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ENERGY FLOWS IN BEAM NETWORKS WITH COMPLIANT JOINTS

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1. INTRODUCTION

Many structures and machines are made up from components (rods, beams, plates) which are joined together by welds, bolts or rivets. The mechanical behaviour of a built-up system is greatly affected by the properties of its joints which usually are compliant and dissipative[1].

In this work, a general solution for the vibrational energy flows through a plane network of beams is sought based on a receptance approach[2, 3]. The joints between any two elements are assumed to act at discrete points and are modelled by three sets of springs and dashpots. This model allows the joints to be compliant and nonconservative in both translational and axial modes[4, 5]. The beams are assumed slender and elastic and the deflections at the joints are assumed small so that conventional linear beam theory may be used in the analysis. The aim of this study is to give greater insight into the problem of energy flows through nonconservative couplings which has not been extensively discussed in the literature[6]. Interest is focused on the effect of damping in the joints on the magnitudes of energy flows between, and energy levels in, each beam. Numerical examples are presented which illustrate some of the behaviour of dissipative joints.

2. THEORY

Consider a network of subsystems which consist of N beams and M couplings. Each beam a has length l_a and mass per unit length ρ_a. The damping c_a is assumed to be viscous and proportional to the mass. The joints between any two beams are assumed to be modelled by three springs of strengths K_x, K_y and K_θ and three dampers of strengths γ_x, γ_y, γ_θ in the global x, y and θ directions, respectively. Let the joint i connect two beams, a and b, at the points A and B, respectively. The vector of displacements of the two ends A and B in the global coordinates are denoted as $\{Y\}_{Ai}$ and $\{Y\}_{Bi}$ and the diagonal matrix which has the joint complex stiffnesses as its diagonal elements is denoted as $[\Omega]_i$. If $\{Y^0\}_A$,

$\{Y^0\}_B$ are the displacements of the points A and B of the beams when uncoupled and due to external forcing only then the compatibility conditions require that the displacements at A and B of the coupled structure be equal to these displacements plus those due to the coupling forces which act at the joint. These conditions can be written as follows

$$\{Y\}_A = \{Y^0\}_A - [G]_A\{P\}_{coup} = \{Y^0\}_A - [A]_A \{\Delta Y\},$$

$$\{Y\}_B = \{Y^0\}_B + [G]_B\{P\}_{coup} = \{Y^0\}_B + [A]_B \{\Delta Y\}.$$

From these two equations, the vector of the relative displacements and therefore the vector of coupling forces can be written in terms of the vector of the relative displacements $\{\Delta Y^0\}$ as $\{\Delta Y\} = [D]^{-1}\{\Delta Y^0\}$ and $\{P\}_{coup} = [\Omega]\,[D]^{-1}\{\Delta Y^0\}$ where $[D]$ is given by $[D]=[I]+[A]_A+[B]_B$.

Energy flow through a joint may then be calculated from the product of force or (moment) and the velocity or (angular velocity) at the joint. Let $\{\Pi\}_{coupAi}$ be the energy flow that enters joint i at end A and let $\{\Pi\}_{coupBi}$ be the energy that enters joint i at end B through the three horizontal, vertical and rotational couplings. In the frequency domain, the vector of energy flow $\{\Pi\}_{coupAi}$ at a specific frequency ω is given in terms of the spectral density of the displacements at A_i, B_i as follows

$$\{\Pi\}_{coupAi} = \mathrm{Re}\{\, i\,\omega\,[\Omega]_i\ \{S_{Y_{Ai}Y_{Bi}}(\omega)\}\} + \omega^2\,[\gamma]_i\ \{S_{Y_{Ai}Y_{Ai}}(\omega)\}.$$

where $\{S_{Y_{Ai}Y_{Bi}}(\omega)\} = \lim_{T\to\infty} [\{Y(\omega)\}^*_{Ai}\ \{Y(\omega)\}^T_{Bi}\,]_{diag}\ \frac{2\pi}{T}$. In order to get the vector of the energy flows through all the joints $\{\Pi\}_{coupA}$, the diagonal elements of the product $\{Y\}^*_A\ \{Y\}^T_B$ and the product $\{Y\}^*_A\ \{Y\}^T_A$ need to be considered. They are given by

