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SOURCE SIGNAL MANIPULATION FOR IMPROVED MEASUREMENTS OF MULTI-DIRECTIONAL MECHANICAL MOBILITIES

M A Sanderson

Dept of Applied Acoustics, Chalmers University of Technology, Gothenburg, Sweden

1. INTRODUCTION

A main hinder in precision measurements of multi-directional mechanical mobilities is the mass and the rotational inertia of the attached exciter configuration which loads the measurement object. In this paper focus is placed on the moment excitation though the theory is equally applicable to force excitation. For moment excitation, it is mainly the mass loading which results in an erroneous force excitation that influences the resulting velocities [1]. This bias error is most apparent for light weight structures and with increasing frequency. For force excitation it is the rotational loading that results in an erroneous moment excitation.

In this preliminary work, the source signals to the two exciters using a T-like configuration [2], are manipulated using a new strategy, in such a manner that the erroneous force excitation, causing the mass loading or any difference between the exciting forces, is compensated for and an improved moment excitation is the outcome. Thus moment and rotational mobilities can be measured with greater accuracy for usage in applications such as; modal analysis, coupled analysis, vibration control, etc. The theory relies on measurable quantities and can be performed using FFT analyzers that allow for programming of the output signals. Theoretical moment mobilities, derived for a free-free light weight Plexiglas beam, are compared with experimental measurements.

2. THEORY

For two DOFs, a translation and a rotation, which is sufficient to describe bending wave motion in a beam, the unloaded and loaded beam behaviour for moment excitation can be determined respectively from

$$\begin{bmatrix}\dot{x}\\ \dot{\gamma}\end{bmatrix}=\begin{bmatrix}Y_{xx} & Y_{x\gamma}\\ Y_{\gamma x} & Y_{\gamma\gamma}\end{bmatrix}\begin{bmatrix}F_x\\ M_\gamma\end{bmatrix} \text{ and } \begin{bmatrix}\dot{x}^M\\ \dot{\gamma}^M\end{bmatrix}=\begin{bmatrix}Y_{xx} & Y_{x\gamma}\\ Y_{\gamma x} & Y_{\gamma\gamma}\end{bmatrix}\begin{bmatrix}F_1-F_2-j\omega m_x\dot{x}^M\\ (F_1+F_2)d-j\omega J_\gamma\dot{\gamma}^M\end{bmatrix} \quad (1, 2)$$

where the superscript M refers to moment excitation, Y_{xx} is the point force mobility, $Y_{x\gamma} = Y_{\gamma x}$ are the reciprocal point cross mobilities, $Y_{\gamma\gamma}$ is the point moment mobility, $2d$ is the distance between excitation forces, m_x is the extra mass, J_γ is the extra rotational inertia, $j = \sqrt{-1}$ and ω is the circular frequency. Harmonic time dependence is assumed.

The rotational velocity at a point on a beam-like structure caused by a force and a moment at the same point can be written using Eq. 1 as

$$\dot{\gamma} = \dot{\gamma}_{F_x} + \dot{\gamma}_{M_\gamma} = Y_{\gamma x} F_x + Y_{\gamma\gamma} M_\gamma \tag{3}$$

assuming that forces and moments in other DOFs can be disregarded.

From Eq. (3) it can be seen that the condition, $F_x = 0$, will give a rotational velocity $\dot{\gamma}$ that depends only on the moment M_γ causing it. The extraneous force F_x arises from the differences between the two forces F_1 and F_2 and more importantly from the mass added by the measurement equipment. Thus, when a force equal to F_x but acting in the opposite direction is applied at the excitation point, the correct cross and moment mobilities can be measured directly as:

$$Y_{x\gamma} = \dot{x}_{M_\gamma} / M_\gamma \text{ and } Y_{\gamma\gamma} = \dot{\gamma}_{M_\gamma} / M_\gamma . \tag{4, 5}$$

To correct for the loading a function is found in terms of directly measurable quantities to calculate the new source signals.

