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THE OPTIMAL WAVE NUMBERS ESTIMATION

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1. INTRODUCTION

The wave numbers estimation of normal modes propagating in an ocean waveguide is a problem of considerable interest in ocean acoustics. Having the wave numbers estimates, it becomes possible to solve a set of important inverse hydroacoustic problems such as ocean acoustic tomography [1, 2], source localization [3] and waveguide characterization [4].

For normal modes wave numbers estimation, various algorithms of spatial processing of the field received by a hydrophone array have been proposed. The matched field processing algorithm was the first to emerge. This algorithm assume usage of the array as a beamformer in the wave number space. The most important property of this algorithm is a simplicity of its technical realization, and the most essential drawback is a low resolution threshold, which inversely proportional to the array's aperture. If the resolution requirement is high but the array length is limited, then a certain kind of algorithms possessing high-resolution ability can be tried for this purpose [5, 6]. So, the Prony method has been suggested for the modal wave numbers estimation problem [4]. But the practical usage of this method is limited by uniformly spaced line arrays and high signal-to-noise ratio. Later, Candy and Sullivan also used high-resolution algorithm MUSIC for this purpose [7]. This algorithm entails eigendecomposition of the array signal correlation matrix and uses the property of its noise subspace vectors [8]. The MUSIC algorithm is applicable to arbitrary array geometry and relatively low signal-to-noise ratio. Simultaneously an accuracy of the wave numbers estimates received by this method essentially depends on the intermodal correlation coefficient [9]. In the case of deterministic waveguide the modes are coherent [10] (fully correlated) and the MUSIC algorithm is not applicable [11]. Chouhan and Anand has proposed to decorrelate modes by horizontally moving the array in the waveguide [10]. However, they have not make clear how the array moving length, i.e., the length on which the array is moved, affect the wave number estimation accuracy and how this accuracy can be improved, if it is impossible to move the array.

The Cramer-Rao bound (CRB) is known to characterize a potential wave numbers estimation accuracy. Under performing some regular conditions, the CRB can be attained by the optimal (in the sense of maximum likelihood ratio) algorithm. In this paper we obtain this optimal algorithm, analyze its accuracy characteristics and compare them with those one for the MUSIC algorithm.

2. STATEMENT OF THE PROBLEM

We consider a plane-parallel waveguide in which a point source and a horizontal receiving array with N elements (hydrophones) are placed. We assume that a random source is located in the Fraunhofer zone (far field) of the array and in the same plane as the array. The Fourier transform of the field $u(t, r_n)$ received by the n-th array element at the time interval $[t + (l-1)T,\ t + lT]$ has the form [12, 13]

$$U_l(\omega; r_n) = \sum_{m=1}^{M} A_{lm}(\omega)(k_m|\rho_n - \rho_o|)^{-1/2} exp(jk_m|\rho_n - \rho_o|) + N_l(\omega; r_n), \quad (1)$$

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where $A_{lm}(\omega) \equiv A_{lm}(\omega; \boldsymbol{r}_o, z_a)$ are the fluctuating modal amplitudes, $\boldsymbol{r}_o = (\rho_o, z_o)$ and $\boldsymbol{r}_n = (\rho_n, z_a)$ are respectively coordinates of the source and the array elements, k_m is the m-th modal wave number, M is the number of propagating in the waveguide modes, $N_l(\omega; \boldsymbol{r}_n)$ is the spectral amplitude of the noise field at the n-th hidrophone, assumed to be uncorrelated with the source signal. It is convenient to rewrite the relation (1) in the following matrix notation

$$\boldsymbol{U}_l = \boldsymbol{S}\boldsymbol{A}_l + \boldsymbol{N}_l, \tag{2}$$

where $\boldsymbol{U}_l = [U_l(\omega;\rho_1), \ldots, U_l(\omega;\rho_N)]^\top$ and $\boldsymbol{N}_l = [N_l(\omega;\rho_1), \ldots, N_l(\omega;\rho_N)]^\top$ are $N \times 1$ vectors, $\boldsymbol{S} = [\boldsymbol{S}(k_1), \ldots, \boldsymbol{S}(k_M)]$ is a $N \times M$ matrix whose m-th column $\boldsymbol{S}(k_m)$ is the array response on the unit amplitude signal propagating in the m-th mode, the elements $S_n(k_m)$ of the vector $S(k_m)$ are equal $S_n(k_m) = (k_m|\rho_n - \rho_o|)^{-1/2} exp(jk_m|\rho_n - \rho_o|)$; $\boldsymbol{A}_l = [A_{l1}(\omega), \ldots, A_{lM}(\omega)]$ is a $M \times 1$ vector of the mode amplitudes.

