Proceedings of the Institute of Acoustics & Belgium Acoustical Society
Noise in the Built Environment, Ghent, 29-30 April 2010

SOURCE-RECEIVER MODEL OF A BUILDING EXCITED BY GROUND BORNE VIBRATION

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1. INTRODUCTION

In this paper, the problem of the vibrational behavior of a building excited through its foundations (structures in contact with ground) by incoming ground borne vibration (such as vibration from railways) is treated using a source-receiver approach applied to a 2D ground-structure configuration. The structures in contact with ground and therefore well coupled to it are naturally considered as the source and the upper structure of the building (above ground) considered as the receiver. This approach not only easily leads to an estimation of the vibrational power injected to the upper structure but also allows separating this power into different wave type components: (i) bending waves, of wavelength of the size of building elements, which are attenuated at junctions between building elements (floors and load bearing walls) and can radiate sound, and (ii) in plane waves, of much longer wavelength, which are less attenuated at junctions, and do not radiate sound. Moreover, once the source parameters have been estimated for a given ground-foundation configuration, then any type of upper structures can be connected, its vibrational behavior being estimated using a (simpler) purely structural model without ground.
First (section 2) the source-receiver approach is presented, its main input parameters introduced and the method used to calculate the vibrational power injected by the source to the receiver described. Then (section 3), the method is applied to a simple ground-structure configuration and its feasibility shown; in the same section, the method is also numerically validated by comparing the total power obtained using the source- receiver approach, to the power directly calculated from a FE model.

2. DESCRIPTION OF THE SOURCE-RECEIVER APPROACH

2.1 Source-receiver model and its input parameters

The 2D simplified ground-building configuration shown in Figure 1a can be split into a one contact source-receiver system (shown in figure 1b), assuming the power injected to the building by the second foundation is negligible.

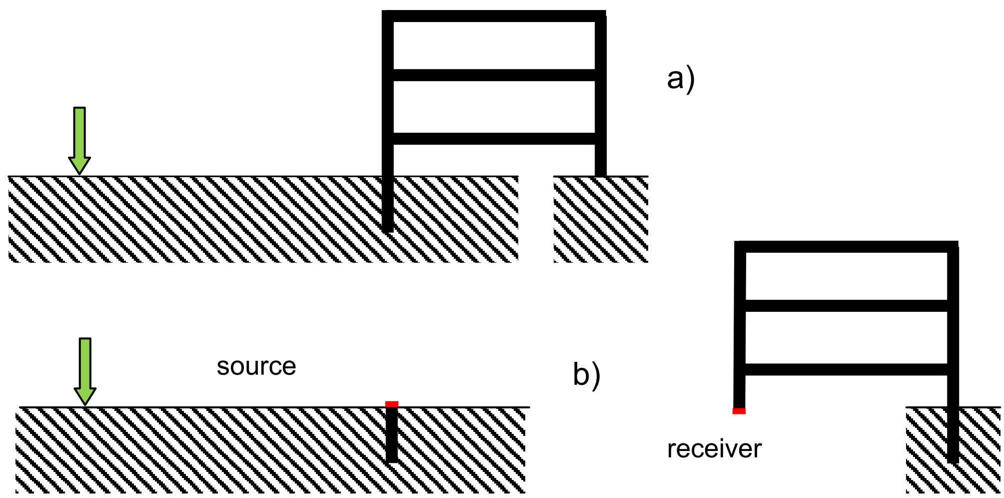

Figure 1: 2D ground-structure configuration (1a) split into a one contact source-receiver system (1b)

In the source model, a force representing an environmental vibration source is applied at a certain distance of the foundation (embedded wall). In the frequency range of environmental vibration (frequencies around 100 Hz and below), the structure thicknesses are small compared to the wavelengths and the contact section (in red in figure 1) stays straight and its movement reduces to 3 degrees of freedom: 2 translations (bending horizontal velocity v_b and longitudinal vertical velocity v_L) estimated from the velocity at the section center and one rotation (angular velocity v_θ) estimated from 2 points in the contact section. In the 2D model used, the source becomes therefore a point source with 3 degrees of freedom.
The source input parameters involved in calculating the power injected to the receiver are the source free velocity and the different source mobilities, which are evaluated using a 2D FEM/BEM ground-structure vibration interaction model developed at CSTB (MEFISSTO software, see [1]) as explained below.

