \qquad k = 0, 1, \cdots, J-1 \tag{9}$$

and ensures that the required frequency response in the desired direction is maintained.

3. Estimate the weights of the time domain structure of Figure 1 using

$$\hat{w}_{m\ell} = \frac{1}{J} \sum_{k=0}^{J-1} \hat{h}_\ell^*(k) e^{-j\frac{2\pi}{J}(m-1)k} \qquad m = 1, 2, \cdots, J \quad \& \quad \ell = 1, 2, \cdots, L. \tag{10}$$

3. PERFORMANCE COMPARISON

In this section, the performance of the time domain broadband beamformer is evaluated when the weights of the processor shown in Figure 1 are estimated by the two methods. The method is referred to as the "time domain" method when the weights are estimated using Equation (2) and it is referred to as the "frequency domain" method when the weights are estimated using equations (4)—(10). The performance of the two schemes is evaluated using a linear array of equi-spaced elements with the desired signal broadside to the array. The angle of an interference is varied from 0° to 180° under various scenarios and results are presented as the output SNR versus the

FFT BASED METHOD

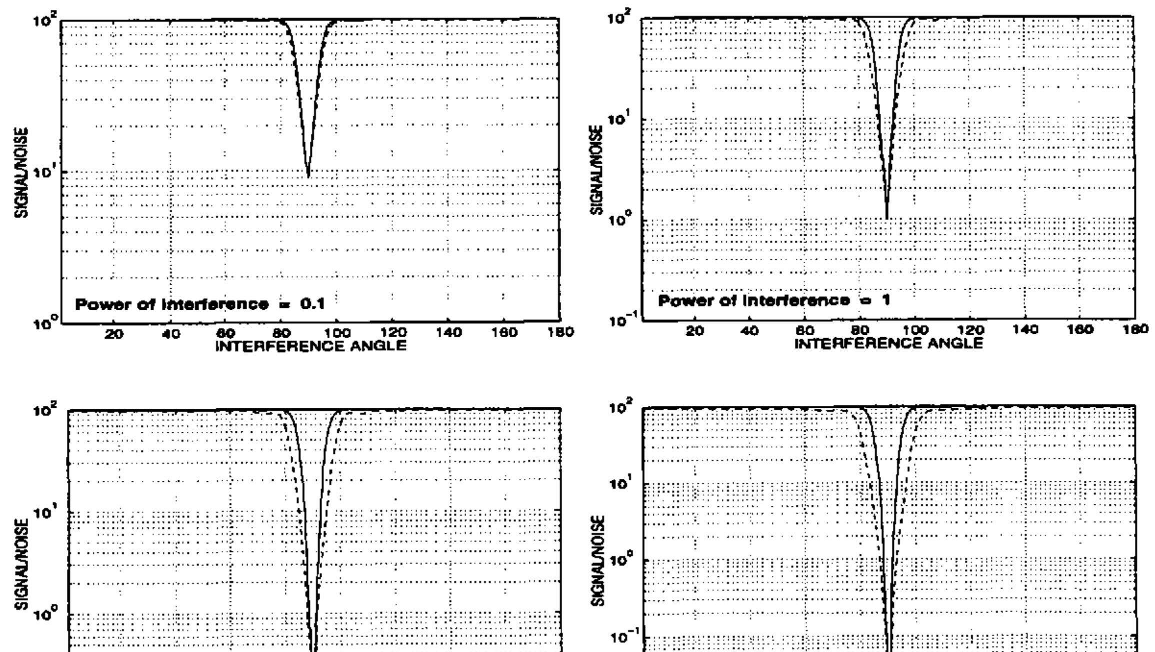
interference angle. The array is constrained to have the all-pass response in the direction of the desired signal by selecting

$$F_i = \begin{cases} 1 & i = \frac{(J+1)}{2} \\ 0 & otherwise \end{cases}$$

with filter length parameter J being an odd integer. Throughout the paper the element spacing is measured in wavelength (λ) at the highest signal frequency and the bandwidth is expressed in terms of the sampling frequency(F_s). All the sources are assumed to have the brick-wall type spectrum.

Figure 2 shows the effect of interference power on the performance of the two processors.