The array signal correlation matrix is then given by

$$\boldsymbol{R}_{SN} = < \boldsymbol{U}_l \boldsymbol{U}_l^+ > = \boldsymbol{S}\boldsymbol{P}\boldsymbol{S}^+ + \boldsymbol{R}_N, \tag{3}$$

where $< \, . \, >$ denotes the expectation operator, the superscript $+$ denotes the Hermitian transpose, $\boldsymbol{P} = < \boldsymbol{A}_l \boldsymbol{A}_l^+ >$ is the mode amplitudes mutual correlation matrix, $\boldsymbol{R}_N = < \boldsymbol{N}_l \boldsymbol{N}_l^+ >$ is the spatial correlation matrix of the waveguide and array noises.

The problem is to estimate the wave numbers k_m, having the observation vectors $\boldsymbol{U}_l$ for the L time intervals $[t + (l-1)T, \, t + lT]$.

A most famous way of solving this problem is to use the matched field processing algorithm, which has the following form

$$F_{conv}(k) = \frac{\boldsymbol{S}^+(k)\boldsymbol{R}_N^{-1} \, \hat{\boldsymbol{R}} \, \boldsymbol{R}_N^{-1}\boldsymbol{S}(k)}{\boldsymbol{S}^+(k)\boldsymbol{R}_N^{-1}\boldsymbol{S}(k)}, \tag{4}$$

where $\boldsymbol{S}(k)$ is a $N \times 1$ vector with elements $S_n(k) = (k|\rho_n - \rho_o|)^{-1/2} exp(jk|\rho_n - \rho_o|)$; $\boldsymbol{R}_N^{-1}$ denotes $N \times N$ matrix, which is inverse to the $\boldsymbol{R}_N$ matrix;

$$\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{U}_l \boldsymbol{U}_l^+ \tag{5}$$

is an estimate of the spatial correlation matrix $\boldsymbol{R}_{SN}$ of the field at the aperture of the array.

In accordance with algorithm (4), the wave numbers can be estimated by locating the peaks of $F_{conv}(k)$. However, if the distance $\triangle k_m = |k_m - k_{m-1}|$ between neighboring wave numbers k_m and k_{m-1} decrease, these two peaks merge into one maxima and the wave numbers are not resolved. In this case, it becomes necessary either to use longer arrays to improve the resolution or to abandon the matched field processing algorithm and employ, so called, high-resolution algorithms [6, 14]. We use the latter approach to solve the problem of wave numbers estimation.

3. SYNTHESIS OF AN OPTIMAL ALGORITHM

From the standard statistical theory of estimation it is known a method of solving the above formulated problem . This is a synthesis of an optimal (by the criterion of the maximum of likelihood function) wave numbers estimation algorithm. We use this approach to synthesize such optimal algorithm.

We begin from the assumption that the observation vectors U_l are statistically independent at different time intervals and have a normal distribution with the parameters $< U_l >= 0$, $< U_l U_m^+ >= R_{SN}\delta_{lm}$, where δ_{lm} is the Kroneker delta.

Then, with the accuracy up to unessential constants, log-likelihood function (LF)

$$\ln \lambda(U|k, P) = -Tr(\ln R_{SN} + R_{SN}^{-1} \hat{R}), \tag{6}$$

where $U = [U_1^T, \ldots, U_L^T]^T$ is the $NL\times 1$ vector formed by the observation U_l vectors, $k = [k_1, \ldots, k_M]^T$ is a vector of estimated parameters, T is the simbol of transpose, $Tr(\cdot)$ is a trace operator.

Besides of interesting for us the wave numbers k_m, the logarithm LF (6), in general, depends on the matrix P. This matrix usually is unknown because the modal spectral power density radiated by the source and intermodal correlation coefficients are unknown.

Following the estimation strategy of maximum likelihood method (MLM) at first we find the maximum likelihood (ML) estimate $\hat{P}$ of the matrix P. We then substitute it into (6) and receive the optimal wave numbers estimation algorithm.