Free velocity
When a force is applied to the ground, the upper section of the embedded wall (in red in figure 1b above) moves with the following three free velocity components: $v_{b\ free}$, $v_{L\ free}$ and $v_{\theta\ free}$.

Source longitudinal mobility
The vertical force distribution shown in figure 2a will generate a vertical velocity v_L, but no other velocity components (horizontal or angular) assuming the embedded wall is the only structure in the ground. The source longitudinal movement can therefore be treated separately from bending, and the source longitudinal mobility $Y_{S,L}$ calculated as:

$$Y_{S,L} = v_L / F_L \qquad (1)$$

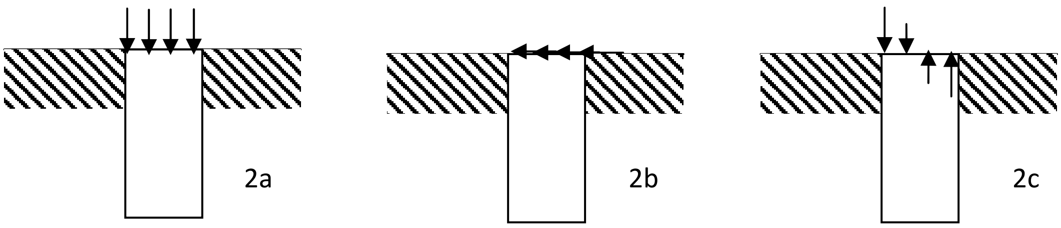

Figure 2: Force distribution used to calculate the source mobilities: longitudinal (2a), transversal / bending (2b) and angular / bending (2c)

Source bending mobilities
The transversal force distribution shown in figure 2b will generate not only a transversal (bending) velocity v_b but also an angular (bending) velocity v_θ, leading to two mobilities:

$$Y_{S,Fb\ vb} = v_b / F_b \quad \text{et} \quad Y_{S,Fb\ v\theta} = v_\theta / F_b \qquad (2)$$

Finally, the moment M generated by the force distribution shown in figure 2c will generate not only a transversal (bending) velocity v_b but also an angular (bending) velocity v_θ, leading to two mobilities:

$$Y_{S,M\ vb} = v_b / M \quad \text{et} \quad Y_{S,M\ v\theta} = v_\theta / M \qquad (3)$$

Receiver mobilities
Five receiver mobilities (index R) can also be defined: $Y_{R,L}$, $Y_{R,Fb\ vb}$, $Y_{R,Fb\ v\theta}$, $Y_{R,M\ vb}$, $Y_{R,M\ v\theta}$
The way of obtaining these mobilities depends on the (purely structural) model used: analytical mobilities can be calculated in simple cases such as infinite or finite single walls; for a more complex receiver, a 2D FEM model can be used, with the same force distributions as in figure 2 applied to the receiver.

2.2 Structural power calculation

Longitudinal structural power

In this case of point source with one degree of freedom (vertical velocity), the structural power injected to the receiver can be expressed as [2]:

$$P_{inj} = \frac{1}{2} |v_{L\,free}|^2 \frac{R(Y_{R,L})}{|Y_{S,L}+Y_{R,L}|^2} \quad (4)$$

Bending structural power

In this case of point source with two degree of freedom (transversal and angular velocities), the structural power injected to the receiver can be expressed as [3]:

$$P_{inj} = \frac{1}{2} R((v_{free})^T [([Y_S] + [Y_R])^T]^{-1} [Y_R^*] \left[[Y_S^*] + [Y_R^*]\right]^{-1} (v_{free}^*)) \quad (5)$$

where *[Y]* represents (2x2) mobility matrices expressed as:

$$[Y] = \begin{pmatrix} Y_{Fb\,vb} & Y_{Fb\,v\theta} \\ Y_{M\,vb} & Y_{M\,v\theta} \end{pmatrix} \quad (6)$$

3 APPLICATION TO A SIMPLE GROUND STRUCTURE CONFIGURATION

In order to show the feasibility of the method, the source receiver approach is applied to the very simple configuration showed in figure 3 and composed of a 3m long embedded wall and an infinite wall as receiving upper structure; in this case no power goes back into ground and no wave conversion bringing bending waves back occurs.

