PREDICTIONS OF SOUND DUE TO A MOVING SOURCE NEAR FINITE IMPEDANCE GROUND

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1. ABSTRACT

A new solution for the calculation of the sound field due to a source in uniform motion above an impedance ground is introduced. It is expressed in a Lorentz coordinate system by means of the Weyl-Van der Pol formula for a stationary source. After transformation back into physical space, the formulation of the sound field takes the form of a modified Weyl-Van der Pol formula calculated at the retarded time and in which the boundary wave uses a modified numerical distance. The latter is a function of the Doppler factor of the source motion. This solution can be extended to the case of arbitrary source motion and two particular cases involving circular paths are presented. These examples show the difference between the influence of the change in source location and the Doppler effect.

2. FORMULATION OF THE SOUND FIELD

2.1 KINEMATICS

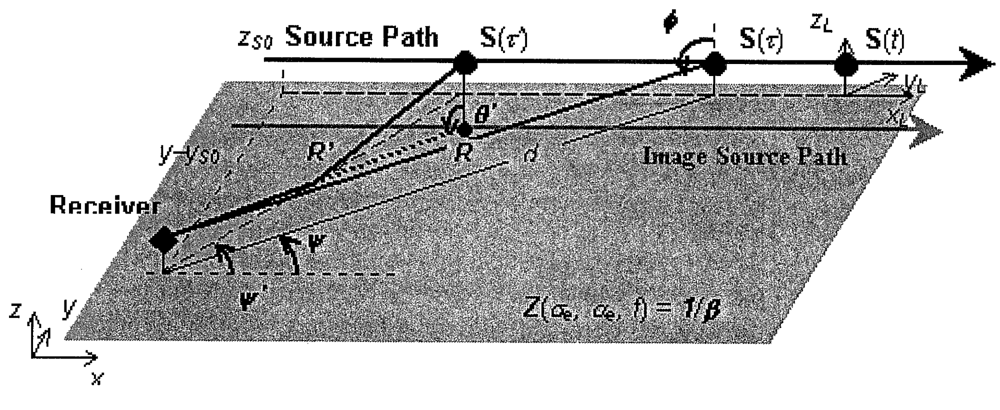


Figure I: Geometry and notations for a source in motion in parallel to the ground

Consider a source S in arbitrary motion charcterised by the Mach number vector $\mathbf{M}(t) = \mathbf{V}(t)/c_0 \equiv (M_x, M_y, M_z)$. The source position at observation time t is:

$$\mathbf{r}_S(t) = \mathbf{r}_{S0} + c_0\mathbf{M}(t) \equiv (x_S(t), y_S(t), z_S(t)) \quad (1)$$

with $\mathbf{r}_{S0} \equiv (x_{S0}, y_{S0}, z_{S0})$ the source initial position.

Consider a receiver O at position $\mathbf{r} \equiv (x, y, z)$

The direct sound perceived at the receiver at time t has been emitted by the source at retarded time τ such that:

$$\tau = t - |\mathbf{r} - \mathbf{r}_S(t)|/c_0 \qquad (2a)$$

Let R be the length of the direct ray: $R = |\mathbf{r} - \mathbf{r}_S(\tau)|$. The emission time τ is solution of the system of equation composed of eq. (2a) and

$$R = c_0(t - \tau) \qquad (2b)$$

In the presence of ground at height z = 0, an interfering reflection occurs. According to the method of image source; the retarded for emission τ' of this ground reflection is solution of

$$\tau = t - |\mathbf{r} - \mathbf{r}'_S(t)|/c_0 \qquad (3a)$$

$$R' = c_0(t - \tau') \qquad (3b)$$

Where the dash stands for the corresponding variables for the interfering reflection.