"Time Domain(——), Frequency Domain(- - - -)"



No. of elements (L) = 10
Element spacing (d) = $\frac{1}{2}$ ($\times\lambda$)
Power of the signal = 1.0
Power of the isotropic noise = 0.0

Length of the tapped delay line filter(J) = 11
Bandwidth (BW): ($f_1 = 0.45, f_2 = 0.5$) ($\times F_s$)
Power of the white noise = 0.1

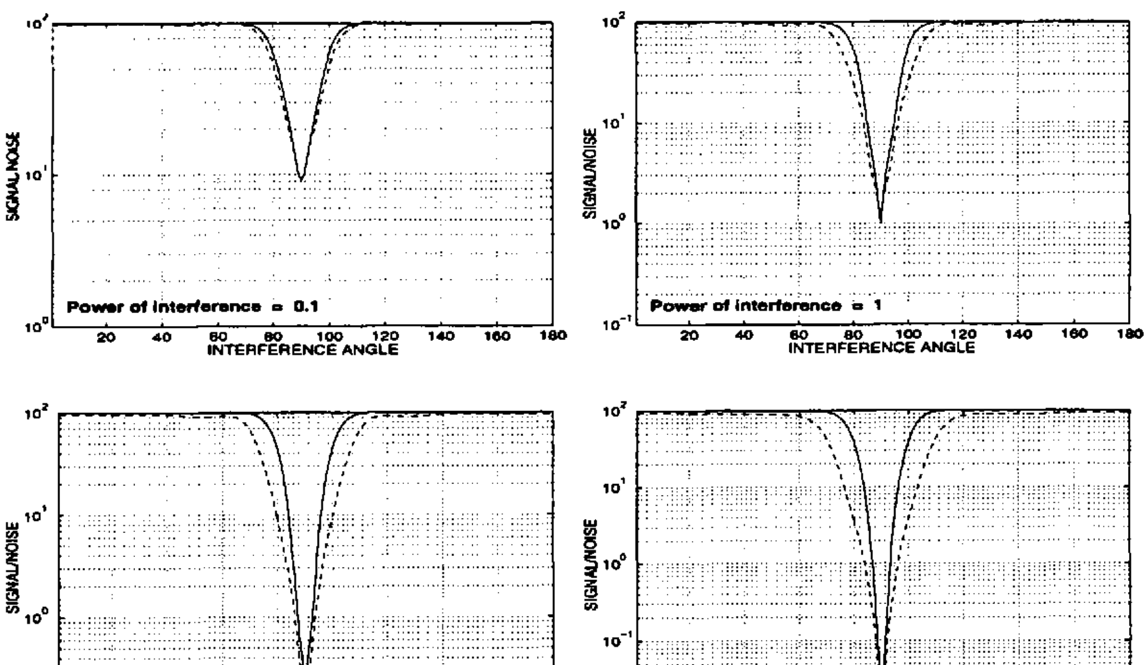
Figure 2

FFT BASED METHOD

One observes from the figure that when the interference power is of the order of the white noise power, the performance of the two processors is very close to each other. However, in the presence of a strong interference (compared to the white noise) the performance of the frequency domain method is worse than that of the time domain method, particularly when the interference is close to the desired signal. The reason of this performance loss is that the frequency domain method minimizes the mean power at frequency bin rather than the total power as is done by the time domain method. However, it is possible to improve the performance of the frequency domain method by increasing the filter length. This aspect is discussed later in the paper.

Figure 3 shows the results when element spacing has been reduced by half. The other parameters are the same as those for Figure 2. A comparison of the two figures reveals that the region of the performance loss of the frequency domain method widens when element spacing is reduced.

"Time Domain(—), Frequency Domain(- - - -)"



No. of elements (L) = 10
Element spacing (d) = $\frac{1}{4}$ ($\times\lambda$)
Power of the signal = 1.0
Power of the isotropic noise = 0.0

