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A NEW PROCEDURE OF ESTIMATION OF THE DEPENDENCE OF THE OPTIMAL ISOLATING SYSTEM FOR A SITTING HUMAN OPERATOR BODY ON THE FORM OF CRITERION OF VIBROISOLATION

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1. INTRODUCTION

The criterion choice is the fundamental problem in the synthesis of optimum vibration isolation systems. In the case of application to biomechanical models of the human-operator body the criterion should contain the principal parameters of vibration causing discomfort and other vibration syndromes. In this paper a general procedure of comparison of different criteria of vibration isolation for a sitting human-operator body has been presented. The procedure has been applied to a 2DOF biomechanical human-body model. For such a model three different kinds of criteria have been chosen and compared. For each criterion and two kinds of random excitations the optimal vibration isolation systems have been analytically synthesized as a result of the application of the Wiener-Hopf filtration theory. The numerical results have been presented graphically.

2. PROBLEM PRESENTATION

2.1. General assumptions.

Basing on the existing models of the sitting human-operator body [1], [3], [5] the following assumptions have been assumed: 1)The human operator body can be represented by a multi-DOF,linear, time invariant, lumped parameter biomechanical model, 2) Input random excitations are specified by power spectral densities of the accelerations of the vibrating base and are assumed to be stationary, normal and ergodic,

2.2. Analytical description of the general human-body model.

The general diagram of the human body model, HBM, and the unknown vibration isolation system, VIS, are presented in Fig.1. *F(t)* is the interaction force between the human-body model and the vibration isolation system. The corresponding transmissibility functions are given by

(1), where $H_{(x_1-x_0)/\ddot{x}_0}(s)$, $H_{\ddot{x}_1/\ddot{x}_0}(s)$, $H_{(x_i-x_k)/\ddot{x}_0}(s)$, $H_{\dddot{x}_i/\ddot{x}_0}(s)$ are the transfer functions for the variables $x_1(t)-x_0(t)$, $\ddot{x}_1(t), \ddot{x}_i(t)$, $x_i(t)-x_k(t)$, $\dddot{x}_i(t)$ and $\ddot{x}_0(t)$. *φ(s)* - the physically realizable function describing the optimal vibration isolation system, which may not have any poles in the right-hand side of the *s*-plane. *L(s)* - the transfer function between the *F(s)* and $x_1(s)$,

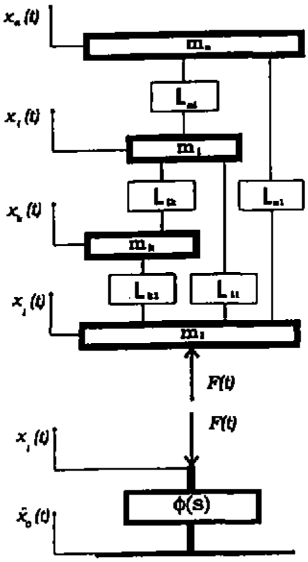

Fig.1

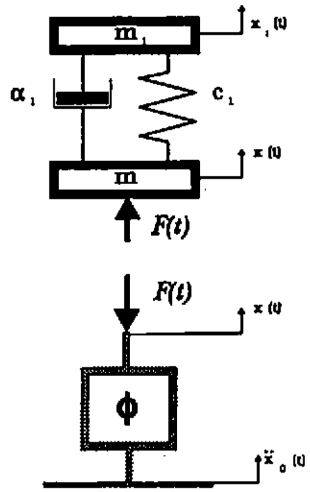

Fig.2

$$\left.\begin{aligned}
&H_{\frac{x_1-x_0}{\ddot{x}_0}}(s)=(\varphi(s)G(s)-1)s^{-2}\\
&H_{\frac{\ddot{x}_i}{\ddot{x}_0}}(s)=L_{i1}(s)\varphi(s)G(s)\\
&H_{\frac{x_i-x_k}{\ddot{x}_0}}(s)=(L_{i1}(s)-L_{k1}(s))\varphi(s)G(s)s^{-1}\\
&H_{\frac{\dddot{x}_i}{\ddot{x}_0}}(s)=sL_{i1}(s)\varphi(s)G(s)
\end{aligned}\right\} \qquad (1)$$

$L_{i1}(s)$-the transfer function between *$x_i(s)$* and *$x_1(s)$*, *$G(s)=s^2 L^{-1}(s)$*.