$$\{Y\}^*_A\{Y\}^T_B = [([I] + [A]_B)^* \mid [A]^*_A] \begin{bmatrix} [D]^{-1} & [0] \\ [0] & [D]^{-1} \end{bmatrix}^* \left\{\frac{\{Y\}^*_{A0}}{\{Y\}^*_{B0}}\right\}$$

$$\times\{\{Y\}^T_{A0} \mid \{Y\}^T_{B0}\} \begin{bmatrix} [D]^{-1} & [0] \\ [0] & [D]^{-1} \end{bmatrix}^T [[A]_B \mid ([I] + [A]_A)]^T$$

where $\left\{\frac{\{Y\}^*_{A0}}{\{Y\}^*_{B0}}\right\}\{\{Y\}^T_{A0} \mid \{Y\}^T_{B0}\} = [gf]^*[S_{FF}][gf]^T$. The matrix $[gf]$ contains the integrals of the Green functions and forces of the individual subsystems taken over the individual subsystems while $[S_{FF}]$ is the matrix of the various cross spectra of the forcing acting on the individual subsystems. If the forces on the different subsystems are spatially incoherent and the components of the vector of forces which act on one subsystem are also incoherent then the matrix $[S_{FF}]$ is diagonal. In this case the following equation is valid

$$[gf]^*[S_{FF}][gf]^T = \sum_{a=1}^{3N} [gf]^*_a[gf]^T_a S_{F_aF_a} = \sum_{a=1}^{3N} [Q]_a S_{F_aF_a}.$$

Finally, the energy flow from the end A through the horizontal, vertical and rotational springs of joint i is given by $\{\Pi\}_{\text{coupA}i} = \sum_{a=1}^{3N} \{H\}_{\text{A}ia} S_{F_a F_a}$, where

$$\{H\}_{\text{A}ia} = \text{Re}\left\{ \text{i}\omega[\Omega]_i \left[[([\text{I}] + [A]_{\text{B}})^* \mid [A]_{\text{A}}^*] \begin{bmatrix} [D]^{-1} & [0] \\ [0] & [D]^{-1} \end{bmatrix}^* \right]_i \right.$$

$$\times [Q]_a \left[\begin{bmatrix} [D]^{-1} & [0] \\ [0] & [D]^{-1} \end{bmatrix}^{\text{T}} [[A]_{\text{B}} \mid ([\text{I}] + [A]_{\text{A}})]^{\text{T}} \right]_i$$

$$+[\gamma]_i \omega^2 \left[[([\text{I}] + [A]_{\text{B}})^* \mid [A]_{\text{A}}^*] \begin{bmatrix} [D]^{-1} & [0] \\ [0] & [D]^{-1} \end{bmatrix}^* \right]_i$$

$$\left. \times [Q]_a \left[\begin{bmatrix} [D]^{-1} & [0] \\ [0] & [D]^{-1} \end{bmatrix}^{\text{T}} [([\text{I}] + [A]_{\text{B}}) \mid [A]_{\text{A}}]^{\text{T}} \right]_i \right\}.$$

The input power due to forcing on subsystem a is calculated in a similar fashion from the product of the vector of forces applied to the subsystem and the vector of velocities at the points of application of the forces.