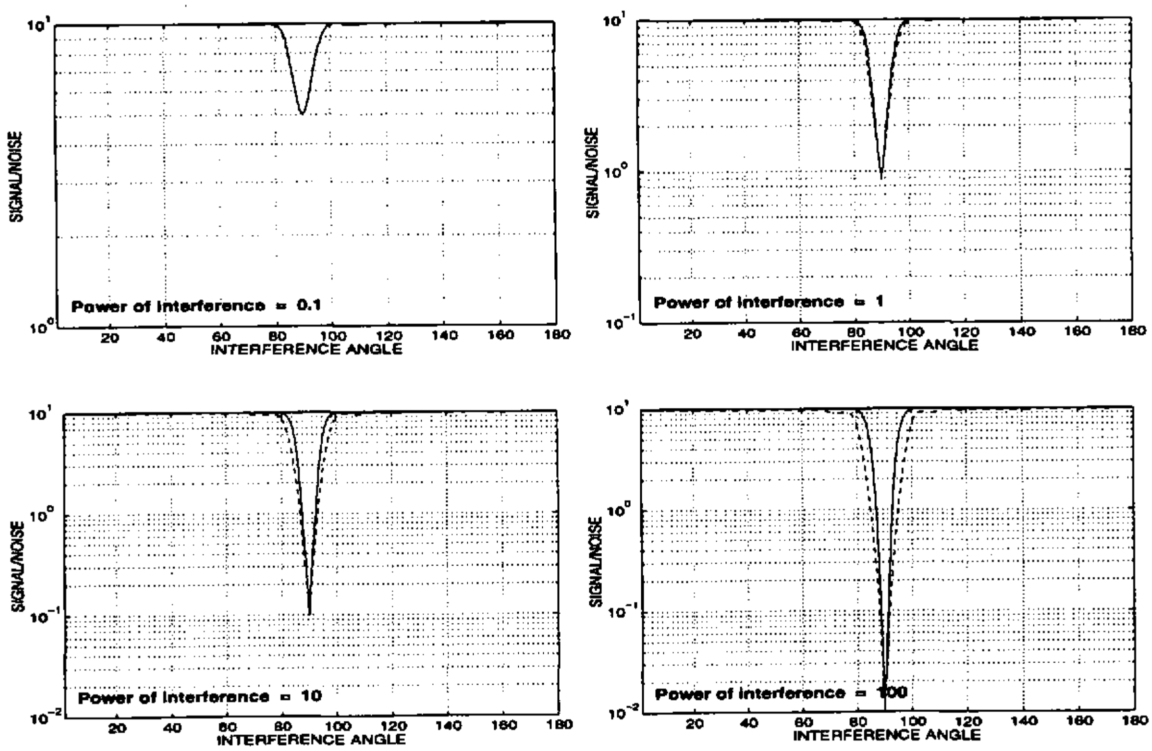
Length of the tapped delay line filter(J) = 11
Bandwidth (BW): ($f_1 = 0.45, f_2 = 0.5$) ($\times F_s$)
Power of the white noise = 0.1

Figure 3

FFT BASED METHOD

Figure 4 examines the effect of white noise power on the comparative performance of the two processors. For this figure the white noise power is raised by ten fold compared to that for Figure 2. The results emphasize the conclusion that the performance of the two processors is almost the same when interference power level is comparable to the white noise power level.

"Time Domain(——), Frequency Domain(- - - -)"



No. of elements (L) = 10
Element spacing (d) = $\frac{1}{2}$ ($\times\lambda$)
Power of the signal = 1.0
Power of the isotropic noise = 0.0