Set the derivative of the function (6) with respect to P equal to zero. In according to the matrix differentiation rules [15] we then obtain the following equation relatively unknown matrix P :

$$\left.\frac{\partial \ln \lambda(U|k, P)}{\partial P}\right|_{P=\hat{P}} = S^+ \tilde{R}_{SN}^{-1} S - S^+ \tilde{R}_{SN}^{-1} \hat{R} \tilde{R}_{SN}^{-1} S = O, \tag{7}$$

where $\tilde{R}_{SN}^{-1}$ denotes the $N \times N$ matrix, which is inverse to the matrix

$$\tilde{R}_{SN} = R_{SN}\Big|_{P=\hat{P}} = S \hat{P} S^+ + R_N. \tag{8}$$

Taking advantage of Vudberi's equality [9]

$$R_{SN}^{-1} = R_N^{-1} - R_N^{-1} S[I + PQ]^{-1} P S^+ R_N^{-1}, \tag{9}$$

where $Q \equiv S^+ R_N^{-1} S$, and solving equuation (7), we have

$$\hat{P} = Q^{-1} S^+ R_N^{-1} \hat{R} R_N^{-1} S Q^{-1} - Q^{-1}. \tag{10}$$

We note that nonsingularity of the matrix Q is necessary for existence of the ML estimate (10). When the matrix R_N is nonsingular, this usually always takes place, the matrix Q will be also nonsingular, if the

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rank of the matrix S is equal to M $(rank(S) = M)$. It is possible, if first, the number M of propagating in the waveguide modes does not exceed the number N of array elements $(M \leq N)$ and, second, the array geometry, has been chosen so that linear independence of the vectors $S(k_m)$, is ensured for arbitrary values of the wave numbers k_m belonging to the interval $[k_{\min}\ k_{\max}]$ of their possible meanings.

Use the obtained estimate (10) and substitude it into the logarithm LF (6). Making several transformations, we have

$$\ln \lambda(U|k) \equiv \ln \lambda(U|k, \hat{P}) = Tr\left\{\Pi_S\, \hat{R}\, R_N^{-1} - \ln(\Pi_S\, \hat{R}\, R_N^{-1})\right\}, \tag{11}$$

where $\Pi_S = SQ^{-1}S^{+}R_N^{-1}$ is a $N \times N$ projection matrix onto the subspace spaned by the vectors $S(k_m)$ associated with the wave numbers k_m.

In accodance with algorithm (11) the optimal wave numbers estimation algorithm assumes estimation of the spatial correlation matrix R_{SN} (5) of the field at the array aperture (using the observatiion vectors U_l), calculation of the projection matrix Π_S, formation of the goal function (11) (taking into account noise spatial correlation matrix R_N) and its global maximizing. The coordinates $\hat{k} = [\hat{k}_1, \ldots, \hat{k}_M]^{\mathsf{T}}$ of this maximum give the required ML estimates of the normal modes wave numbers.

4. POTENTIAL ACCURACY OF THE OPTIMAL ESTIMATES

From the general theory of estimation it is known that an important characteristic of the ML estimates $\hat{k}$ of the wave numbers k is the correlation matrix

$$K_{ML} = <(\hat{k} - k)(\hat{k} - k)^{\mathsf{T}}>. \tag{12}$$

Under some regularity conditions the ML estimates are known to be asymptotically $(L \to \infty)$ efficient, or in other words, their correlation matrix of the errors (12) attains the limit correlation matrix K_{RK} called the Cramer-Rao bound.

The essential regularity condition is consistency of the ML estimates $\hat{P}$ and $\hat{k}$. This condition is fulfiled in the case considered by us. As a result, the formula

$$LK_{RK} = \lim_{L \to \infty} (LK_{ML}), \tag{13}$$

which is very useful for callculating CRB, is valid [9].

Introduce the following notation : $F(k) \equiv \ln \lambda(U|k)$. Since the vector k of the ML estimates maximize the function $F(k)$, the gradient $F'(k) = [\partial F(k)/\partial k_1, \ldots, \partial F(k)/\partial k_M]^{\mathsf{T}}$ of this function calculated at the point $\hat{k}$ is equal to zero, i.e., $F'(\hat{k}) = O$.

A Taylor series expantion of the gradient $F'(k)$ around the true vector k leads to

$$F'(k) + F''(k)(\hat{k} - k) + \ldots = 0, \tag{14}$$

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where $F''(k)$ is the $M \times M$ Hessian matrix with elements $[F''(k)]_{nm} = \partial^2 F(k)/\partial k_n \partial k_m$ calculated at the point of true values of the wave numbers. In an asymptotic analysis $(L \to \infty)$ it is possible to neglect in (14) by the terms containing the factor $(\hat{k} - k)$ in the power higher than the first. Similarly, the Hessian $F''(k)$ in (14) can be replaced by $F_0''(k) = \lim_{L \to \infty} F''(k)$ without affecting the asymptotic $(L \to \infty)$ properties of k. Thus, neglecting all the terms in (14) that tend to zero faster than $(\hat{k} - k)$, as $L \to \infty$, we can write

$$(\hat{k} - k) = -[F_0''(k)]^{-1} F'(k), \tag{15}$$

assuming the inverse matrix exist (that is true under weak conditions [9]).