In the case of uniform subsonic motion along the x-axis, $\mathbf{M} \equiv (M\,0\,0)$. The retarded time equations for τ and τ' can be calculated analytically, and hence,

$$R = \left[M(x - x_{S0} - c_0 Mt) + R_1\right]/\left(1 - M^2\right) \qquad (4a)$$

with $$R_1 = \sqrt{(x - x_{S0} - c_0 Mt) + (1 - M^2)[(y - y_{S0})^2 + (z - z_{S0})^2]}$$
(4b)

$$R' = \left[M(x - x_{S0} - c_0 Mt) + R'_1\right]/\left(1 - M^2\right) \qquad (4c)$$

with $$R'_1 = \sqrt{(x - x_{S0} - c_0 Mt) + (1 - M^2)[(y - y_{S0})^2 + (z + z_{S0})^2]}$$
(4d)

In more complex situations, the evaluation of the retarded times must be carried out numerically. For subsonic motion however, the solutions are unique and hence simple numerical schemes converge fast.

2.2 VELOCITY POTENTIAL FIELD

2.2.1 UNIFORM MOTION

Using a Lorentz transformation [1],

$$x_L = \gamma^2(x - x_{S0} - c_0 Mt)\,;\; y_L = \gamma(y - y_{S0})\,;\; z_L = \gamma z\,;\; t_L = \gamma^2\left[t - M(x - x_{S0})/c_0\right]$$

$$h_L = \gamma z_{S0}\,;\; \gamma = \left(1 - M^2\right)^{-1/2}$$
(5)

The governing equations for the velocity potential field for a harmonic source in uniform motion above a ground with admittance β become, in the transformed coordinates,

$$\nabla_L^2\varphi - \frac{1}{c_0^2}\frac{\partial^2}{\partial t_L^2}\varphi = \gamma^2 e^{-i\omega\left(t_L + \frac{M}{c_0}x_L\right)}\delta(x_L)\delta(y_L)\delta(z_L - h_S) \qquad (6a)$$

$$\left.\frac{1}{c_0}\frac{\partial\varphi}{\partial t_L}\right|_{z_L=0} - M\left.\frac{\partial\varphi}{\partial x_L}\right|_{z_L=0} - \left.\frac{1}{\gamma\beta}\frac{\partial\varphi}{\partial z_L}\right|_{z_L=0} = 0 \qquad (6b)$$

The wave equation given in eq. (6a) is similar to that for a stationary source. The boundary condition in eq. (6b) is somewhat different from the motionless situation. Nevertheless, following in-line the derivation of the Weyl-Van der Pol formula for a stationary point source [2], the total velocity potential field can be expressed as [3],

$$\varphi(\mathbf{r}_L, t_L) = \gamma^2\frac{\exp(-i\omega t_L)}{4\pi}\left\{\frac{\exp(ik_0 R_L)}{R_L} + \left[\Re_p + (1 - \Re_p)F(w_L)\right]\frac{\exp(ik_0 R'_L)}{R'_L}\right\}$$
(7)

Where the plane wave reflection coefficient $\Re_P$ remains unaffected by the transformation:

$$\Re_P = \frac{\cos\theta_L - \beta_L}{\cos\theta_L + \beta_L} = \frac{\cos\theta' - \beta}{\cos\theta' + \beta} \qquad (8)$$

with θ_L and θ' are the incidence angles of the reflected wave in the Lorentz space and in the physical space respectively. β_L is an effective admittance [3], to which we shall refer as *modified admittance*. It follows from eq. (8) that

$$\beta_L = \gamma\beta(1 + M\cos\psi_L \sin\theta_L) = \beta\sqrt{1-M^2}\,/(1 - M\cos\psi'\sin\theta')$$

(9)

ψ_L and ψ' are the azimuthal angles of the reflected ray in the Lorentz and physical space respectively. $(1 - M\cos\psi'\sin\theta')^{-1}$ is the Doppler factor for the reflected wave [3]. F is the usual boundary loss function, with argument the *modified numerical distance* w_L

$$F(w_L) = 1 + i\sqrt{\pi}w_L e^{-w_L^2}\operatorname{erfc}(-iw_L) \qquad (10a)$$

$$w_L = \sqrt{\tfrac{1}{2}ik_0 R_L'}\,(\beta_L + \cos\theta_L) = w'/\sqrt{1 - M\cos\psi'\sin\theta'}$$

(10b)

Where w' is the numerical distance for a stationary source at the position corresponding to the interfering reflection. The boundary loss factor takes the same form as that for a stationary source, except for a correction coefficient equal to the square root of the Doppler factor.

