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DYNAMICS OF CYLINDRICAL SHELLS IN CONTACT WITH SUBSONIC FLOW - FINITE ELEMENT MODELLING

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1. INTRODUCTION

Following the pioneer works of Warburton [1], Bolotin [2] and Chen [3], recently the researchers interest has been directed towards considering compressible flow with the corresponding fluid-structure coupling [4,5,6]. Despite the interesting findings in the above studies the main results still need additional analytical, numerical or experimental verification. The present work is intended to cast more light on the acoustic-structure coupling of light flow-shell systems using a numerical approach, namely 3D FE modelling.

2. FINITE ELEMENT MODELLING

Shell-Gas System for Quiescent Acoustic Fluid

Finite Element Model of an Elastic Circular Cylindrical Shell and an Annular Acoustic Layer. The structure under consideration is a circular cylindrical shell of given length and mean-surface radius and the compressible fluid is confined in the annulus between the flexible shell and a rigid wall. The thin elastic shell is discretised by using SHELL 63 elements and the acoustic fluid layer is modelled by FLUID 30- interfacing structure (KEYOPT 2=0), and FLUID 30- general (KEYOPT 2=1) possessing only one degree of freedom per node - the pressure(PRESS).

To switch on the coupling matrices creating a fluid-structure-interface (FSI) flag is issued for all shell and fluid elements in contact. The rigid wall confining the annulus is modelled by suppressing all displacements for the associated fluid elements. The fluid front and back boundaries are considered open (PRES=0).The elastic shell ends are specified to be simply supported - all translational degrees of freedom UX,UY,UZ=0 along the front and back edges of the elastic tube.

Assembled Governing Equations. Upon creating the FE model for mode-frequency analysis the final assembled set of fluid structure equations to be solved takes the following shape:

$$\begin{bmatrix} [M] & [0] \\ [M^{fs}] & [M^P] \end{bmatrix} \begin{Bmatrix} \{\ddot{U}\} \\ \{\ddot{P}\} \end{Bmatrix} + \begin{bmatrix} [K] & [K^{fs}] \\ [0] & [K^P] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \qquad (1)$$

Where $[M]$=structural assembled mass matrix, $[K]$=structural assembled stiffness matrix, $[M^P]$=fluid equivalent assembled "mass" matrix, $[K^P]$=fluid equivalent assembled "stiffness" matrix, $[M^{fs}]$=fluid-structure assembled coupling "mass" matrix, $[K^{fs}]$=fluid-structure assembled coupling "stiffness" matrix.

Eqs.(1) lead to unsymmetric matrices requiring unsymmetric eigenvalue extraction procedure. The FE programme ANSYS offers one unsymmetric routine (see [7])- the Lanczos' solver (UNSYM).

Shell-Fluid System for Flowing Acoustic Medium

The aerodynamic perturbation pressure acting on the elastic shell surface could be expressed as follows:

$$p(r) = -\rho_f\left(\frac{\partial\varphi}{\partial t} + U\frac{\partial\varphi}{\partial x}\right) \qquad (2)$$

Where $\varphi=\varphi(x,r,\theta,t)$ = perturbation velocity potential, U=mean steady flow velocity

Perturbation Velocity Pressure and the ANSYS FE Model. In order to realistically include the unsteady velocity pressure into the model it was decided to utilise the ANSYS prestressing capability along with a relatively complex pre-processing procedure intended to analytically determine and apply the relevant aerodynamic pressure pattern onto the thin-walled cylindrical shell. The coupled set of Eqs.(1), already takes into consideration the aerodynamic pressure term due to the quiescent fluid layer (quiescent pressure) by means of the fluid-structure coupling "mass" and "stiffness" matrices. This is the reason to consider Eq.(2) in the form $p_u(r) = -\rho_f U\frac{\partial\varphi}{\partial x}$. The perturbation velocity potential $\varphi(x,r,\theta)$ is to satisfy the wave (Poisson's) equation. For inviscid fluid we impose the impermeability boundary conditions at $r=R$ and $r=R_r$:

$$\left\{\frac{\partial\varphi}{\partial r}\Big|_{r=R} = \frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x} \quad and \quad \frac{\partial\varphi}{\partial r}\Big|_{r=R_r} = 0 \right. \qquad (3)$$

where R=elastic shell radius, R_r=rigid shell (wall) radius, $w(x,\theta,t)$=radial displacement function. We seek the unknown functions in the following form:

$$\begin{bmatrix} w \\ \varphi \end{bmatrix} = \begin{bmatrix} W \\ \Phi(r) \end{bmatrix} \exp[i(\omega t - \alpha x - n\theta)] \qquad (4)$$

where $\Phi(r)$=velocity potential amplitude, W=radial displacement wave coefficient, $\alpha=m\pi/L$=longitudinal wave parameter, n=circumferential wave parameter, m=axial halve wave number, L=shell length, ω=circular frequency of free vibration of the fluid-shell system. The amplitude function $\Phi(r)$ is to satisfy the modified Bessel's equation. Finally the expression for the perturbation velocity pressure in terms of the shell natural modes of vibration $W(x,\theta)$ is obtained as follows:

$$p_u = \rho_f R\mu(\beta, R, R_r)\left((\alpha U)^2 - 2\alpha U\omega\right) \; W(x,\theta)\exp i\omega t \qquad (5)$$

The parameter β is determined by $\beta = \sqrt{\left|\alpha^2 - \left(\frac{\omega - \alpha U}{c}\right)^2\right|}$.