From (12), (13) and (15) it follows that

$$\lim_{L \to \infty} (LK_{ML}) = [F_0''(k)]^{-1} C [F_0''(k)]^{-\top}, \tag{16}$$

where matrix

$$C = \lim_{L \to \infty} \left\{ L < [F'(k)][F'(k)]^{\top} > \right\}, \tag{17}$$

$[\cdot]^{-\top}$ denotes the transpose of an inverse matrix.

The Hessian $F_0''(k)$ and the matrix C have the following form:

$$F_0''(k) = -C^{\top} = -2Re\left\{ (D^+ R_N^{-1} \Pi_0 D) \odot (PS^+ R_{SN}^{-1} SP)^{\top} \right\}, \tag{18}$$

where $D = [d_1, \ldots, d_M]$ is a $N \times M$ matrix whose m-th column d_m is the $N \times 1$ vector with elements $d_{nm} = \partial S_n(k)/\partial k$ calculated at the point $k = k_m$; $\Pi_0 \equiv I - \Pi_S$; $\odot$ denotes element-by-element multiplication of two matrices.

We now substitute the obtained matrices $F_0''(k)$ and C into (16) and take into consideration the equality (13). Thus, we finally have

$$K_{RK} = \frac{1}{2L} \left[Re\left\{ H \odot (PS^+ R_{SN}^{-1} SP)^{\top} \right\} \right]^{-1}, \tag{19}$$

where

$$H \equiv D^+ R_N^{-1} \Pi_0 D. \tag{20}$$

The formula (19) permits to calculate the limiting wave numbers estimation accuracy, which can be attained asymptotically by the optimal algorithm (11). In other words, from (13) and (19) it follows that

$$K_{ML} = K_{RK} + o(L^{-1}), \tag{21}$$

where $o(L^{-1})$ denotes a matrixy which elements have the order higher than L^{-1}.

The relation (19) permits to execute the comparison on the estimation eccurasy of the optimal algorithm with the MUSIC algorithm.

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5. MUSIC ALGORITHM

We now direct our attention to the MUSIC algorithm, which is one of the most popular high-resolution algorithm, and elucidate how much it concede in the wave numbers estimation accuracy to the optimal algorithm (11).

The MUSIC algorithm is based on the property of noise eigenvectors of the equation

$$R_{SN}\varphi_n = \lambda_n R_N \varphi_n, \tag{22}$$

where $\lambda_1, \ldots, \lambda_N$ are eigenvalues arranged in the monotonically nonincreasing order ($\lambda_n \geq \lambda_{n+1} > 0$); $\varphi_1, \ldots, \varphi_N$ are assosiated eigenvectors.

It is known [11, 14], that if the number N of the array elements greater than the number M of propagating in the waveguide modes, then, by virtue of the matrix R_{SN} structure (3), eigenvectors $\varphi_1, \ldots, \varphi_N$, can be separeted in two groups: the signal eigenvectors $\varphi_1, \ldots, \varphi_{\widetilde{M}}$ corresponding to the largest eigenvalues, and noise eigenvectors $\varphi_{\widetilde{M}+1}, \ldots,$
φ_N, corresponding to the smallest eigenvalues $\lambda_{\widetilde{M}+1} = \ldots = \lambda_N = 1$ (moreover, $\lambda_{\widetilde{M}} > \lambda_{\widetilde{M}+1}$). The bound $\widetilde{M}$ of this separation is determined by the rank of the matrix P, i.e., $\widetilde{M} = rank(P)$.

If among the modes there are no coherent ones and $rank(P) = M = \widetilde{M}$, then the ortogonality condition

$$S^+(k_m)\varphi_n = 0, \quad m = 1, \ldots, M;\ n = M+1, \ldots, N. \tag{23}$$

fulfilled [6, 11]. This condition means that any noise eigenvector φ_n is ortogonal to any column $S(k_m)$ of the matrix S.