2.2.2 ARBITRARY MOTION

The expression for the direct wave φ_1 is well know as the free field solution and can be derived by means of the Green function of the problem. For subsonic motion [4],

$$\varphi_1 = \exp(-i\omega\tau)/4\pi R(1 - M_r) \qquad (11)$$

Where $M_r = \mathbf{M}\cdot(\mathbf{r} - \mathbf{r}_S(\tau))/R$ is the component of the Mach number along the direct wave path ($M_r = M\cos\psi\sin\phi$ for uniform motion). $(1 - M_r)^{-1}$ can be identified as the Doppler factor for the direct wave.

The reflected wave is also straightforward, by means of the method of image source. Extending the results for uniform motion given in Sec. 2.2.1, the plane wave reflection coefficient is not affected by source motion. Similarly, the ground wave takes the usual form except for the argument of the boundary loss function. this one is the numerical distance for a stationary source corresponding to the interfering reflection corrected by the square root of the Doppler factor for the reflected wave . Hence the total velocity potential field due to a source in arbitrary motion can be written in the form of the *modified Weyl-Van der Pol formula*:

$$\varphi = \frac{\exp(-i\omega\tau)}{4\pi R(1 - M_r)} + \left[\Re_P + (1 - \Re_P)F\left(\frac{w'}{\sqrt{1 - M_r'}}\right)\right]\frac{\exp(-i\omega\tau')}{4\pi R(1 - M_r')}$$

(12)

Where the dash stands for the corresponding variables for the interfering reflection. The same result is obtained for uniform motion by means of a Lorentz transformation [3].

2.3 ACOUSTIC PRESSURE

Moving sources are of different nature whether they satisfy the wave equation for the velocity potential or that for the acoustic pressure [5,6]. Only the first have physical relevance as the fluid emitted is then convected with the source along its motion [5]. The second model describes an line of intermittent stationary source. As a result, the acoustic pressure in a homogeneous atmosphere being proportional to the time derivative of the velocity potential [1], it satisfies

$$\left[\nabla^2 - \frac{1}{c_0^2}\frac{\partial}{\partial t^2}\right]p = \exp(-i\omega t)\left[1 - \frac{i}{k_0}\left(\mathbf{M}\cdot\vec{\nabla}\right)\right]\left[\delta(x - x_S(t))\,\delta(y - y_S(t))\,\delta(z - z_S(t))\right]$$

(13)

Where the term inversely proportional to the wave number k_0 in the Right hand side of eq. (13) characterises radiation of sound from a dipole laying along the tangent of the source path. This extra contribution results from source convection as it is proportional to the Mach number. The boundary condition is the same as that for the velocity potential. In the case of uniform motion, the solution in the Lorentz space is straightforward, by means of the modified admittance defined in eq. (9) and knowing the solution for ground reflection of the dipole field [7].

$$p(\mathbf{r}_L, t_L) = \gamma^4\frac{\exp(-i\omega t_L)}{4\pi}\left\{\left(1 + i\frac{Mx_L}{k_0 R_L}\frac{1 - ik_0 R_L}{R_L}\right)\frac{\exp(ik_0 R_L)}{R_L} + \right.$$
$$\left. + \left[\Re_P + (\Re_P)F(w_L)\right]\left(1 + i\frac{Mx_L}{k_0 R_L'}\frac{1 - ik_0 R_L'}{R_L'}\right)\frac{\exp(ik_0 R_L')}{R_L'}\right\}$$

(14)

For arbitrary source motion, although the Lorentz transformation cannot be used, the pressure field can be derived differentiating the velocity potential field as expressed in eq. (12). Considering that variations of the reflection coefficient for long ranges and low source speeds are small and may hence be neglected,

$$p = \frac{\exp(-i\omega t)}{4\pi}\left\{\left[\left(1 + \frac{1}{ik_0 R}\frac{M^2 - M_r}{1 - M_r} + i\frac{\dot{M}_r}{\omega(1 - M_r)}\right)\right]\frac{\exp(ik_0 R)}{4\pi R(1 - M_r)^2} + \right.$$
$$\left.\left[\Re_P + (1 - \Re_P)F\left(\frac{w'}{\sqrt{1 - M_r'}}\right)\right]\left[\left(1 + \frac{1}{ik_0 R'}\frac{M'^2 - M_r'}{1 - M_r'} + i\frac{\dot{M}_r'}{\omega(1 - M_r')}\right)\right]\frac{\exp(ik_0 R')}{4\pi R'(1 - M_r')^2}\right\}$$