3. DETERMINATION OF OPTIMUM VIBRATION ISOLATION SYSTEM

3.1 General form of vibration isolation criterion.

As a general form of criterion of optimal vibration isolation synthesis for a One-Input-Multi-Output dynamical multibody system, the following expression was assumed:

$$J_G = \sigma^2_{x_1-x_0}(W) + \sum_{i=1}^{n} \lambda_i \sigma^2_{\ddot{x}_i}(W) + \sum_{i,k=1}^{n} \chi_{ik} \sigma^2_{x_i-x_k}(W) + \sum_{i=1}^{n} \mu_i \sigma^2_{\dddot{x}_i}(W) \tag{2}$$

where: $\sigma^2_{x_1-x_0}$ - mean square value of the relative displacement $x_1(t)-x_0(t)$, $\sigma^2_{\ddot{x}_i}$ - mean square value of the acceleration $\ddot{x}_i(t)$, $\sigma^2_{x_i-x_k}$ - mean square value of the relative displacement $x_i(t)-x_k(t)$, $\sigma^2_{\dddot{x}_i}$ - mean square value of the "jerk" $\dddot{x}_i(t)$ of the mass "i", and a set of lagrangian multipliers $W = \lambda_1, \dots \lambda_i, \dots, \lambda_n, \chi_{1,2}, \dots \chi_{i,k}, \dots \chi_{n-1,n}, \mu_1, \dots \mu_i, \dots \mu_n$, for $i = 1, \dots n, k = 1, \dots n, k \neq i$. The mean square values $\sigma^2_{x_1-x_0}$, $\sigma^2_{\ddot{x}_i}$ $\sigma^2_{x_i-x_k}$, $\sigma^2_{\dddot{x}_i}$ can be calculated as follows:

$$\sigma^2_y = \frac{1}{i} \int_{-i\infty}^{+i\infty} |H_y(s)|^2 S_{\ddot{x}_0}(s) ds \tag{3}$$

where according to formula (2) the index y signifies $x_1(t)-x_0(t)$, $\ddot{x}_i(t)$, $x_i(t)-x_k(t)$ or $\dddot{x}_i(t)$, respectively. $S_{\ddot{x}_0}(s)$ is the power spectral density of acceleration of excitation, which can be factorized as follows

$$S_{\ddot{x}_0}(s)s^{-4} = S_0 \psi(s) \psi(-s). \tag{4}$$

3.2 Determination of the optimum vibration isolation system.

To obtain the optimum vibration isolation system the Wiener-Hopf theory [2], [4] has been applied. By inserting expressions (1) and (3) into (2) one obtains

$$\begin{aligned} J = \tfrac{1}{i} \int_{-i\infty}^{i\infty} \Big\{ & |\varphi(s)G(s) - 1|^2 + \sum_{i=1}^{n} \lambda_i s^4 |L_{i1}(s)\varphi(s)G(s)|^2 + \\ & \sum_{i,k=1}^{n} \chi_{ik} |(L_{i1}(s) - L_{k1}(s))\varphi(s)G(s)|^2 - \sum_{i=1}^{n} \mu_i s^6 |L_{i1}(s)\varphi(s)G(s)|^2 \Big\} s^{-4} S_{\ddot{x}_0}(s) ds \end{aligned} \tag{5}$$

The function $\varphi(s)$ in eqs.(1) and (5) is unknown. Using variational calculus, $\varphi(s)$ can be calculated in such a way that criterion (5) takes on the minimum value. The resulting function can be written as follows

$$\varphi(s) = \frac{1}{R(s)G(s)\psi(s)} \left[\frac{\psi(s)}{R(-s)} \right]_+ \tag{6}$$

where symbol $[\ ]_+$ is the component of the internal function, which has its poles and zeros on the left half-plane of the complex variable s, and

$$\begin{aligned} R(s) = \Big[& 1 + \sum_{i=1}^{n} \lambda_i s^4 L_i(s) L_i(-s) - \sum_{i=1}^{n} \mu_i s^6 L_i(s) L_i(-s) + \\ & + \sum_{i,k=1}^{n} \chi_{ik} (L_{i1}(s) - L_{k1}(s))(L_{i1}(-s) - L_{k1}(-s)) \Big]_+ \end{aligned} \tag{7}$$

