THE LOCATION AND IDENTIFICATION OF DEFECTS IN PIPES AND DUCTS BY MEANS OF RESONANCE AND ANTI-RESONANCE SHIFTS

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Abstract
A technique is presented for determining the blockage area function of a pipe or duct using resonance and anti-resonance frequencies determined from a single measurement of sound pressure within a duct. The technique has also been successfully applied to the detection, location and sizing of small holes in duct walls. The position of a hole in the duct wall is revealed as the beginning of an apparent gradual expansion emanating from an initial positive DC shift in the reconstructed blockage area function. Once the hole has been located, an impedance model of the duct incorporating the hole location and the measured wave number at the first order duct resonance allows the determination of the radius of the side hole from a simple quadratic equation.

1. INTRODUCTION

This paper describes a technique developed by the authors for blockage detection in pipes or ducts based upon resonance frequency shifts. It is demonstrated that the technique can also be successfully applied to the location and sizing of holes in the walls of ducts. The technique requires that the duct termination conditions be of a closed-open nature with a driver located at the closed rigid end.

2. DETERMINATION OF THE BLOCKAGE AREA FUNCTION

Earlier work [1-3] has demonstrated that the internal profile or *blockage area function* of a duct can also be determined by utilising the high noise immunity of a Maximum Length Sequence (MLS) measurement system to reveal the resonance and anti-resonance frequencies of the duct by means of a single measurement of the loudspeaker voltage to sound pressure transfer function within the duct.

The blockage area function $A_b(x)/A_0(x)$ (where $A_b(x)$ is the blockage cross-sectional area, $A_0(x)$ is the cross-sectional area of the unblocked duct) of a duct with closed-open end conditions has been shown by de Salis and Oldham to be given by:

$$A_b(x)/A_0(x) = \left[1 - \exp\left(\sum_{n=1} \left[\frac{L_e}{n\pi}\right]^2 \mu_{(a)n} \cos\left(\frac{2n\pi x}{L_e}\right) - \sum_{n=1} \left[\frac{2L_e}{(2n-1)\pi}\right]^2 \chi_n \cos\left(\frac{(2n-1)\pi x}{L_e}\right) - a_0 \right) \right] \quad (1)$$

where L_e is the end corrected duct length for the closed-open duct and a_0 is an added DC component which is equal to the ratio of blockage to duct volume. As detailed in Reference 1, the value χ_n is the n[th] blockage induced duct resonance value shift and is calculated from the measured resonance frequencies $f_n^{(b)}$ of a partially blocked duct and the calculated resonance frequencies $f_k^{(cu)}$ of a theoretical unblocked duct of uniform cross-section, identical length and approximately identical volume where

$$\chi_n = \frac{4\pi^2}{c^2}\left(\left[f_n^{(b)}\right]^2 - \left[f_k^{(cu)}\right]^2\right), \quad n = k. \quad (2)$$

The frequencies $f_k^{(cu)}$ are determined from the measured values of $f_n^{(b)}$ as follows:

$$N \to \infty,\ f_k^{(cu)} = \frac{(2k-1)}{N^2}\sum_{n=1}^{N} f_n^{(b)}\ ,k = 1,2,3,.. \tag{3}$$

where N is the number of longitudinal resonance frequencies inclusive from N=1 used in the calculation.

Similarly, $\mu_{(a)n}$ is the n^{th} blockage induced anti-resonance value shift of the duct and is calculated from the measured anti-resonance frequencies in the partially blocked duct $f_{(a)n}^{(b)}$ and the calculated anti-resonance frequencies $f_{(a)k}^{(cu)}$ of a theoretical unblocked duct of identical length and approximately identical volume but of uniform cross-section,

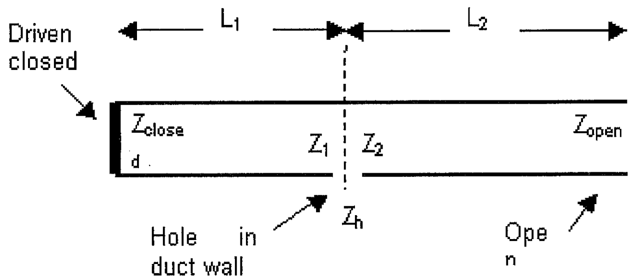
$$\mu_{(a)n} = \frac{4\pi^2}{c^2}\left(\left[f_{(a)n}^{(b)}\right]^2 - \left[f_{(a)k}^{(cu)}\right]^2\right)\ ,\ n = k. \tag{4}$$