(15)

3. NUMERICAL CALCULATIONS

3.1 UNIFORM MOTION

Figure II shows calculations of the instantaneous Excess Attenuation due to a source in uniform motion at Mach number 0.3 above ground modelled by means of a two parameter model [8]. This diagram shows the influence of motion on the ground effect dip.

When the source approaches the receiver, the latter is shifted towards lower frequencies and hence the ground is seen as softer. On the other hand, when the source recedes, the ground is seen as harder. For the chosen geometry and the displayed source positions, the shift in frequency of the ground effect dip is of the order of 100 Hz.

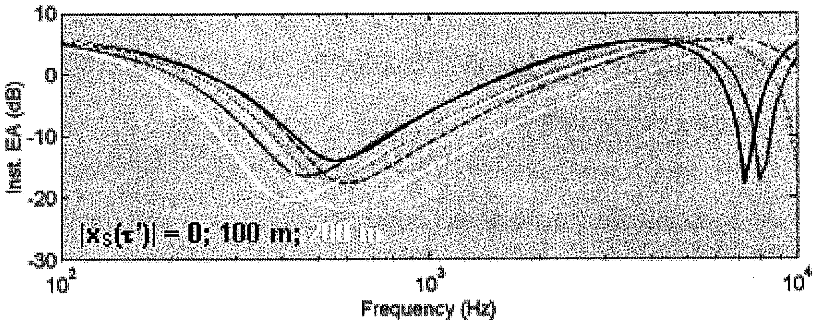
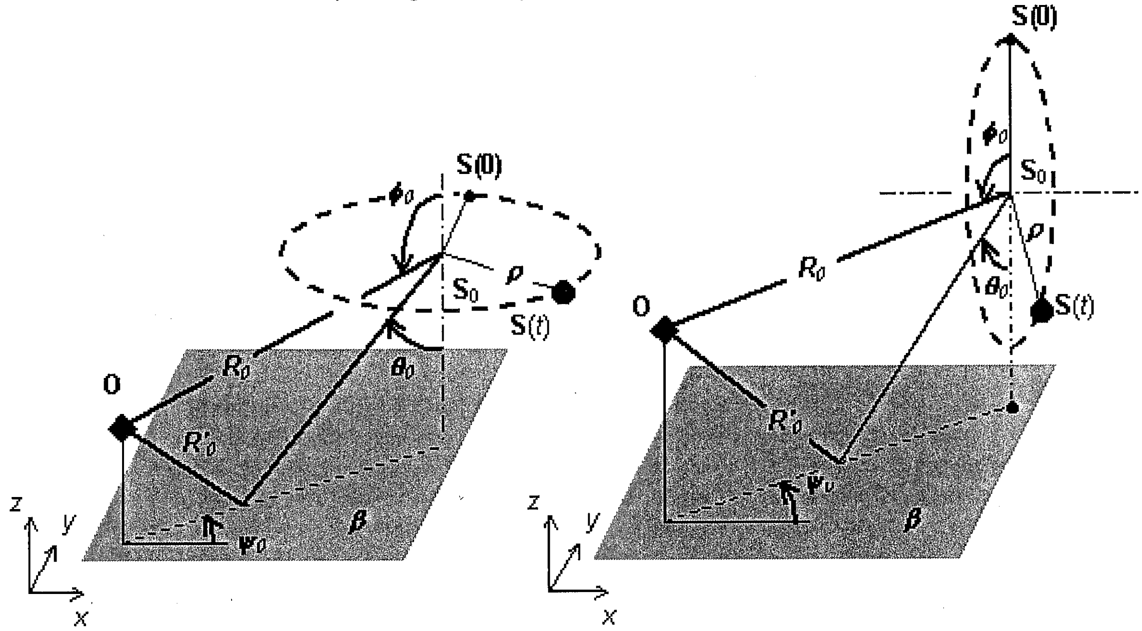