Rearranging Eq. (2) and using the fact that $[Z] = [Y]^{-1}$, gives

$$\begin{bmatrix} Z_{xx}^L & Z_{x\gamma}^L \\ Z_{\gamma x}^L & Z_{\gamma\gamma}^L \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} F_x \\ M_\gamma \end{bmatrix} \text{ where } \left[Z^L\right] = [Z] + j\omega \begin{bmatrix} m_x & 0 \\ 0 & J_\gamma \end{bmatrix} . \tag{6, 7}$$

The superscript L signifies that the quantity is influenced of loading by the measurement equipment. Eq. (6) can be rewritten as

$$\begin{bmatrix} \dot{x}^M \\ \dot{\gamma}^M \end{bmatrix} = \begin{bmatrix} Y_{xx}^L & Y_{x\gamma}^L \\ Y_{\gamma x}^L & Y_{\gamma\gamma}^L \end{bmatrix} \begin{bmatrix} F_x \\ M_\gamma \end{bmatrix} \text{ with } \begin{cases} F_x = F_1 - F_2 , \\ M_\gamma = (F_1 + F_2)d \end{cases} \text{ and } \left[Y^L\right] = \left[Z^L\right]^{-1} \tag{8-11}$$

where F_1, F_2 and d are measurable and therefore known quantities.

The manner in which Eq. (8) is formulated is advantageous since it leads to a simple expression for a compensatory force function, ΔF^F, in terms of the measured quantities. By inspection of Eq. (9), $\Delta F^F = F_2 - F_1$.

The mobilities in Eq. (8) compared with those in Eq. 1 now include the effects of mass and rotational loading and are thus labelled as loaded mobilities. These mobilities are functions of the unloaded mobilities, frequency, mass and rotational inertia. The attractiveness of formulating the problem in this manner is seen in the following. If $\Delta F^F / 2$ is added to F_1 and subtracted from F_2 then the erroneous force excitation is negated and the moment is left unchanged. Eq. (2) can thus be written as

$$\begin{bmatrix} \dot{x}^M \\ \dot{\gamma}^M \end{bmatrix} = \begin{bmatrix} Y_{xx} & Y_{x\gamma} \\ Y_{\gamma x} & Y_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} 0 \\ (F_1 + F_2)d - j\omega J_\gamma \dot{\gamma}^M \end{bmatrix} . \tag{12}$$

By comparing Eqs. (8) and (12) the link between the loaded and unloaded mobilities can be seen. By removing F_x and extracting the rotational inertia term from the matrix, the loaded mobilities are converted back into the unloaded mobilities. The mobilities due to a moment excitation can be found and they are not contaminated by mass loading.

Successful introduction of such a compensatory force function can be accomplished and is demonstrated by first measuring the two exciting forces and thereafter filtering the original output signals, by a factor determined by $\Delta F^F/2$, to give new source signals that compensate for the loading of the measurements equipment's mass. Note that there still exists rotational loading, but this only affects the moment excitation and is adjusted by subtracting $j\omega J_\gamma \dot{\gamma}$ from the measured moment. The translational and rotational velocities are not contaminated by mass loading but are somewhat smaller in value because of rotational loading.

3. MEASUREMENT PROCEDURE

The chosen beam material was Perspex because of its relatively low density and high internal damping. It had a mass of 0.5 kg while the attached measuring equipment had a mass of 0.07 kg and a rotational inertia $1 \cdot 10^{-4}$ kgm^2. This gave approximately a seven to one mass ratio between beam and exciter configuration which would normally be considered as bad measurement practice.

The experimental set-up of the T-like configuration is similar to that depicted in reference [2]. The free-free Perspex beam has the following data: length 0.0932 m, width 0.0455 m, thickness 0.0095 m, density 1245 kg/m^3, Young's modulus $5.25 \cdot 10^9$ Pa and loss factor 0.08.