The frequencies $f_{(a)k}^{(cu)}$ are determined from the measured values of $f_{(a)n}^{(b)}$ as follows:

$$N \to \infty,\ f_{(a)k}^{(cu)} = \frac{k}{N!}\sum_{n=1}^{N} f_{(a)n}^{(b)}\ ,k = 1,2,3.. \tag{5}$$

where N is the number of longitudinal anti-resonance frequencies inclusive from N=1 used in the calculations. Using the single measurement technique to determine the resonance and anti-resonance frequencies of the partially blocked duct it was found that taking N = 15 in equation (3) and (5) gave a sufficiently accurate approximation to make the reconstruction process valid.

3. LOCATION AND SIZING OF A HOLE IN A DUCT WALL



The work of Sharp and Campbell[4] involving the reconstruction of the internal area function of small bore pipes, has shown that a hole in the pipe wall gives rise to an apparent gradual expansion of the bore emanating from the position of that hole. If the hole in the wall of a duct with closed-open ends is known to be of approximately circular cross-section then the resonance and anti-resonance characteristics may be used to size the hole usir nall hole in the wall as shown in Fig. 1.

Figure 1: Impedance Model of Pipe with a Hole

The left hand section is bounded by the plane of the driver and the plane of the hole while the right hand section is bounded by the plane of the hole and the plane of the open end. The complex impedance at the end of the left hand section is denoted by Z_1 and the input impedance of the right hand section by Z_2. The impedance of the wall orifice Z_h may then be expressed as:

$$\frac{1}{Z_h} = \frac{1}{Z_1} - \frac{1}{Z_2} \tag{6}$$

This can be rearranged to give:

$$Z_h = \frac{Z_1 Z_2}{Z_2 - Z_1} \tag{7}$$

The impedance $Z_1$of the initial section of duct may be determined from the impedance Z_{closed} of the fluid at the driver end using the impedance model for a section of duct with impedance terminations. Adapting the expression for mechanical impedance of a section of duct then the acoustic impedance Z_1 can be written as:

$$Z_1 = \frac{\rho c}{S_{duct}} \left[\frac{Z_{closed} - j\left(\frac{\rho c}{S_{duct}}\right) tan(kL_1)}{\left(\frac{\rho c}{S_{duct}}\right) - j\, Z_{closed}\, tan(kL_1)} \right] \tag{8}$$

where k is the acoustic wave number, L_1 is the longitudinal distance from the closed driver end of the duct to the orifice, r is the density of air, c is the speed of sound in air and S_{duct} is the cross-sectional area of the duct.

The value of the fluid impedance Z_{closed} at the driver is required to evaluate Z_1. At resonance the impedance of the fluid in the duct adjacent to the driver closely matches the impedance of the driver which maximises power transmission into the duct. Making the assumption that the impedance of the closed driver end is very large when compared to the characteristic impedance of free air leads to the assumption that the impedance Z_{closed} of the fluid next to the driver also becomes very large as the system becomes resonant. Equation (8) now becomes:

$$Z_1 = \frac{\rho c}{S_{duct}} \left[\frac{j}{tan(k_n L_1)} \right] \tag{9}$$

where k_n is the wavenumber of the duct system at resonance.

At anti-resonance the pressure close to the driver goes to a minimum. The assumption this time is that the impedance Z_{closed} of the fluid next to the driver end is poorly matched to the large impedance of the closed end and becomes very small as the system approaches anti-resonance. Equation (8) then becomes:

$$Z_1 = -\frac{\rho c}{S_{duct}} \left[j\, tan(k_{(a)n} L_1) \right] \tag{10}$$

where $k_{(a)n}$ is the wavenumber of the duct system at anti-resonance.