Finally, if the vector of energies leaving the subsystems is denoted by $\{\Pi\}_{\text{OUT}}$ then this vector can be related to the vector of energies flowing through the coupling systems at the ends A and B by the relation, $\{\Pi\}_{\text{OUT}} = [CON] \begin{Bmatrix} \{\Pi\}_{\text{coupA}} \\ \{\Pi\}_{\text{coupB}} \end{Bmatrix}$ where the elements of the connectivity matrix $[CON]$ are either 1 or 0, depending on the topology of the system. The energy dissipated within each subsystem due to damping can then easily be deduced from the energy balance for each subsystem as $\{\Pi\}_{\text{DISS}} = \{\Pi\}_{\text{IN}} - \{\Pi\}_{\text{OUT}}$. Finally, the energy levels for each subsystem can be related to the energy dissipated due to damping by the well known relation $\{E\} = [c]^{-1}\ \{\Pi\}_{\text{DISS}}$ where $[c]$ is the diagonal matrix of damping constants of each subsystem and the damping of each subsystem is assumed to be viscous and proportional to mass per unit length.

3. EXAMPLES

It is commonly the case that the joints in a built-up structure dissipate more energy than material damping does[7]. It is therefore of interest to see the effects of variations in joint damping on the magnitudes of the energy transferred to, or dissipated within each system.

In what follows, R denotes the ratio of the power dissipated at the joint to the power input by the external forcing. These ratios are very small when the joint damping is either very weak or very strong. In the first case because the damper is too weak to have much effect and in the second because the damper is so stiff that it is hardly deflected[8]. It is therefore expected that these ratios are maximized for certain levels of the coupling damping, as discussed below.

Consider two beams at right angles connected together by three springs and three dampers in the three degrees of freedom, the parameters adopted for the study are given in table (1). Let $Z_x=K_x/\omega+i\gamma_x$ be the joint impedance in the x direction, $Z_y=K_y/\omega+i\gamma_y$ be the joint impedance in the y direction and $Z_\theta=K_\theta/\omega+i\gamma_\theta$ be the joint impedance in the θ direction. If C_B represents the bending wave speed, the dimensionless ratios $Z_x'=Z_x/(\rho_2 C_{B2})$, $Z_y'=Z_y/(\rho_1 C_{B1})$ and $Z_\theta'=(C_{B1} Z_\theta)/EI_1$ are then the ratios of the impedances of the joint and the bending wave impedances. First the case of beams coupled in the x direction only is considered. The first beam is driven by a horizontal force at the end away from the coupling,which excites the flexural modes of the second beam. Figure (1) shows the variation of the ratio R with the driving frequency ω for increasing values of the damper strength γ_x while the spring stiffness is constant and weak. It is obvious that R increases as the damper strength increases until it reaches a maximum value for a certain value of γ_x then it begins to fall again. This figure also shows that the ratio always drops at the flexural natural frequencies of the undriven beam. The values of the damper strength for which the ratio is maximised always yield impedance ratios Z_x around unity, see figure(2). Figure (3) shows the variation of the ratio R versus the impedance ratio Z_x' when the damper strength is varied, for a constant value of the driving frequency ω. It is obvious that the value of the maximum ratio R depends on the value of the spring strength K_x and that for a stiff spring this maximum ratio R is very low because the damper is virtually blocked and unable to dissipate much energy.

Next, the case of two beams coupled in the y direction only is considered. The first beam is now driven by a vertical load which excites the longitudinal modes in the second beam. Figure (4) shows the variation of the ratio R with the driving frequency for different values of the damper strength. Again it is obvious that at a certain value of the damper strength γ_y, the ratio R is maximised and the dissipated power at the junction is then 80-90% of the input power. This means that by the careful selection of the damper strength it is easy to arrange for the bulk of the input power to be dissipated at the damper, an idea which is employed in many vibration control problems. Because the natural frequencies of the axial

Table 1 - Parameters used in the Examples

Parameter	Beam 1	Beam 2	Units
Mass density (ρ)	56.16	39.78	kg/m
Length (l)	1.200	1.00	m
Rigidity(EA)	$1.512E^9$	$1.071E^9$	N
Bending Rigidity(EI)	18144	8925	Nm^2
Damping ratios	0.01	0.01	-

modes lies well beyond the frequency range of interest, R doesn't show any marked dips, instead varying rather smoothly over this frequency range. When the ratio R is plotted against Z'_y, a plot similar to that in figure(3) is obtained which shows that the optimum value of γ_y is associated with a value of Z'_y again in the proximity of unity. Further, when the spring becomes very stiff, the ratio R once more becomes small regardless of the damper strength, since the damper is then rigidly blocked and hardly dissipates any energy.