The function μ represents the fluid influence onto the aerodynamic velocity pressure and thus onto the dynamics of the coupled system:

$$\text{for } \alpha > \left|\frac{\omega - \alpha U}{c}\right| \rightarrow \mu(\beta, R, R_r) = \frac{1}{\beta}\frac{Y_n(\beta R)J_n'(\beta R_r) - J_n(\beta R)Y_n'(\beta R_r)}{Y_n'(\beta R)J_n'(\beta R_r) - J_n'(\beta R)Y_n'(\beta R_r)} \tag{6}$$

$$\text{for } \alpha < \left|\frac{\omega - \alpha U}{c}\right| \rightarrow \mu(\beta, R, R_r) = \frac{1}{\beta}\frac{K_n(\beta R)I_n'(\beta R_r) - I_n(\beta R)K_n'(\beta R_r)}{K_n'(\beta R)I_n'(\beta R_r) - I_n'(\beta R)K_n'(\beta R_r)}$$

3. RESULTS

The finite element model developed presents a shell-gas system of following geometrical and material parameters- tube length L= 0.5m, 0.75m, 1m; elastic shell mean radius R= 0.3m; rigid wall radius R_r= 0.15m, 0.21m, 0.27m.; elastic tube material-aluminium of density ρ= 2710kg/m^3, Young's modulus E= 71GPa and Poisson's ratio ν= 0.34; light fluid - air of density ρ_f= 1.293kg/m^3, and speed of sound c= 331.3 m/s.

The elastic external shell is discretised by using 6 divisions over a quarter of the circumference and 12 divisions in axial direction - 228 SHELL 63 elements in total. The annular fluid layer is modelled by 144 uniform FLUID 30 elements- 6 divisions in radial direction and 24 in circumferential one. The presstresing of the elastic shell representing the unsteady perturbation velocity pressure is considered proportional to shell mode 1/8 - first longitudinal, eight circumferential (m=1, n=8) taken out of the single shell spectrum.

Fig.1 shows a comparison between the single shell, pure acoustic and shell-gas system structural and acoustic spectra. An excellent coincidence in acoustic frequencies is evident. The first axial shell frequencies are hardly affected by the presence of the acoustic layer but it is not the case about higher ones. All structural frequencies decrease in comparison to the single shell ones. In Fig.2 the acoustic, structural and acoustic/structural spectra are depicted. The so-called combined modes (high pressure and high displacement patterns) are clustered around the corresponding acoustic ones in the regions of many close-to-each-other structural modes. Fig. 3 and Fig.4 picture one aspect of the velocity influence onto the structural and acoustic spectra in the regions of strong shell-fluid coupling (structural modes near an acoustic one).The flow brings down the shell frequencies. The critical flow velocity for diversion is reached when the coefficient $\mu \rightarrow \infty$ at $\beta \rightarrow 0$. The corresponding acoustic spectrum in proximity to the structural mode in consideration is hardly affected by the flow. In conclusion, it is clear that the presence even of a very light compressible flow affects to a great degree the overall dynamic behaviour of the interacting system with a stronger effect on the structural and acoustic-structural (combined) spectra rather then on the acoustic one. The critical flow velocity (U_{cr}) for diversion could be predicted by solving the equation $\alpha = \left|\frac{\omega - \alpha U}{c}\right|$.

Structural and Acoustic Spectra for Single Shell, Acoustic Layer and Shell-Gas System of Parameters: L=0.5m, R=0.3m, Rr=0.15m

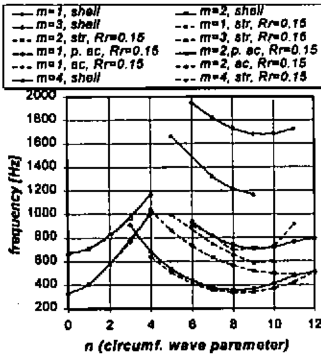

Fig. 1

Structural, Acoustic and Combined Spectra for Shell-Gas System Parameters: L=0.5m, R=0.3m, Rr=0.15m

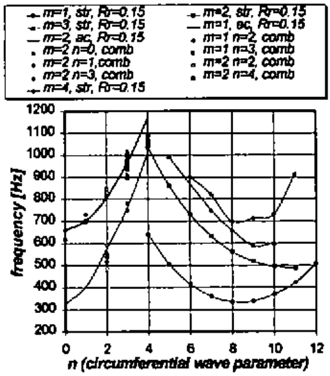

Fig.2

Acoustic Spectrum for Shell Lengths L=1m, 0.75m, 0.5m and Rr=0.21m, 0.27m

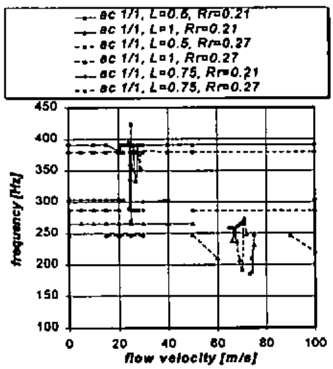

Fig. 3

Structural Spectrum for Shell Lengths L=1m, 0.75m, 0.5m and Rr=0.21m, 0.27m

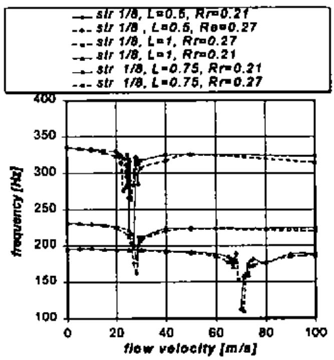

Fig. 4

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