Figure II: *Instantaneous Excess Attenuation for a source approaching (solid lines) and receding (dashed lines) at uniform Mach number 0.3 above ground charaterised by σ_e = 140 kPa.s.m^{-2}; α_e = 35 m^{-1}. The source and receiver height are 2 m and 1.2 m respectively. The separation y–y_0 is 100 m. The dotted line show calculations for the corresponding stationary source.*



3.2 CIRCULAR MOTION

Figure III: *Geometry and notations for circular source motions (a) about the vertical axis; b) about the horizontal axis).*

3.2.1 CIRCULAR MOTION ABOUT THE VERTICAL AXIS

Consider a source in motion about the vertical axis (see fig. III) at constant tangential speed. The source position vector is

$$\mathbf{r}_S(t) = \begin{pmatrix} x_S(t) \\ y_S(t) \\ z_S(t) \end{pmatrix} = \mathbf{r}_0 + \begin{pmatrix} \rho \sin\Omega t \\ \rho\cos\Omega t \\ 0 \end{pmatrix} \tag{16}$$

where $\mathbf{r_0} \equiv (x_0, y_0, z_0)$ is the coordinate vector of the centre S_0 of the source path and ρ its radius.

$\Omega = c_0 M/\rho$ is the rotational speed of the source. The time of emission of the direct wave τ is solution of

$$\tau = t - \tau_0\sqrt{1+\eta^2 - 2\eta\sin(\Omega\tau + \psi_0)\sin\phi_0} \quad \text{with } \tau_0 = R_0/c_0 \;\; ; \;\; \eta = \rho/R_0 \;\; ; \;\; R_0 = |\mathbf{r} - \mathbf{r_0}|$$

(17) **a)** **b)**

Since equati , cannot be solved analytically, τ must t ated numerically in the interval

$$t - \frac{R_0 + \rho}{c_0} \leq \tau \leq t - \frac{R_0 - \rho}{c_0} \qquad (18)$$

The component of the Mach number in the Source – Receiver direction is thence

$$M_r = \frac{\rho\Omega}{c_0}\frac{R_0}{R}\sin\phi_0\cos(\Omega_H\tau + \psi_0) \qquad (19)$$

For long ranges, the incidence angle ϕ_0 (and θ_0) is close to $\pi/2$ and hence the Doppler factor $(1 - M_r)^{-1}$ shows important variations in the range $(1 + M)^{-1}$ to $(1 - M)^{-1}$.

Relations analogous to those in eq. (16) to (19) in are found for the reflected wave, substituting the corresponding variables.

The total pressure field can hence be calculated by means of eq. (15). Plots of the Excess attenuation for a source in circular motion with radius 3 m are shown in figure IV.a.

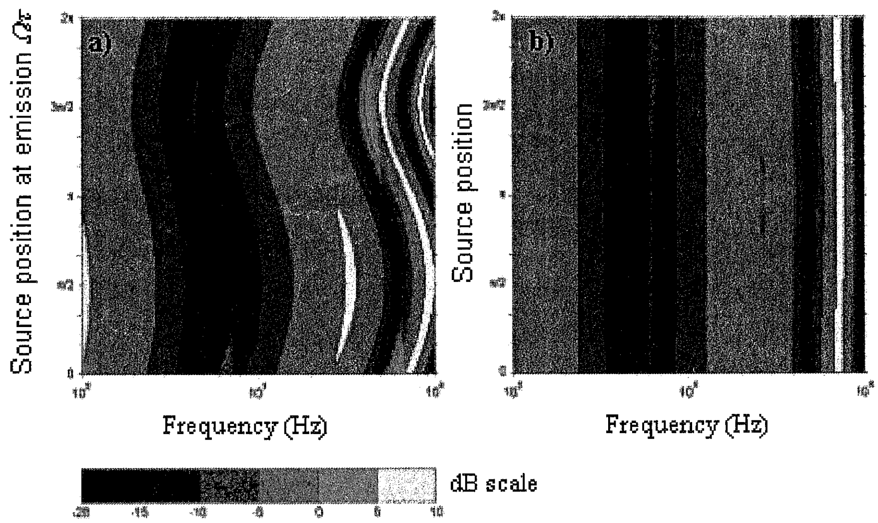


Figure IV: *a) instantaneous Excess Attenuation for a source in circular motion about the vertical axis at $M = 0.3.\rho = 2$ m; $\mathbf{r_0} \equiv (0, 0, 3$ m); $\mathbf{r} \equiv (0, 100$ m, 1.2 m); $\sigma_e = 140$ kPa.s.m^{-2}; $\alpha_e = 35$ m^{-1}. b) Calculation for the corresponding stationary source.*