Lastly, it is worth noting that for the case of two beams coupled through the rotational degree of freedom and when the beam is driven by a vertical force (or a moment), the behaviour is very similar to the first case of two beams coupled through a horizontal spring and damper.

4. CONCLUSIONS

Energy flow and dissipation in a joint between two beams modelled by springs and dampers are studied and expressions for the various energy receptances based on receptance theory established in the frequency domain.

It is then shown that the ratio of the energy dissipated in a dissipative joint to that input to the system is strongly controlled by the strength of the joint damper. This ratio reaches high levels at values of the damping which depend on the driving frequency and the coupling spring strength, while the ratio is very small for either small or large values of the coupling damping. By selecting the correct value of coupling damping it is easy to arrange for most of the energy injected into the system to be dissipated in the joint. However, when the coupling spring is stiff, only a small ratio is dissipated at the joint regardless of the damper strength. It is also shown that the joint impedance to wave impedance ratio is a useful measure of when the joint is able to dissipate maximum energy.

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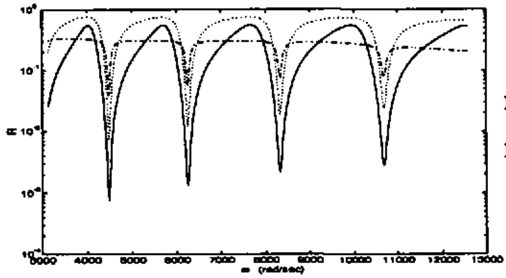

Figure (1) $-.-\gamma_x=10^3$ NS/m, $\cdots=10^4$ NS/m, $—=10^5$ NS/m; $K_x=10^5$ N/m.

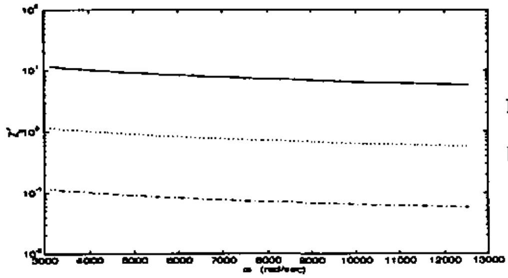

Figure (2) $-.-\gamma_x=10^3$ NS/m, $\cdots=10^4$ NS/m, $—=10^5$ NS/m; $K_x=10^5$ N/m.

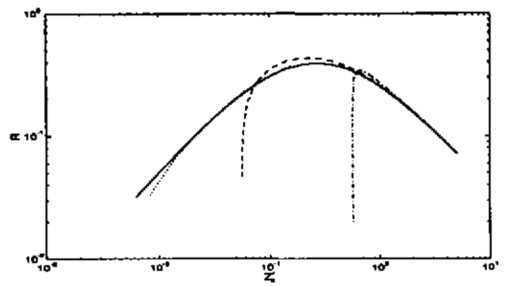

Figure (3) $—K_x=10^5$ N/m, $\cdots=10^6$ N/m, $--=10^7$ N/m, $-.-=10^8$ N/m; $\omega=11{,}000$ rad/s.

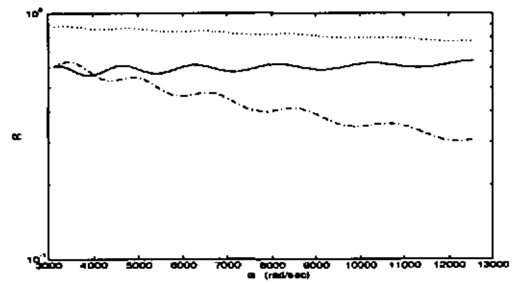

Figure (4) $-.-\gamma_y=10^3$ NS/m, $\cdots=10^4$ NS/m, $—=10^5$ NS/m; $K_y=10^5$ N/m.