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A SIMPLIFIED MODEL FOR SOUND ABSORPTION IN RIGID-FRAME GRANULAR MATERIALS

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1. INTRODUCTION

Sound propagation in rigid-frame porous media has been studied for many years. Biot [1-2] developed the theory of propagation of elastic waves in a fluid-saturated porous solid and later generalized his theory [3] to explain acoustic propagation in porous dissipative media. Chandler and Johnson [4] studied the equivalence of quasi-static flow in fluid-saturated porous media and Biot's slow wave in the limit of zero frequency. Johnson *et al* [5] extended his previous work and derived the dynamic tortuosity and permeability in fluid-saturated porous media. Their formulations have been used by Allard *et al.* [6] and Tizianel *et al.* [7] to study sound propagation in air-saturated random packing of beads/sand. A comprehensive text on the acoustic properties of porous materials has been written by Allard [8], to which the reader is referred for a relatively up-to-date review of the field.

As regards the prediction of the acoustic properties of rigid-frame porous materials there are two currently polarised situations. Simple empirical expressions exist for noise control engineers to predict the performance of porous materials, such as those given in [9], and there are relatively complicated expressions to be found in the literature, for example in references [1-8]. These expressions contain a number of variables, which generally need to be measured or can be predicted from approximate models, such as those given in [8]. Moreover, because of the complexity of the expressions, it is difficult to gain insight into the dominant physical properties of the material at a given frequency.

In this paper simplified non-dimensional expressions for the acoustic wavenumber and characteristic impedance of a rigid-frame porous material are derived. The starting point is the complicated expressions developed by Johnson *et al* [5] and Lafarge *et al* [10] and by using the concepts of acoustic mass, stiffness and damping these expressions are simplified. The losses due to thermal effects are neglected for simplicity.

2. CURRENT MODEL

Provided that a rigid-frame porous material is locally reacting then the two characteristics that describe its acoustic behaviour are the acoustic wavenumber k_c and the characteristic acoustic impedance Z_c , which are given by:

$$k_c = \omega\left(\frac{\rho_e}{B_e}\right)^{\frac{1}{2}}, \qquad Z_c = (\rho_e B_e)^{\frac{1}{2}} \tag{1a,b}$$

where ρ_e and B_e are the effective density and effective bulk modulus of the fluid contained within the porous material respectively. Johnson *et al* [5] have developed an expression for the effective density by deriving high and low frequency expressions for the effective density and then finding an expression to match both of these. The resulting expression is given by:

$$\rho_e = \rho_o\alpha_\infty\left(1 + j\frac{\eta\phi}{\rho_o\alpha_\infty k_o\omega}\left(1 - j\frac{4\rho_o\alpha_\infty^2 k_o^2\omega}{\eta\phi^2\Lambda^2}\right)^{\frac{1}{2}}\right) \quad (2)$$

where ρ_o and η are the density and the viscosity of the fluid, ϕ and α_∞ the porosity and the tortuosity of the porous material, k_o and Λ the viscous permeability and the viscous characteristic dimension of air in the material, and ω is circular frequency. Following the arguments proposed by Johnson et al [5], Lafarge *et al* [9] showed that the effective bulk modulus of the fluid within a porous material is approximated by:

$$B_e = \frac{\gamma P}{\gamma - (\gamma - 1)\left(1 + j\frac{\eta\phi}{\rho \Pr k_o'\omega}\left(1 - j\frac{4\rho \Pr k_o'^2\omega}{\eta\phi^2\Lambda'^2}\right)^{\frac{1}{2}}\right)^{-1}} \quad (3)$$

where γ is the ratio of specific heats, P the atmospheric pressure, Pr the Prandtl number, and k_o' and Λ' the thermal permeability and thermal characteristic dimension of the fluid.

If a harmonic propagating wave of the form $e^{-j(k_c x - \omega t)}$ is assumed (which is of opposite sign to that assumed by Johnson *et al* [5] and Lafarge *et al* [8]), then the effective density and bulk modulus can be written in different forms. The effective density described in equation (2) normalised by $\rho\alpha_\infty$ can be written as:

$$\frac{\rho_e}{\rho\alpha_\infty} = \left(1 + \frac{1}{j\Omega}\left(1 + j\Omega\frac{M}{2}\right)^{\frac{1}{2}}\right) \quad (4)$$