4.GENERAL PROCEDURE OF COMPARING THE CRITERIA

Let's select a subset of masses m_l, m_r, m_s where $l, r, s \in n$. The following subset of Lagrangian multipliers can be written for that subset of masses

$$M \in \lambda_l, \lambda_r, \lambda_s, \chi_{l,r}, \chi_{l,s}, \chi_{r,s}, \mu_l, \mu_r, \mu_s$$

In this case criterion (4) takes the following form

$$J_{Mi} = \sigma^2_{x-x_0}(M) + \lambda_l \sigma^2_{\ddot{x}_l}(M) + \lambda_r \sigma^2_{\ddot{x}_r}(M) + \lambda_s \sigma^2_{\ddot{x}_s}(M) + +\chi_{l,r} \sigma^2_{x_l - x_r}(M) + \chi_{l,s} \sigma^2_{x_l - x_s}(M) + \chi_{r,s} \sigma^2_{x_r - x_s}(M) + \mu_l \sigma^2_{\dddot{x}_l}(M) + \mu_r \sigma^2_{\dddot{x}_r}(M) + \mu_s \sigma^2_{\dddot{x}_s}(M) \tag{8}$$

To compare this particular criterion with the general criterion J_G the following accompanying criterion expression needs to be introduced

$$J_{GM} = \sigma^2_{x-x_0}(M) + \lambda_l \sigma^2_{\ddot{x}_l}(M) + \lambda_r \sigma^2_{\ddot{x}_r}(M) + \lambda_s \sigma^2_{\ddot{x}_s}(M) + +\chi_{l,r} \sigma^2_{x_l - x_r}(M) + +\chi_{l,s} \sigma^2_{x_l - x_s}(M) + \chi_{r,s} \sigma^2_{x_r - x_s}(M) + \mu_l \sigma^2_{\dddot{x}_l}(M) + \mu_r \sigma^2_{\dddot{x}_r}(M) + \mu_s \sigma^2_{\dddot{x}_s}(M) + + \sum_{l=1; l \neq l,r,s}^{n} \lambda_l \sigma^2_{\ddot{x}_l}(M) + \sum_{l,k=1; k \neq l,r,s}^{n} \chi_{l,k} \sigma^2_{x_l - x_k}(M) + \sum_{l=1; l \neq l,r,s}^{n} \mu_l \sigma^2_{\dddot{x}_{l}}(M) \tag{9}$$

The relationship J_{GM} / J_G allows the comparison of these two criteria.

5. APPLICATION TO A 2DOF HUMAN BODY MODEL

5.1 Model description.

To illustrate the general procedure, the 2DOF model of the sitting human - operator body [5] given in fig.2, is considered. This model can be represented by the function

$$G(s) = sZ^{-1}(s) = (m_1 s^2 + \alpha_1 s + c_1)/[m m_1 s^2 + (m + m_1)(\alpha_1 s + c_1)] \tag{10}$$

5.2 Excitations.

Two kinds of random acceleration excitations with the power spectral densities given in table 1, were considered.

5.3 2DOF HBM - comparison of impedance and transmitance criteria.

