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EFFECT OF PERMEABILITY ON ACOUSTIC PROPERTIES OF DOUBLE-LEAF MEMBRANES

M Kiyama (1), K Sakagami (2) & M Morimoto (2)

(1) Graduate School of Science & Technology, Kobe University,Japan and Nippon Sheet Glass, Environmental Amenity Ltd, Osaka, Japan, (2) Environmental Acoustics Laboratory, Faculty of Engineering, Kobe University, Japan

1. INTRODUCTION

Though membrane-type absorption has been supposed to occur only in structures which include a massive back wall, the authors have pointed out that absorption of this type occurs also in light-weight impermeable double-leaf membrane [1]. It is noted, though, that permeable membranes give higher absorption at high frequencies than do impermeable ones in single-leaf membrane-type absorbers [2]. The same is expected in double-leaf membranes.

In this paper, permeability in one of the leaves of a double-leaf membrane is considered to be variable, theoretical solutions to reflected and transmitted sound pressure are derived, and effects of parameters on acoustic properties are discussed.

2. THEORETICAL CONSIDERATIONS

Assume that a plane wave impinges on a double-leaf membrane consisting of two infinite membranes (Fig. 1) at an angle of incidence θ. Only the first membrane at the incident side of the two membranes is permeable. The surface density and tension of the membranes are m_1, T_1 and, m_2, T_2, respectively. The thickness of the air cavity is d. The permeability of the first membrane is characterized by flow resistivity Rh, where h is the thickness of the first membrane. The specific acoustic admittance of the incident side and transmission side surfaces of the second membrane are A_3 and A_4, respectively.

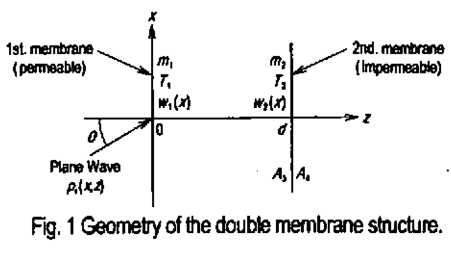

Fig. 1 Geometry of the double membrane structure.

The displacement of each membrane is $w_1(x)$ and $w_2(x)$. The time factor, $e^{i\omega t}$, is suppressed throughout.

The sound pressure on the front and back sides of the first membrane, $p_1(x,z)$ and $p_2(x,z)$, and similarly, $p_3(x,z)$ and $p_4(x,z)$ in the second membrane, can be expressed, using the pressure difference Δp between sides of the first membrane, as follows:

$$p_1(x,z) = 2p_i(x,0) + \frac{i}{2}\int_{-\infty}^{\infty}[\rho_0\omega^2 w_1(x_0) + ik_0 A_M \Delta p(x_0)]H_0^{(1)}(k_0|x-x_0|)dx_0\,, \tag{1}$$

$$p_2(x,z) = \frac{-2i\rho_0 c_0\omega w_2(x)\cos\theta + [i\rho_0 c_0\omega w_1(x) - A_M\Delta p(x)][(\cos\theta - A_3)e^{i\varphi} + (\cos\theta + A_3)e^{-i\varphi}]}{\cos\theta[(\cos\theta - A_3)e^{i\varphi} - (\cos\theta + A_3)e^{-i\varphi}]}\,, \tag{2}$$

$$p_3(x,z) = \frac{-i\rho_0 c_0\omega w_2(x)(e^{i\varphi} + e^{-i\varphi}) + 2[i\rho_0 c_0\omega w_1(x) - A_M\Delta p(x)]}{(\cos\theta - A_3)e^{i\varphi} - (\cos\theta + A_3)e^{-i\varphi}}\,, \tag{3}$$

$$p_4(x,z) = -\frac{i}{2}\int_{-\infty}^{\infty}[\rho_0\omega^2 w_2(x_0) - ik_0 A_4 p_3(x_0,z)]H_0^{(1)}(k_0|x-x_0|)dx_0\,, \tag{4}$$

where the acoustic wave-number $k_0=\omega/c_0$, ω is the angular frequency, c_0 the sound speed, ρ_0 the air density, $A_M = \rho_0 c_0/Rh$, and $\varphi = k_0 d\cos\theta$.

Using the unit response of membrane $u_j(x)$ (j=1,2), the membranes' displacement are:

$$w_1(x) = \int_{-\infty}^{\infty}\Delta p(\xi)u_1(x-\xi)d\xi\,, \tag{5}$$

$$w_2(x) = \int_{-\infty}^{\infty}[p_3(x,z) - p_4(x,z)]u_2(x-\xi)d\xi\,. \tag{6}$$