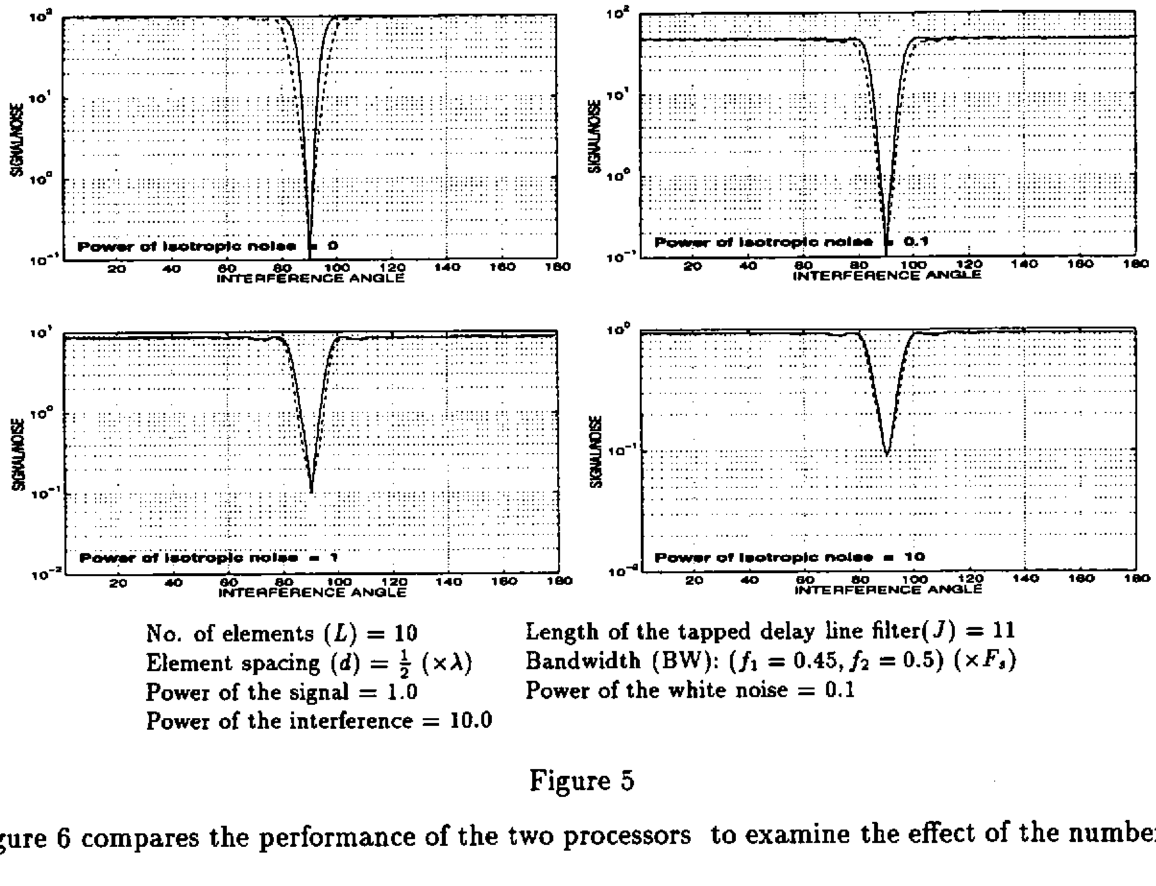
Length of the tapped delay line filter(J) = 11
Bandwidth (BW): ($f_1 = 0.45, f_2 = 0.5$) ($\times F_s$)
Power of the white noise = 1.0

Figure 4

Figure 5 studies the effect of spherical isotropic noise [4] on the comparative performance of the two processors. One observes from the figure that as the isotropic noise level is raised, the performance of the two processors approaches each other every where.

FFT BASED METHOD

"Time Domain(—), Frequency Domain(- - - -)"



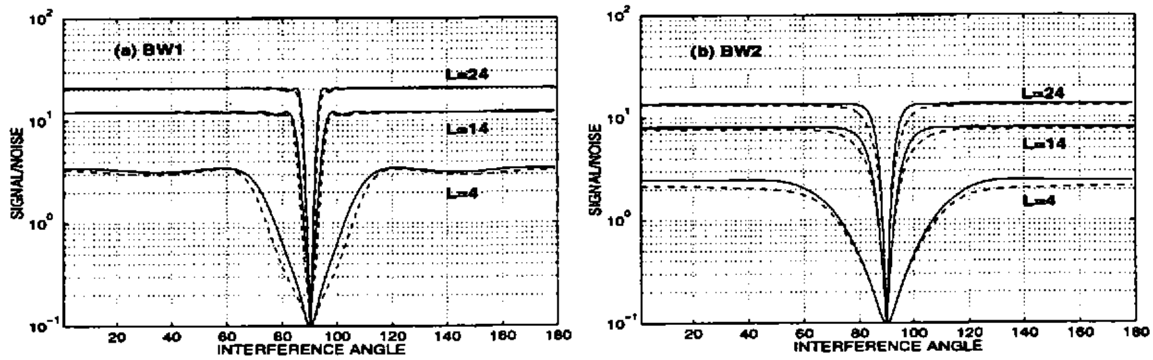
No. of elements (L) = 10
Element spacing (d) = $\frac{1}{2}$ ($\times\lambda$)
Power of the signal = 1.0
Power of the interference = 10.0

Length of the tapped delay line filter(J) = 11
Bandwidth (BW): ($f_1 = 0.45, f_2 = 0.5$) ($\times F_s$)
Power of the white noise = 0.1

Figure 5

Figure 6 compares the performance of the two processors to examine the effect of the number of

"Time Domain(—), Frequency Domain(- - - -)"



Length of the tapped delay line filter(J) = 11
Bandwidth (BW): BW1=($f_1 = 0.45, f_2 = 0.5$) ($\times F_s$)
Power of the signal =1.0
Power of the interference = 10.0