Having used the condition (23), the MUSIC algoritm was syntethized in Refs. [6, 7]. By this algorithm, the wave numbers estimates $\hat{k}_m$ can be obtained as the coordunates of the M highest peaks of the function

$$F_M(k) = [S^+(k)\,\hat{\Psi}_N \hat{\Psi}_N^+\, S(k)]^{-1}, \tag{24}$$

where $\hat{\Psi}_N^+ = [\hat{\varphi}_{M+1}, \ldots, \hat{\varphi}_N]$ is the $N \times (N-M)$ matrix maked up from estimates $\hat{\varphi}_n$ of the noise eigenvalues φ_n.

The estimates $\hat{\varphi}_n$ can be derived from the equation

$$\hat{R}\hat{\varphi}_n = \hat{\lambda}_n\, R_N\, \hat{\varphi}_n, \tag{25}$$

in which $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \ldots \geq \hat{\lambda}_N$ are the noise egenvalue estimates arranged in the decreasing order.

It can be shown by analogy with Ref. [9], that the estimates $\hat{k}_m$ obtained by the MUSIC algorithm (25) are asymptopically unbiased (i.e., $\lim_{L\to\infty} \hat{k}_m = k_m$) and have the following correlation matrix

$$\begin{aligned} K_M &= \langle (\hat{k} - k)(\hat{k} - k)^{\mathrm{T}} \rangle \\ &= \frac{1}{2L}[Re\{H \odot I\}]Re\{H \odot (P^{-1} + P^{-1}QP^{-1})^{\mathrm{T}}\}[Re\{H \odot I\}] + o(L^{-1}). \end{aligned} \tag{26}$$

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This formula generalizes the results obtained in Ref. [9] on the case of arbitrary spatially correlated noise. When the noise is spatially uncorrelated, i.e., $R_N = g_N I$, formula (26) coincide with those received in Ref. [9].

We now assume that the source is placed in a deterministic waveguige. Then, as it has been shown in Ref. [10], the ortogonality condition (23) is not fulfilled ($rank(P) = 1 < M$) and the MUSIC algorithm becomes unapplicable for wave numbers estimation. To change these situation and make possibile the MUSIC algorithm application, it has been offered [10] to decorrelate the modes at the expence of towing of the array. It can be shown, that if during towing, the array remains far enough from the source then the matrix P is transformed into the smoothed correlation matrix

$$P_C = P \odot W_C, \tag{27}$$

where W_C is the $M \times M$ smoothing matrix with elements

$$[W_C]_{nm} = \frac{\sin(L_b(k_n - k_m)/2)}{L_b(k_n - k_m)/2} \exp(jL_b(k_n - k_m)/2); \tag{28}$$

L_b denotes a towing interval.

The degree of modal decorrelation is obvious to depend on the value L_b of the towing interval. Nevertheless, the rank of the matrix P remains equal to M, the condiion (23) is fulfilled and for wave numbers estimation we can use the MUSIC algorithm.

We now use the derived formulas (19), (26) and numerically investigate the estimation accuracy of the optimal algorithm (11) and the MUSIC algorithm (24).

6. NUMERICAL EXAMPLE

In this section, we study the performance of the algorithms described above through computer simulation.

Let a horizontal equidistant linear array of $N = 6$ hydrophones and a source radiating a tone signal at a frequancy $f_o = 40\ Hz$ are placed in a plane-paralell Pekeris waveguide. Numerical values assined to the parameters of the array and the waveguide are taken from Ref. [10] and are as follows: channel depth $h = 50\ m$, source depth $z_o = 25\ m$, array depth $z_a = 25\ m$, uniform inter-hydrophone spacing $|\rho_{n-1} - \rho_n| = 12.5\ m$, sound speed in water $c_W = 1500\ m/s$, sound speed in sediment $c_S = 2000\ m/s$, density of the water layer $\rho_W = 1.0\ g/sm^3$, density of the sediment half-space $\rho_S = 1.1\ g/sm^3$. For the assined values there are two propagating modes ($M = 2$), which wave numbers are equal to $k_1 = 0.1592$ and $k_2 = 0.1331$. The distance $|\rho_1 - \rho_o|$ between the first (the nearest to the source) hydrophone and the source is equal to $1000\ m$. The noise is asummed to be delta-correlated in space (spatially uncorrelated), so that the spatial correlation matrix $R_N = g_N I$, where g_N is the spectral power density of the noise.