The impedance criterion.

$$J_1 = \sigma^2_{x_1 - x_0}(\lambda_1) + \lambda_1 \sigma^2_{\ddot{x}_1}(\lambda_1) \tag{11}$$

The transmitance criterion.

$$J_2 = \sigma^2_{x_1 - x_0}(\lambda_1, \lambda_2, \chi_{1,2}) + \sum_{l=1}^{2} \lambda_l \sigma^2_{\ddot{x}_l}(\lambda_1, \lambda_2, \chi_{1,2}) + \chi_{1,2} \sigma^2_{x_2 - x_1}(\lambda_1, \lambda_2, \chi_{1,2}) \tag{12}$$

The accompanying criterion of comparison.

$$J_{12} = \sigma^2_{x_1 - x_0}(\lambda_1) + \sum_{l=1}^{2} \lambda_l \sigma^2_{\ddot{x}_l}(\lambda_1) + \chi_{1,2} \sigma^2_{x_2 - x_1}(\lambda_1) \tag{13}$$

5.4 2DOF HBM - comparison of impedance and jerk criteria.

The jerk criterion.

$$J_3 = \sigma^2_{x_1 - x_0}(\lambda_2, \mu_2) + \lambda_2 \sigma^2_{\ddot{x}_2}(\lambda_2, \mu_2) + \mu_2 \sigma^2_{\dddot{x}_2}(\lambda_2, \mu_2) \tag{14}$$

The accompanying criterion of comparison.

$$J_{13} = \sigma^2_{x_1 - x_0}(\lambda_1) + \lambda_2 \sigma^2_{\ddot{x}_2}(\lambda_1) + \mu_2 \sigma^2_{\dddot{x}_2}(\lambda_1) \tag{15}$$

Table 1

	FORM OF OPTIMAL $\varphi_{ij}(s)$	
CRITERION	Power spectral density of white noise acceleration $S_{\ddot{x}_0}(s)=\frac{\sigma_0^2}{2\pi}$	Power spectral density of narrow-band noise acceleration $S_{\ddot{x}_0}(s)=\frac{\alpha\sigma_0^2}{\pi}\frac{\Omega^2-s^2}{(\Omega^2-s^2)^2-4\alpha^2s^2}$
impedance criterion	$\varphi(s)=\frac{(M_{11}s+N_{11})Z(s)}{(A_1s^2+B_1s+C_1)s}$	$\varphi(s)=\frac{(M_1s^3+N_1s^2+P_1s+Q_1)Z(s)}{(A_1s^2+B_1s+C_1)s(\Omega+s)}$
transmitance criterion	$\varphi(s)=\frac{(M_{21}s+N_{21})N(s)}{D_2(s)}$	$\varphi(s)=\frac{(M_2s^3+N_2s^2+P_2s+Q_2)N(s)}{D_2(s)(\Omega+s)}$
jerk criterion	$\varphi(s)=\frac{(M_{31}s+N_{31})N(s)}{D_3(s)}$	$\varphi(s)=\frac{(M_3s^3+N_3s^2+P_3s+Q_3)N(s)}{D_3(s)(\Omega+s)}$

where: $N(s)=mm_1s^2+(\alpha_1s+c_1)(m+m_1)$, $D_k(s)=A_ks^4+B_ks^3+C_ks+D_k$, for k=1,2. The constants in the table must be calculated as described in [4].

5.5 Numerical results.

The data concerning the HBM taken from [6] and the parameters of the power spectral density excitation were assumed as follows: $\Omega=27.17[s^{-1}]$ $\alpha=0.1[s^{-1}]$, $m_1=27.5[kg]$, $m=29.5[kg]$, $c_1=22955.4[kgs^{-2}]$, $\alpha_1=353.16[kgs^{-1}]$
In Fig.3 the relations J_{12}/J_2 and J_{13}/J_3 have been presented for white noise and narrow band noise acceleration excitations. These relations were presented as a function of Lagrangian multipliers λ_1,λ_2,χ, given by the expressions $\lambda_1=\lambda_{10}\rho_0/(1-\rho_0)$, $\lambda_2=\lambda_{20}\rho_1/(1-\rho_1)$, $\chi=\chi_0\rho_2/(1-\rho_2)$, where $\lambda_{10}=\lambda_{20}=\chi_0=1$ and $\rho_0,\rho_1,\rho_2\in(0,1)$.