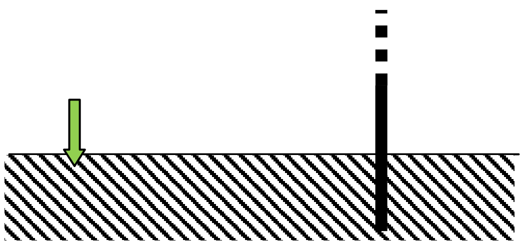

Figure 3: Simple configuration studied

The dynamic characteristics of the concrete are the following: E = 26 GPa, η = 0.40, ρ = 2400 kg/m^3 and μ = 0.3 ; the loss factor is (on purpose) very high so that the infinite wall (upper structure) can be easily modeled using FEM as a finite, 100 m long, wall. The ground has the following (not too stiff and not too soft) characteristics: E = 200 MPa, η = 0.1, ρ = 1600 kg/m^3 and μ = 0.25. A 1N force is applied to the ground surface at 10 m from the structure in a 0-250 Hz frequency range.

3.2 Source and receiver mobilities

First the source mobilities are estimated using MEFISSTO applied to the source configuration of figure 1b, using the force distribution shown in figure 2 and giving the results presented in figure 4. All the source mobilities are shown, including, $Y_{S,FL\,vb}$, $Y_{S,Fb\,vL}$ and $Y_{S,M\,vL}$. The upper graph shows in particular that a longitudinal excitation mainly generates a longitudinal movement ($Y_{S,FL\,vb}$ negligible) and the other graphs that a bending excitation (transversal force or moment) mainly generate a bending movement; bending and longitudinal movements can indeed be separated.

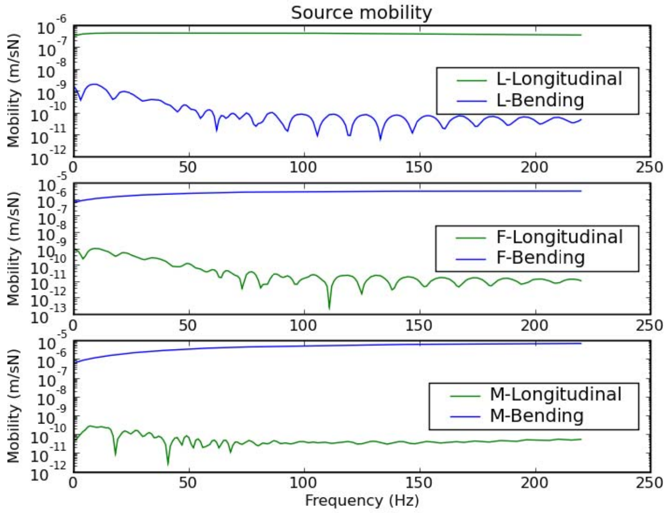

Figure 4: Source mobilities obtained using the MEFISSTO software

In the case considered of an infinite wall, the receiver mobilities can easily be analytically calculated (see [4] for example); the mobility ratios receiver / source obtained are given in figure 5, showing that the source mobilities are much lower than the receiver mobilities (case of velocity source) only at very low frequencies (below 50 Hz).

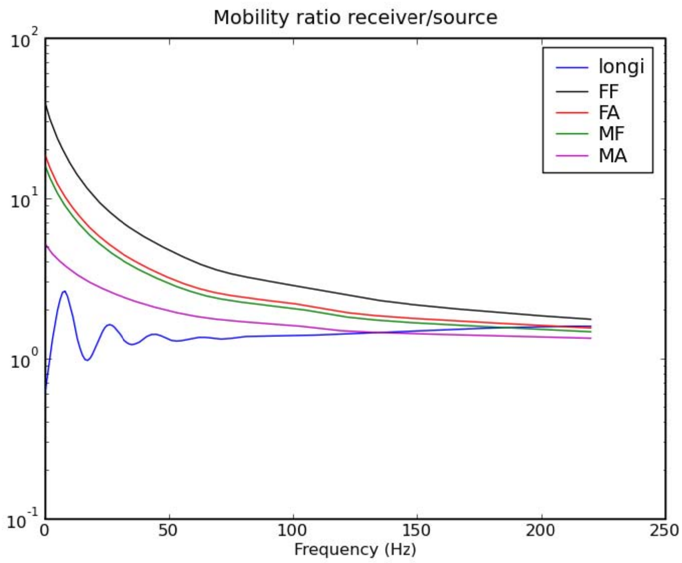

Figure 5: Mobility ratio receiver / source

3.3 Structural power calculation

The longitudinal and bending structural power injected into the receiver, calculated using equations (4) and (5) respectively, are given in figure 6, showing that the longitudinal power is dominant in this case.