The motion takes place in the horizontal plane, in which variations of the reflection coefficient are very small - particularly at long range. This is illustrated by the results for the sound field due to a stationary source shown in figure IV.b where the Excess Attenuation is almost the same for all source positions. As a result, the effects of motion on the sound field result mostly from the presence of the Doppler factor in the expression of the sound field. These are far from being negligible as the interference pattern shown in figure IV.a oscillates around the values calculated for the corresponding stationary source. For the chosen geometry, the location of the dip in instantaneous Excess Attenuation varies in a range of about 250 Hz.

3.2.2 CIRCULAR MOTION ABOUT THE HORIZONTAL AXIS

In the case of circular motion in the vertical plane (fig. III), the source path can be described by

$$\mathbf{r}_S(t) = \mathbf{r}_0 + \begin{pmatrix} \rho \sin \Omega t \\ 0 \\ \rho \cos \Omega t \end{pmatrix} \qquad (20)$$

The retarded time τ is hence solution of

$$\tau = t - \tau_0 \sqrt{1 + \eta^2 - 2\eta(\cos\psi_0 \sin\phi_0 \sin\Omega\tau + \cos\phi_0 \sin\Omega\tau)}$$

(21)

Eq. (21) must be solved numerically in the interval defined in eq. (18).
The component of the Mach number in the receiver direction is then

$$M_r = \frac{\rho\Omega}{c_0}(\cos\psi_0 \sin\phi_0 \cos\Omega\tau - \cos\phi_0 \sin\Omega\tau) \qquad (22)$$

Again, analogous relations are found for the reflected wave by means of the method of image source, substituting the corresponding variables into (20) to (22).
If the receiver is located in the mid-plane perpendicular to the source path ($x - x_0 = 0$) the azimuthal angle ψ_0 is equal to $\pi/2$. At long range, the incidence angle ϕ_0 (and θ_0) is close to $\pi/2$ and hence the Doppler factor is almost 1. The variations in the resulting instantaneous Excess Attenuation are shown in figure V.a. The interference pattern is then very similar to that due to a stationary source shown in figure V.b where a deeper ground effect dip is observed at lower source heights. The effects of motion are thus mainly related to the changes in source position.