Similarly, the impedance Z_2 in Fig. 1 may be expressed in terms of Z_{open}. Utilising the standard equation for an unflanged open ended duct, Z_2 may be expressed as follows:

$$Z_2 = \frac{\rho c}{S_{duct}} \left[\frac{\frac{1}{4}(k\, r_{duct})^2 + j\left[0.6(k\, r_{duct}) + tan(kL_2)\right]}{\left[1 - 0.6(k\, r_{duct}) tan(kL_2)\right] + j\left(\frac{1}{4}(k\, r_{duct})^2\right) tan(kL_2)} \right] \tag{11}$$

where r_d is the cross-sectional radius of the duct.

Substituting (9) or (10) and (11) into (7) yields the impedance of the orifice Z_h.

The representation for the impedance of an orifice as a side branch in a duct wall where the dimensions of the slit are small compared to acoustic wavelength is given by:

(12)

$$Z_h = \frac{\rho c \mathbf{k}_h^2}{4\pi} + j\frac{\rho c L' \mathbf{k}_h}{\pi r_h^2}$$

where r_h is the radius of the orifice, $\mathbf{k}_h$ is the complex wave number of propagation in the side orifice,

$$\mathbf{k}_h = k - j\frac{\alpha}{r_h} \quad (13)$$

the term $\frac{\alpha}{r_h}$ accounts for viscous losses in the hole and has the form:

$$\alpha = \frac{1}{c}\left[\sqrt{\frac{\mu\omega}{2\rho}} + (\gamma - 1)\sqrt{\frac{\kappa\omega}{2\rho c_p}}\right] \quad (14)$$

Where μ is the gas viscosity ($1.8*10^{-5}$ kg $m^{-1}s^{-1}$ for air at 293 K), γ is the ratio of specific heats (1.4 for air), κ is the thermal conductivity ($3*10^{-6}$ $Wm^{-1}k^{-1}$ for air) and c_p is the specific heat of air at constant pressure (1004.15 J Kg^{-1} K^{-1} for air).
The length L' in equation (12) is equal to the thickness of the duct wall L_h plus additional end corrections terms, and may be expressed as:

$$L' = L_h + r_h\left[1.5 - 0.58\left(\frac{r_h}{r_d}\right)^2\right] \quad (15)$$

where r_d is the radius of the main duct section.

The bracketed term in r_h on the right hand side of equation (15) evaluates the end correction of the side hole. It is apparent that the second term inside the brackets in equation (15) will tend to zero as r_h becomes small in comparison to r_d. Therefore, for small values of r_h e.g. $\frac{r_h}{r_d} < 0.2$ the simplified expression for the length L' of an orifice in a plane pipe wall may be used where:

$$L' = L_h + 1.5 r_h \quad (16)$$

Evaluating and equating the imaginary parts of equation (7) gives:

$$Im(Z_h) = \frac{Im(Z_2)\left[(Re(Z_1))^2 + (Im(Z_1))^2\right] - Im(Z_1)\left[(Re(Z_2))^2 + (Im(Z_2))^2\right]}{\left[Re(Z_1) - Re(Z_2)\right]^2 + \left[Im(Z_1) - Im(Z_2)\right]^2} \quad (17)$$

Substituting these results into equation (17) evaluates *Im*(Z_h). To evaluate the hole radius r_h using this result we substitute equations (13) and (16) into equation (12) which yields:

$$Im(Z_h) = \rho ck\left(-\frac{\alpha}{2\pi r_h} + \frac{1.5 r_h + L_h}{\pi r_h^2}\right) \quad (18)$$

which may be rearranged thus:

$$\frac{\pi}{\rho ck} Im(Z_h) r_h^2 - \left(1.5 - \frac{\alpha}{2}\right) r_h - L_h = 0 \quad (19)$$

It may be noted that even at near ultra-sonic frequencies $\frac{\alpha}{2}$ evaluated from (14) makes up less than 0.5% of the total term in r_h. Therefore it may be deduced that viscous losses are insignificant in any evaluation of Im(Zh) or r_h and equation (18) now simplifies to:

$$\frac{\pi}{\rho c k} Im(Z_h) r_h^2 - 1.5 r_h - L_h = 0 \tag{20}$$

Equation (20) is a quadratic equation and may be solved in the standard way to yield r_h