Element spacing (d) = $\frac{1}{2}$ ($\times\lambda$)
BW2=($f_1 = 0.15, f_2 = 0.5$) ($\times F_s$)
Power of the white noise =1.0
Power of the isotropic noise =1.0

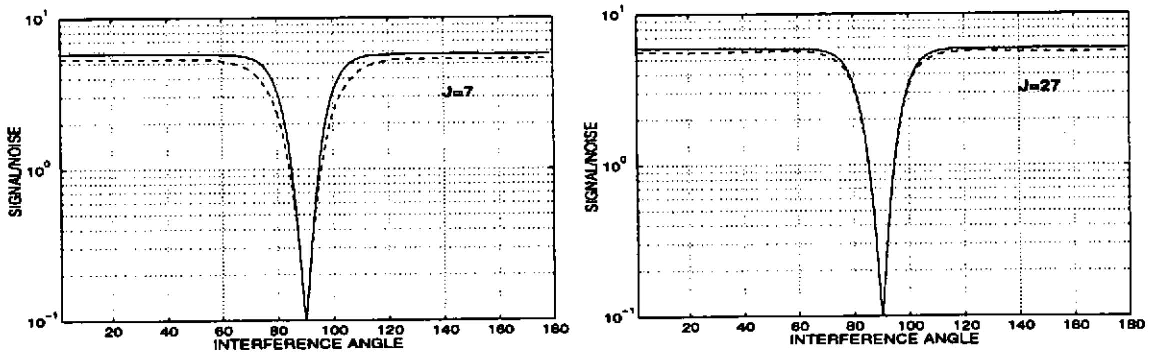
Figure 6

FFT BASED METHOD

elements and the bandwidth. It appears from the figure that the performance of the two processors approaches near to each other as number of elements in the array are increased. By comparing figure 6a and 6b where the bandwidth of the directional sources and the isotropic noise has been increased substantially, one observes the increased loss in the performance of the frequency domain method for the later case over a wider area.

Figure 7 examines the effect of the filter length and it is clear from the results of this figure that when a long filter is used, the performance of the frequency domain method improves substantially, and it may approach to that of the time domain method in the limit.

"Time Domain(—), Frequency Domain(- - - -)"



No. of elements (L) = 10
Element spacing (d) = $\frac{1}{2}$ ($\times\lambda$)
Power of the signal =1.0
Power of the interference = 10.0

Bandwidth (BW): ($f_1 = 0.15, f_2 = 0.5$) ($\times F_s$)
Power of the white noise =0.1
Power of the isotropic noise = 1.0

Figure 7

4. COMPUTATIONAL COMPLEXITY

Figure 8 shows the ratio of floating point operations required to calculate weights by frequency domain method to the time domain method versus the number of elements for various filter lengths. Figure 8a shows the results when the frequency domain method does not exploit the inherent parallel structure and figure 8b shows the result using the parallel processing. The block diagrams for the two algorithms are shown in Figure 9.

One observes from these figures that the frequency domain method is able to estimate the weights much faster than the time domain method even without exploiting the parallel nature of it structure. For a 40 elements array using J =37, the time saved by the frequency domain method in