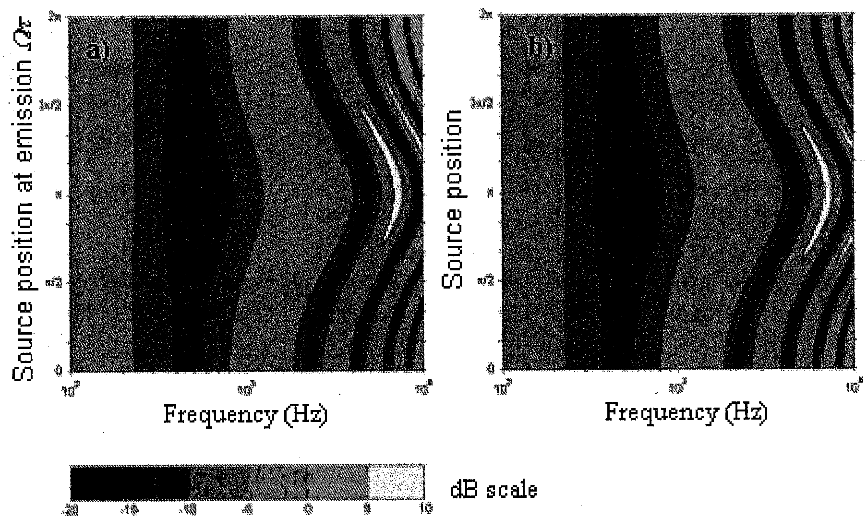
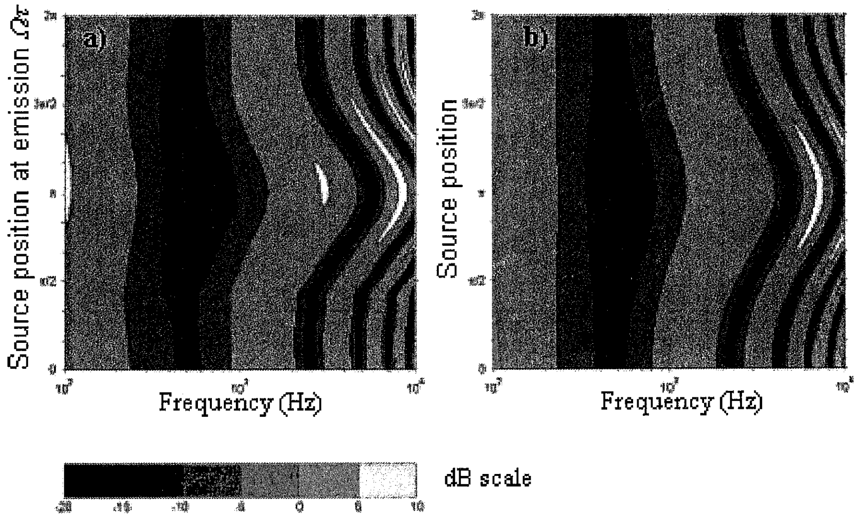


Figure V: *instantaneous Excess Attenuation for a source in circular motion about the horizontal axis at $M = 0.3$. $\rho = 2$ m; $\mathbf{r_0} \equiv (0,0,\ 3\ m)$; $\sigma_e = 140$ kPa.s.m^{-2}; $\alpha_e = 35$ m^{-1}. a) Receiver in the mid-plane of the source motion $\mathbf{r} \equiv (0,\ 100\ m,\ 1.2\ m)$ b) Calculation for the corresponding stationary source.*



If the receiver is located in the same plane as that of the source motion ($y-y_0 = 0$), the azimuthal angle is equal to zero and hence the Doppler factor varies within the range $(1+M)^{-1}$ to $(1-M)^{-1}$. In this configuration, the effects of motion are linked to the Doppler factor as well as the changes in source position. This case can hence be seen as an intermediate situation for those presented in figures IV and V. The corresponding results are shown in figure VI.a.

Figure VI: *instantaneous Excess Attenuation for a source in circular motion about the horizontal axis at $M = 0.3$.* $\rho = 2$ m; $\mathbf{r_0} \equiv (0,0,3\text{ m})$; $\sigma_e = 140$ kPa.s.m^{-2}; $\alpha_e = 35$ m^{-1}. *a) Receiver in the axis of the source motion* $\mathbf{r} \equiv (100\text{ m}, 0, 1.2\text{ m})$ *b) Calculation for the corresponding stationary source.*

Comparing with the sound field due to the corresponding stationary source given in figure VI.b, a slight oscillation pattern is observed. Nevertheless, it is obvious that motion effects are mainly related to the variations in sound wave incidence resulting from the evolution in the source location in the vertical plane.

4. CONCLUSIONS

By means of considerations for a source in uniform motion, the sound field due to a source in arbitrary motion above an impedance ground is expressed in the form of a modified Weyl-Van der Pol formula. In this expression, a correction proportional to the Doppler factor accounts for the effects of source motion on the boundary wave. Numerical calculations have been carried out for uniform motion in parallel to the ground and for circular revolution in the horizontal plane and in the vertical plane. These situations allows insight into the effects of source motion on the sound field. When the source approaches the receiver, the ground is seen as softer. It is seen as harder when the source recedes. As a result, when the source is in circular motion in the horizontal plane, the interference pattern somewhat oscillates. When the source is motion about the horizontal axis, its location in the vertical plane varies. The resulting changes in the incidence of direct and reflected waves have stronger effects on the sound field than the Doppler shift.

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