In theory the compensatory force, for the moment excitation, is calculated while driving a moment. The idea is to compensate for the mass loading by the measuring equipment using both force signals to calculate new source signals. This in actuality means that the force equal to the mass loading must be measured. However, initial experiments showed unacceptable results around resonance- and anti-resonance frequencies. This problem was overcome, by instead, determining the compensatory force while driving the exciters in phase, i.e. a *force* excitation. The signals are originally amplified to give a force quotient centering on unity. Transfer functions using one of the output signals as a references, with the two force signals F_1 and F_2, the translational acceleration and the rotational acceleration were measured.

A pseudo random output signal (20-300 Hz) was used for two separate output channels. After the initial measurement, the two output signals, s_1 and s_2, were individually modified. At low frequencies the original source signals were used since the force quotient magnitude was close to unity and the source signal filtering gave poorer results indicating that at lower frequencies loading becomes less important.

For the case of moment excitation, the compensatory force function and the new output signal, in the frequency domain, respectively are

$$\Delta F^F = F_2^F + F_1^F \ , \ F_1^{M,new} = F_1^F - \frac{\Delta F^F}{2} \ \text{and} \ F_2^{M,new} = F_2^F - \frac{\Delta F^F}{2} \qquad (13\text{-}15)$$

where superscript F refers to force excitation.

This way the force caused by mass loading will sum to zero and a *pure* moment excitation will be obtained. The filters called $H_1^M(\omega)$ and $H_2^M(\omega)$ must then fulfil:

$$H_1^M(\omega)=\frac{F_1^{M,new}}{F_1^F}=\frac{F_1^F-F_2^F}{2F_1^F} \text{ and } H_2^M(\omega)=\frac{F_2^{M,new}}{F_2^F}=\frac{F_2^F-F_1^F}{2F_2^F}. \quad (16\text{-}17)$$

The calculations are made in the frequency domain but usually the output signal has to be specified as a time signal so that

$$s_1^M(t)=\text{IFFT}\left[H_1^M(\omega)\text{FFT}(s_1(t))\right] \text{ and } s_2^M(t)=\text{IFFT}\left[H_2^M(\omega)\text{FFT}(s_2(t))\right]. \quad (18\text{-}19)$$

4. RESULTS AND DISCUSSION

Results from a measurement compared with theory are seen in Fig 1. The relatively low mass of the beam compared to the measuring equipment showed that the technique is robust.

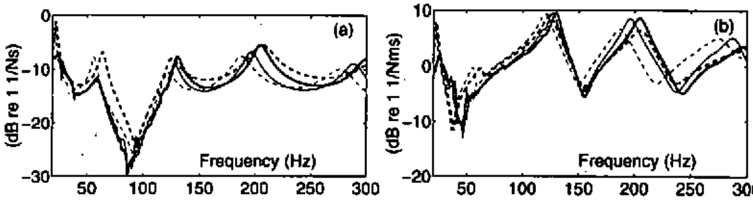

Fig 1. Magnitude of driving point a) cross- and b) moment mobilities of the free-free beam. ▬▬, source signal filtered measured, - - -, theoretical (Euler-Bernoulli) ——, directly measured and – – –, theoretical directly 'measured' (loaded).

This initial study shows that the source filtering technique is feasible but further work is required to optimize and fine tune the procedure. Stiffer levers or a new stiffer excitation configuration can be developed so that measurements can be made to higher frequencies. The signal processing can be improved and techniques such as signal shaping that supply a higher level of force and moment at the anti-resonance frequencies and less at the resonance frequencies can be applied, see for example [3]. The principles of the technique can be expanded for other structures such as plates or structures where mobilities in all 6 DOFs are needed. In theory, an exciter is required for each DOF, however, for plates, 4 exciters would be practical to excite two moments and one force corresponding to bending motion.

5. ACKNOWLEDGMENTS

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6. REFERENCES

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