4. EXPERIMENTAL ANALYSIS

A series of experiments were carried out to validate the theoretical approach described above. The duct used in the experiments was a 2 m long plastic duct of 0.1 m diameter and wall thickness of 5.5 mm. The duct characteristics were determined using the Maximum Length Sequence (MLS) analysis. The substantial noise immunity and versatility of the MLS system has also been utilised to good effect in previous work by the authors[1-3]. The duct had a cut-on frequency for transverse modes of approximately 1900 Hz. The duct was excited by a piezo-electric driver attached to one end while the other end of the duct was left open. The excitation signal applied consisted of a 16384 point maximum length sequence (MLS) of 2 kHz bandwidth and 8 kHz sampling rate. A microphone situated in the wall of the duct close to the driver was used to record the distorted MLS signal emanating from the driver allowing generation of the frequency transfer function of loudspeaker voltage to sound pressure in the duct. In each case four averages of the MLS signal were taken making a total measurement time of approximately 8 seconds.

The measured duct transfer function was imported into the MATLAB numerical processing package and a simple routine was utilised to select the resonance and anti-resonance frequencies of the duct. The resonance and anti-resonance frequencies of the duct with unperforated walls were subsequently calculated from equations (3) and (5), and the resonance and anti-resonance value shifts were processed using equations (2) and (4). The internal area function of the duct was then calculated using equation (1) to reveal the position of the obstruction or hole in the duct wall. The end corrected length of the duct L_e in equation (1) was determined from the fundamental resonance frequency of the un-obstructed and un-perforated closed-open duct f_1^0 from equation (3) as follows:

$$L_e = \frac{1}{4}\frac{c}{f_1^0} \tag{21}$$

where the speed of sound c was estimated using the standard expression

$$c = 331.45 \sqrt{(1 + T/373.16)} \tag{22}$$

With T being the measured ambient temperature in Kelvin.

The first fifteen resonance and anti-resonance values determined using equations (2) and (4) respectively were found to be adequate for the reconstruction of the internal area function of the duct to enable successful location of a side hole. The resonance and anti-resonance frequencies required to obtain these values were located well below the cut-on frequency.

5. BLOCKAGE DETECTION

Figure 2 shows an example of the results obtained with a large blockage inside the duct. The blockage re-construction function follows the form of the actual blockage very closely apart from at the regions close to the end of the duct.

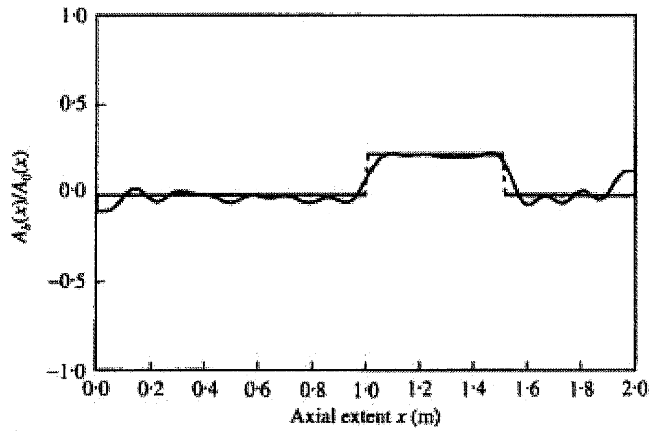
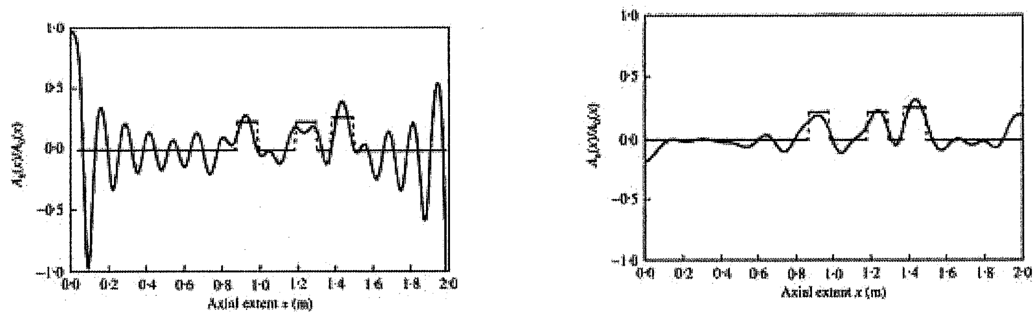


Figure 2: Comparison of reconstructed blockage area function (continuous line) with actual function (broken line)

Figure 3 shows an example of the techniques used to locate and size multiple small obstructions in the duct. It was found that spurious noise made it difficult to determine the minima of the transfer function with the result that the calculated blockage area function differed greatly from the real blockage area function as shown in Figure 3a. However, the use of a simple low pass filter algorithm reduced the problem as can be seen from Figure 3b.