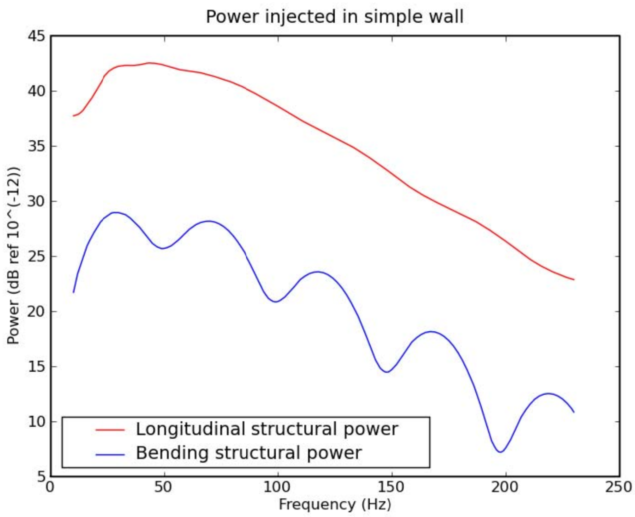

Figure 6: Longitudinal and bending structural power injected to the upper structure, obtained using the source-receiver method

3.4 Numerical validation

The whole ground structure configuration given in figure 3 is now modeled using MEFISSTO and the total structural power flow at the source-receiver contact section is directly expressed from the normal structural intensity calculated at each node *k* of the contact section from the normal and tangential stresses (σ and τ respectively) as:

– (7)

In figure 7 (middle curve), this numerically obtained power is compared to the total (mainly longitudinal) power shown in figure 6; the two curves are the same (maximum 2 dB difference) and the source-receiver approach therefore validated.

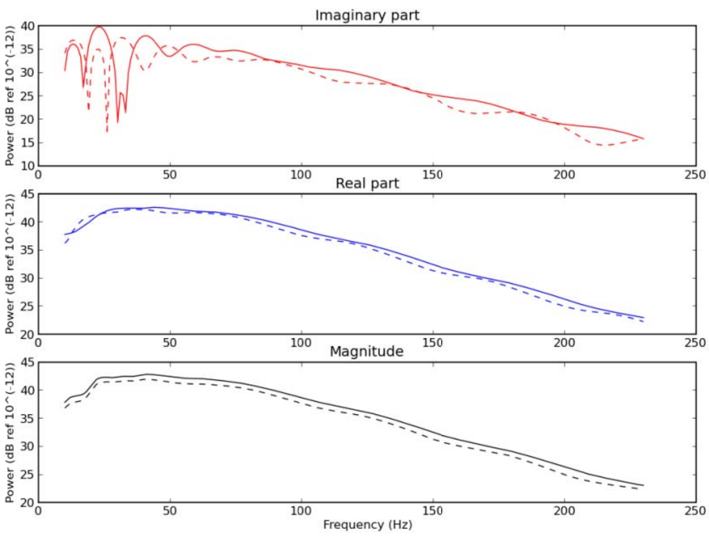

Figure 7: Total structural power injected to the receiver (middle curve), using the source- receiver approach (continuous line) and directly obtained from MEFISSTO (dotted line)

4 CONCLUSION

In this paper, a source-receiver approach is used to estimate the structural power injected by underground structures (source) into upper structures (receiver). The approach is applied to a 2D ground structure configuration. The method not only easily leads to an estimation of the vibrational power injected to the upper structure but also allows separating the power into wave type components: longitudinal and bending waves. Once the source parameters (source free velocities and input mobilities) have been estimated for a given ground-foundation configuration, then any type of upper structures with known input mobilities can be connected and its vibrational behavior estimated using any (purely structural) model

Only the case of a simple receiver (infinite wall) has been presented, but the method can be extended to more complicated cases:

(i) 2D wall / floor structures: the wave conversion between bending and longitudinal waves in the upper structure requires the use of (3x3) mobility matrices ; however, separating the power into wave types is still possible
(ii) 2D multi-foundation building (source-receiver with several contact points): this case requires checking the importance of cross mobilities in estimating the power
(iii) 2D ½ configurations: the same source receiver method can be applied, using line mobilities in the wave number domain instead of point mobilities
(iv) 3D configurations: the contact line between source and receiver can be considered as N discrete contact points (the nodes of a FEM model)

5 BIBLIOGRAPHY

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