Equations (1) to (6) can be solved for w and p in the wave-number space using, a Fourier transform. The solutions are substituted in to the Helmholtz integral to obtain reflected sound pressure $p_r(x,z)$ and transmitted sound pressure $p_t(x,z)$, which are:

$$p_r(x,z) = \frac{N\cos\theta - A_M(M+N) + i\rho_0 c_0\omega N\Gamma_1(k_0\sin\theta) - 2i\rho_0 c_0\omega A_M\Gamma_2(k_0\sin\theta)}{N\cos\theta - A_M(M-N)}e^{i(k_0 x\sin\theta - k_0 z\cos\theta)}\,, \tag{7}$$

$$p_t(x,z) = \frac{-i\rho_0 c_0\omega}{\cos\theta + A_4}\Gamma_2(k_0\sin\theta)e^{i[k_0 x\sin\theta + k_0(z-d)\cos\theta]}\,, \tag{8}$$

where $B(k) = (k_0^2 - k^2)^{1/2}$, the transferred unit response of the membranes $U_j(k) = [2\pi(T_j k^2 - m_j\omega^2)]^{-1}$ (j=1,2), and

$$\Gamma_1(k) = [R_1(k)\gamma_2(k) - R_2(k)\beta_2(k)]/[\beta_1(k)\gamma_2(k) - \beta_2(k)\gamma_1(k)]\,, \tag{9}$$

$$\Gamma_2(k) = [R_1(k)\gamma_1(k) - R_2(k)\beta_1(k)]/[\beta_2(k)\gamma_1(k) - \beta_1(k)\gamma_2(k)]\,, \tag{10}$$

$$\beta_1(k) = Q(k) - 2\pi i\rho_0 c_0\omega[k_0 N\cos\theta - B(k)M]U_1(k)\,, \tag{11}$$

$$\beta_2(k) = -4\pi i\rho_0 c_0\omega B(k)\cos\theta U_1(k)\,, \tag{12}$$

$$\gamma_1(k) = 4\pi i\rho_0 c_0\omega[B(k) + k_0 A_4]\{Q(k) - A_M[k_0 N\cos\theta - B(k)M]\}U_2(k)\,, \tag{13}$$

$$\gamma_2(k) = -NQ(k)[B(k) + k_0 A_4] + 2\pi i\rho_0 c_0\omega\{k_0 NQ(k) - 2\cos\varphi Q(k)[B(k) + k_0 A_4] - 4A_M B(k)\cos\theta[B(k) + A_4]\}U_2(k)\,, \tag{14}$$

$$R_1(k) = 4\pi B(k)N\cos\theta U_1(k)\,, \tag{15}$$

$$R_2(k) = 8\pi A_M B(k)N\cos\theta[B(k) + k_0 A_4]U_2(k)\,, \tag{16}$$

$$Q(k) = B(k)N\cos\theta + k_0 A_M N\cos\theta - B(k)MA_M\,, \tag{17}$$

$$M = (\cos\theta - A_3)e^{i\varphi} + (\cos\theta + A_3)e^{-i\varphi}\,, \tag{18}$$

$$N = (\cos\theta - A_3)e^{i\varphi} - (\cos\theta + A_3)e^{-i\varphi}\,. \tag{19}$$

3. RESULTS AND DISCUSSION

The field-incidence-averaged absorption and transmission coefficients, α and τ, are calculated from eqs. (7) and (8). The absorption coefficient includes the effect of the transmitted wave (expressed by the transmission coefficient), which is not appropriate for calculating actual energy loss in double-leaf membranes. Therefore, the difference $\alpha-\tau$ is used in the following to describe the real absorption. In the examples ρ_0=1.2[kg/m^3], c_0=340[m/s], $T_1=T_2$=1.0[N/m], $A_3=A_4$=0.026, and h=0.005[m] are assumed throughout. It has been confirmed that tension has no effect on acoustic characteristic [3].

Effects of Flow Resistivity

Figure 2 shows the effect of Rh on $\alpha-\tau$, when $R=\infty$[MKS-rayl/m], a tendency similar to that of the rigid wall membrane-type is seen, which is also similar to that of impermeable double-leaf membranes [2]. When 2×10^7[MKS-rayl/m]$\geq R \geq 2\times10^5$[MKS-rayl/m], the absorption at high-frequencies increases, making a plateau with slight fluctuations, typical of porous absorbents. The absorption reaches its maximum when $R=2\times10^6$[MKS-rayl/m]. This suggests the existence of an optimum Rh for maximum absorption, just as is the case for porous absorbents. As R decreases further, the absorption characteristics approach those of an impermeable single membrane.

Effects of Surface Density

Figure 3 & 4 show the effect on $\alpha-\tau$ of m_1 and m_2, respectively. In the frequency range below the dip at middle-frequencies, the absorption increases as m_1 increases, but the absorption decreases as m_2 increases. The dip frequency dose not vary with changes in m_1, but shifts to lower frequencies as m_2 increases. At frequencies above 2kHz, neither m_1 nor m_2 affect the absorptivity.

Effects of Cavity Depth

Figure 5 shows the effect of cavity depth d on $\alpha-\tau$, the peak value of $\alpha-\tau$ observed near 3kHz at d=0.05 [m], moves to lower frequencies as d increases while its maximum value decreases.