6.CONCLUSIONS

From the numerical results presented in figure 3 it can be seen, that if the criterion structure is more complex the vibration isolation is better. However, the differences between the criteria depend on the values of Lagrangian multipliers ρ_0,ρ_1,ρ_2 . For some sets of the multipliers these differences are very small or even negligible. If the differences are negligible the application of the simpler criterion is preferable. In general, the simpler the criterion, the simpler is the structure of vibration isolation system. This fact is directly connected with the costs of construction of such a system.

References.

[1] Garg, D.P., Ross, M.A., (1976), Vertical Mode Human Body Vibration Transmissibility,IEEE Transactions on Systems, Man, and Cybernetics, No.2, February 1976, pp.102-112.

[2] Goupta, S.C., Hasdorff, L., (1981), Fundamentals of Automatic Control, Edited by John Willey and Sons, Inc, New York.

[3] Griffin, M.J.,(1990), Handbook of Human Vibration, Edited by Academic Press, London.

[4] Książek, M.,(1994), Comparison of Optimal Passive and Active Vibration Isolation Systems for a Sitting Human Operator Body, In Proceedings of the 5th International Conference on Structural Dynamics - Recent Advances, Institute of Sound and Vibration, University of Southampton, England, July 18-21, 1994, pp.924-933, Edited by N.S.Fergusson, H.F. Wolfe, C.Mei.

[5] Khatchaturov, A.A., et al.,(1976), Dinamika Sistiemy: Doroga-Schina-Avtomobil-Voditiel, Izd.Maschinostroienie, Moskva.

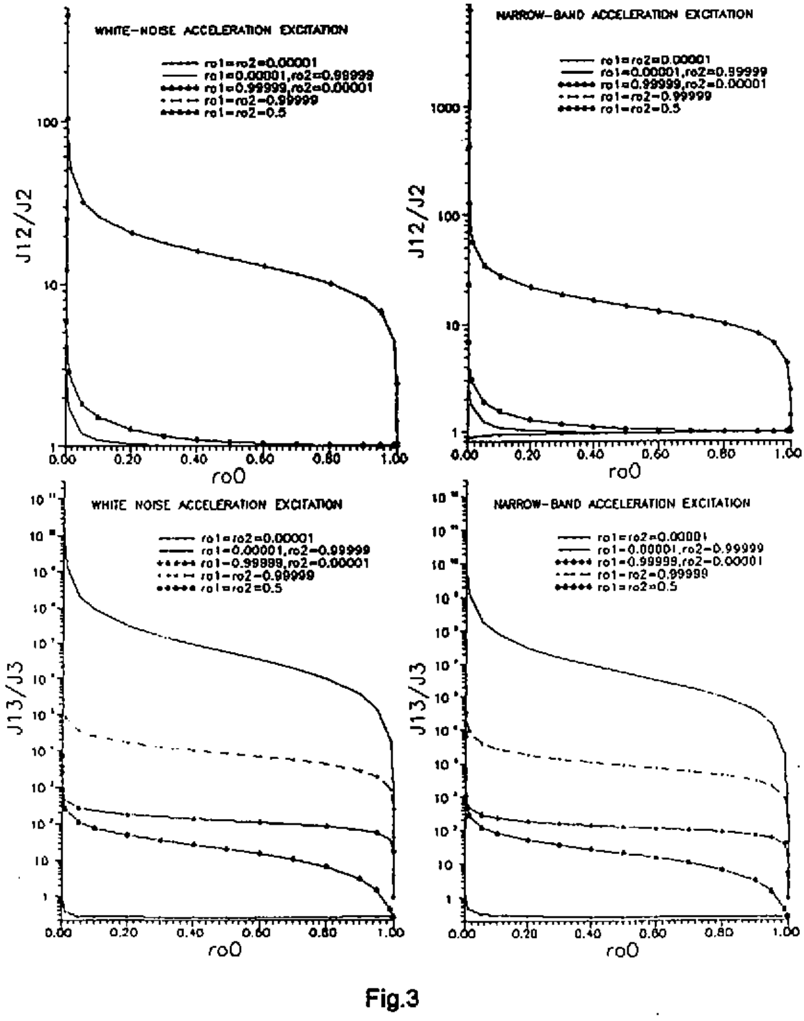

Fig.3