FFT BASED METHOD

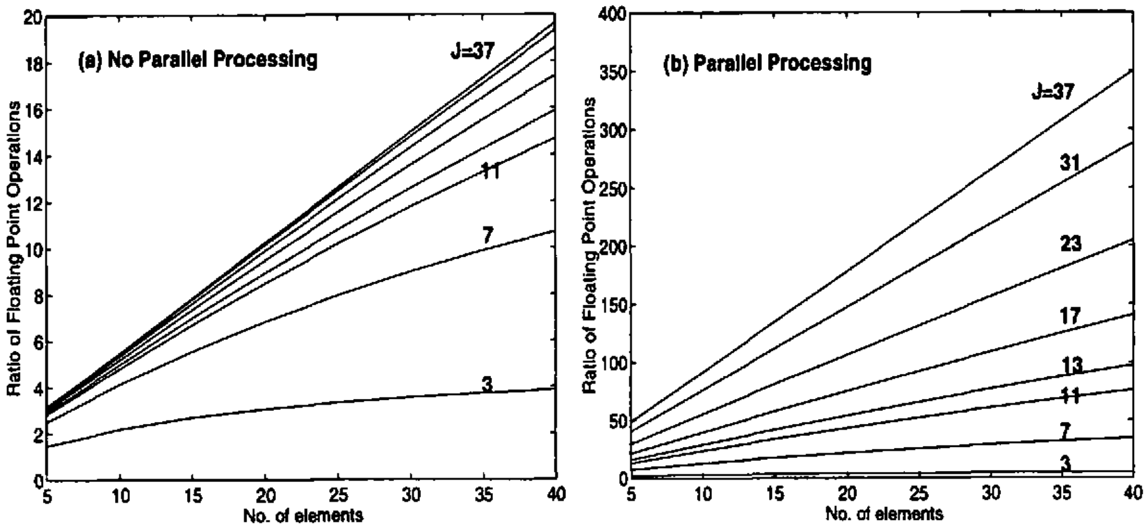
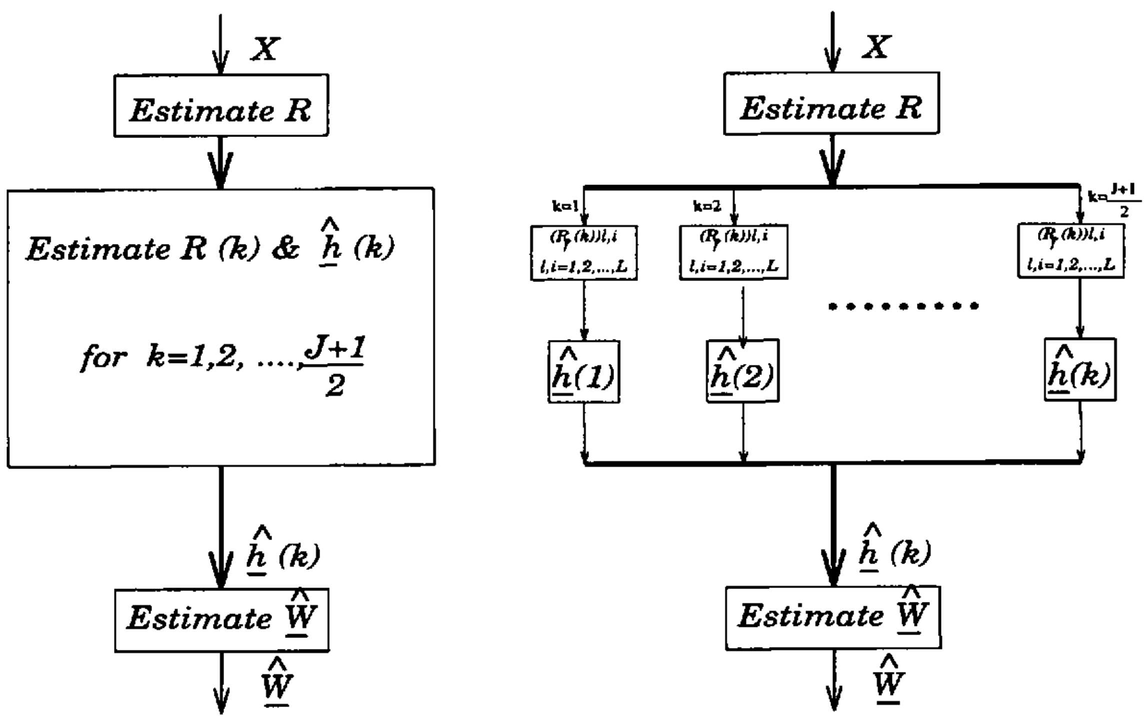


Figure 8: Ratio of floating point operations versus the number of elements for various filter lengths.



No Parallel Processing

Parallel Processing

Figure 9: Block diagrams of parallel processing & no parallel processing schemes.

FFT BASED METHOD

calculating the weights is of the order of 20 without parallel processing and of the order of 300 with the parallel processing.

It should be noted that the computation efficiency of the frequency domain method increases linearly with the number of elements in the array. In fact the efficiency increases faster with the increase in the number of elements than with the increase in the filter length.

5. CONCLUSIONS

The paper has compared the performance of a time domain method and a frequency domain method when the optimal weight of the broadband time domain constrained beamformer using a tapped delay line structure are estimated using the two method and shows that even though the frequency domain method is suboptimal, its performance could be made close to that of the time domain method by using a long filter. It also shows that the frequency domain method is much more computationally efficient than that of the time domain method and suits better for the real time signal processing compared to the other one.

6. REFERENCES

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