Using the equations (19), (26), the dependence of the root-mean-squared (RMS) error σ_m of the wave number estimates from the array towing interval L_b has been calculated. The results of these calculations corresponding to the first wave number are $k_1 = 0.1592$ shown in Fig.1. The curve 1 is obtained for the optimal algorithm (11) ($\sigma_1 = [K_{RK}]_{11}^{1/2}$), the curve 2 is obtained for the MUSIC algoritm (24) ($\sigma_1 = [K_M]_{11}^{1/2}$). Performing calculations based on the equation (26), the smoothed correlation matrix P_C was used insted of matrix P. The signal to noise ratio (SNR) g_S/g_N was equal to $60\ dB$.

Here, g_S is the source spectral power densities. The average value of the SNR on the hydrophones $\tilde{g}_m = S^+(k_m)S(k_m)p_{mm}/Ng_N$ $(m = 1,2)$ was equal to 15.5 dB for the first mode and 5.5 dB for the second one. The number of observation vectors U_l utilized for estimating $\hat{R}$ were equal to 100, i.e., $L = 100$.

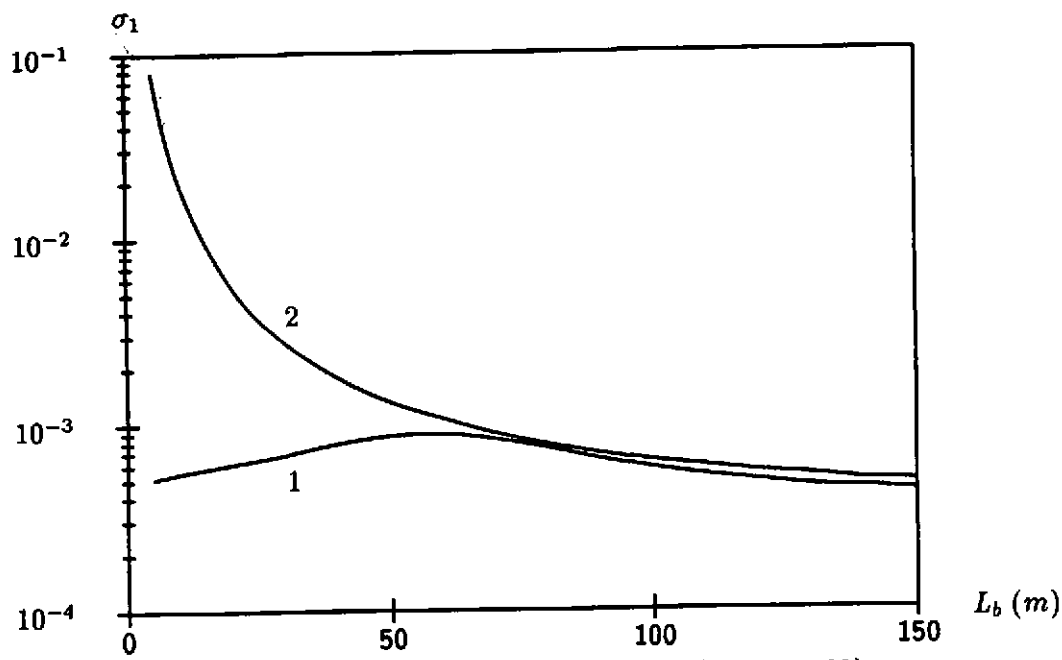

Fig.1 RMS error of the first wave number ($k_1 = 0.1592$) as a function of towing interval

An analysis of the curves displayed in Fig.1 shows that the optimal algorithm gives more accurate wave numbers estimates than the MUSIC algorithm. The distinction in RMS error between these two algorithms increases as the towing interval and, as a consiquence, the intermodal correlation coefficient decreases.

7. SUMMARY AND CONCLUSIONS

The present article is devoted to investigation of structure and efficiency of normal modes wave numbers estimation algorithms.

The synthesis and analysis of an optimal (by the criterion of the maximum likelihood function) algorithm for normal modes wave numbers estimation has been perfotmed. The optimal algorithm has a high-resolution property, does not require prior modal decorrelation, is suitable for a horizontal array of any configuration and permits to attain the potentinal wave numbers estimation accuracy limit (Cramer-Rao bound).

As opposed to the MUSIC algorithm, the optimal algorithm makes possible to estimate wave numbers of coherent (fully correlated) modes without their preliminary decorralation. But if the such decorrelation is done, for example, by the means of array towing, then the optimal algorithm permits to estimate the wave numbers with much higher accuracy (in terms of root-mean-square error) than it does the MUSIC algorithm. Moreover, this difference in performance between the algorithms can be significant at small towing intervals when impossible to decorrelate the modes well enough.

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