(a) (b)

Figure 3: Comparison of Reconstructed blockage area function (continuous line) with actual function (broken line) for multiple blockages (a) unfiltered signal; (b) filtered signal

6.

At each measurement of transfer function in the duct, the internal bore sizing expression of equation (1) was used to locate the position of a hole in the duct wall. In all bore profile reconstructions, the DC correction term a_0 in equation (1) was excluded as the notion of a blockage in the duct was now defunct. Once the reconstruction had been completed, the effect of the hole in the duct wall was immediately noticeable and its position could be determined. The bore profile reconstruction of the duct of 50 mm radius with a hole of 5 mm radius in the wall at a position of 1.42 m from the driver end of the duct is shown in Fig. 4.

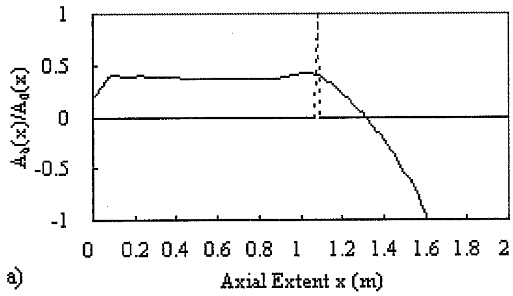


Figure 4: Reconstruction of Duct Area Function: dotted line shows position of the hole

A substantial positive DC shift above the zero plane is apparent leading up to the position of the hole. A positive DC shift of this type is associated with expansion in the duct as in earlier work negative DC shifts were found to occur in ducts with constrictions. The DC shift shown is typical of a large expansion in the duct which is effectively how the hole appears in the bore reconstruction process. At the position of the hole a sharp point of inflexion occurs and from this point the curve drops steadily, crossing the zero line and continuing into what appears to be a flaring expansion towards the end of the duct. This negative gradient is distinguished from other negative gradients in the reconstruction by its axial extent and also by its continuation below the zero expansion line of the reconstruction.

7. SIZING OF HOLES

The sizing of cylindrical holes in the duct wall was undertaken using the technique set out above employing the measured wave number values both at resonance k_n and at anti-resonance $k_{(a)n}$ determined from the duct transfer function and the values L_1 and L_2 relating to the hole position in Fig. 1. The wave numbers at first order resonance were generally found to be the most accurate for use in the hole sizing process. Fig. 5 shows the estimation of radius r_h for holes at 1.42 m from the driver end using the above approach.

These estimated values were plotted against the actual values of the hole radius. Fig. 3 shows estimations using equation (9). The estimated values shown in Fig. 3 are highly accurate, all being within 10 % of the actual hole size.

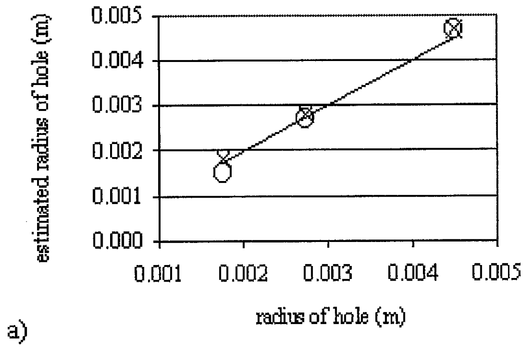


Figure 5: Comparison of Actual and Hole Diameters Estimated Using Wavenumbers at First Order Resonance and Anti-resonance

8. CONCLUSIONS

A technique has been developed for the detection, location and sizing of small apertures in a duct wall using the resonance and anti-resonance frequencies of the duct determined from a single transfer function measurement The technique reconstructs the apparent internal area function of the duct from which the position of the hole may be ascertained by eye or detected automatically using a simple computational routine. Following the location of a hole it may be sized using an impedance model of the duct. The location of the hole using this method is shown to be highly accurate and subsequent sizing of the hole is found to be most accurate using the measured first order resonance wave number of the duct.

9. REFERENCES

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