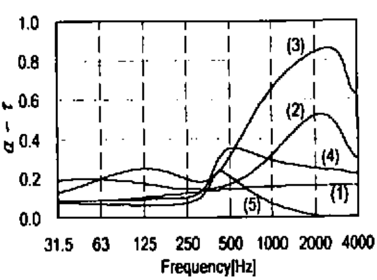

Fig. 2 Effect of the flow resistivity of the 1st membrane (R) on $\alpha-\tau$: R = 2×10^3(1), 2×10^5(2), 2×10^6(3), 2×10^7(4), ∞(5) [MKS-rayl/m] : m_1 = m_2 = 1.0[kg/m^2], d = 0.05[m] throughout.

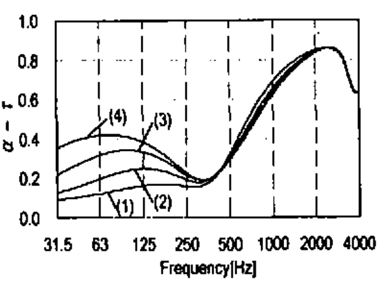

Fig. 3 Effect of the surface density of the 1st membrane (m_1) on $\alpha-\tau$: m_1 = 0.5(1), 1.0(2), 2.0(3), 4.0(4) [kg/m^2] : R= 2×10^6[MKS-rayl/m] m_2= 1.0[kg/m^2], d= 0.05[m] throughout.

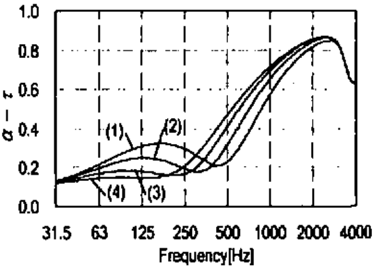

Fig. 4 Effect of the surface density of the 2nd membrane (m_2) on $\alpha-\tau$: m_2 = 0.5(1), 1.0(2), 2.0(3), 4.0(4) [kg/m^2] : R= 2×10^6[MKS-rayl/m], m_1 = 1.0[kg/m^2], d= 0.05[m] throughout.

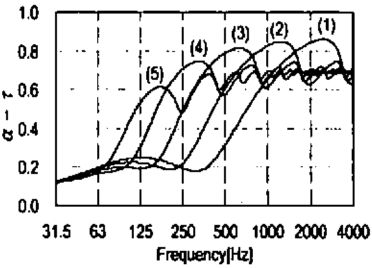

Fig. 5 Effect of the cavity depth (d) on $\alpha-\tau$: d= 0.05(1), 0.10(2), 0.20(3), 0.40(4) 0.80(5) [m] : R = 2×10^6[MKS-rayl/m], m_1 =m_2 = 1.0[kg/m^2], throughout.

4. SOUND ABSORPTION MECHANISM

At the low frequencies, when $d \ll \lambda$ (λ : wavelength), the sound pressure in the air cavity can be considered nearly constant. Therefore, the impedance of the system can be approximated to $[1/Rh+1/(-i\omega m_1)-i\omega m_2]^{-1}$, and the $\alpha-\tau$ to the normal incidence is expressed as follows:

$$\alpha-\tau=\left(4\frac{\rho_0 c_0}{Rh}\right)\Bigg/\left[1+2\frac{m_2}{m_1}+\left(\frac{m_2}{m_1}\right)^2+\left(\frac{\omega m_2}{Rh}\right)^2+4\left(\frac{\rho_0 c_0}{\omega m_1}\right)^2+4\left(\frac{\rho_0 c_0}{Rh}\right)^2+4\frac{\rho_0 c_0}{Rh}\right], \qquad (20)$$

This formula shows that $\alpha-\tau$ increases as m_1 increases, and decreases as m_2 increases. When m_1=∞, eq. (20) is expressed as follows:

$$\alpha-\tau=\left(4\frac{\rho_0 c_0}{Rh}\right)\Bigg/\left[1+\left(\frac{\omega m_2}{Rh}\right)^2+4\left(\frac{\rho_0 c_0}{Rh}\right)^2+4\frac{\rho_0 c_0}{Rh}\right], \qquad (21)$$

where $\omega m_2/Rh \approx 0$ in the low-frequency region. Therefore, eq. (21) becomes identical to the equation for $\alpha-\tau$ of a single permeable membrane; $\alpha-\tau$ tends toward the characteristics of a single permeable membrane at low frequencies as m_1 increases, and converges to a value determined by Rh only. On the contrary, $\alpha-\tau$ decreases with increasing m_2. However,$\alpha-\tau$ characteristics are similar to those in a membrane with a rigid wall, because of the effect of the air cavity.

Because it hardly vibrates in the high frequency region, the second membrane tends to act as a rigid wall. In other words, in the high frequency region, it becomes the characteristic of a permeable membrane with a rigid wall. The middle frequency range may be considered as a transient stage of change in characteristics from those of a permeable double-leaf membrane to those of a permeable membrane with a rigid wall.

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