where $M = \frac{8\alpha_\infty k_o}{\phi\Lambda^2}$ is a non-dimensional shape factor that depends on the microstructure of the material, and $\Omega = \frac{\omega}{\omega_t}$, where $\omega_t = \frac{\phi\eta}{k_o\rho_o\alpha_\infty}$. The non-dimensional shape factor, M, is related to the dynamic and static shape factors described by Attenborough [12], and is given by $M = \frac{n^2}{s}$ where n is the 'dynamic shape factor' and s is the 'static shape factor'. There is some discussion as to the value of M for different classes of materials. Attenborough [12] suggests that $0.25 \le M \le 1$ for both granular and porous materials, Johnson *et al* [5] and Pride *et al* [10] suggest that M=1, and Allard *et al* [6] and Tizianel *et al* [7] have found M=5 and 6 respectively for two different granular materials. Thus there seems to be uncertainty with this parameter and this should be taken into account when assessing the accuracy of the simplified expressions derived in the next section.

The bulk modulus given in equation (3) normalised by γP can be written as follows:

$$\frac{B_e}{\gamma P} = \frac{1}{\gamma - (\gamma - 1)\left(1 + \frac{1}{j\bar{\Omega}}\left(1 + j\bar{\Omega}\frac{M'}{2}\right)^{\frac{1}{2}}\right)^{-1}} \quad (5)$$

where $M' = \frac{8k_o'}{\phi\Lambda'^2}$ is a thermal dimensionless coefficient which is analogous to M [11] and $\bar{\Omega} = \Omega\frac{A}{M}$

where $A = \frac{Pr\, k_o'}{\alpha_\infty \eta}$. It can be seen that to determine the effective density using equation **(4)** six parameters are required, and to determine the effective bulk modulus a further five parameters are required. It is shown in the next section that this number of parameters can be significantly reduced by making some simplifying assumptions.

3. SIMPLIFIED EXPRESSIONS

Another way of representing the acoustic properties of a porous material is in terms of its acoustic density, stiffness and damping. The mass and damping characteristics are encompassed in the effective density, and the stiffness is associated with the bulk modulus. There are two loss mechanisms with a porous material; viscous losses due to the flow resistance and thermal losses as heat is transferred from the fluid to the rigid frame. In this paper the thermal losses are neglected.

To determine the acoustic mass and damping, equation **(4)** can be expanded in terms of its real and imaginary components to give:

$$\frac{\rho_e}{\rho_a} = 1 + \left\{\frac{\left(1+\left(\frac{\Omega M}{2}\right)^2\right)^{\frac{1}{2}} - 1}{2\Omega^2}\right\}^{\frac{1}{2}} - j\left\{\frac{\left(1+\left(\frac{\Omega M}{2}\right)^2\right)^{\frac{1}{2}} + 1}{2\Omega^2}\right\}^{\frac{1}{2}} \quad \textbf{(6)}$$

where $\rho_a = \rho_o \alpha_\infty$ is defined as the acoustic mass per unit volume. The low frequency limit of equation **(6)** can be written as:

$$\left\{\frac{\rho_e}{\rho_a}\right\}_{\lim \Omega \to 0} = \left(1+\frac{M}{4}\right) - \frac{j}{\Omega} \quad \textbf{(7a)}$$

where higher order terms containing Ω have been neglected. The high frequency limit of equation **(6)** can be written as:

$$\left\{\frac{\rho_e}{\rho_a}\right\}_{\lim \Omega \to \infty} = 1 + \frac{1}{2}\left(\frac{M}{\Omega}\right)^{\frac{1}{2}} - \frac{j}{2}\left(\frac{M}{\Omega}\right)^{\frac{1}{2}} \quad \textbf{(7b)}$$

To relate the acoustic properties to the mechanical properties of mass and damping it is useful to write equations **(7a,b)** in the following from:

$$\{\rho_e\}_{\lim \omega \to 0} = \rho_a\left(1+\frac{M}{4}\right) + \frac{C_a}{j\omega} \quad \textbf{(8a)}$$

$$\{\rho_e\}_{\lim \omega \to \infty} = \left(\rho_a + \frac{1}{2}\left(\frac{C_a M \rho_a}{\omega}\right)^{\frac{1}{2}}\right) + \frac{1}{j2}\left(\frac{C_a M \rho_a}{\omega}\right)^{\frac{1}{2}} \quad \textbf{(8b)}$$

where $C_a = \phi\eta / k_o$ is the acoustic damping per unit volume which has units of NsM^{-4}. (It should be noted that this is related to the flow resistivity, σ, because $\sigma = \eta / k_o$).

The real part of the effective density relates to the mass of the fluid contained within the pores of the material and the imaginary part relates to the viscous damping properties of the fluid interacting with the porous material. From equation **(8a)** it can be seen that damping dominates at low frequencies and that mass dominates at high frequencies. The frequency at which the mass-like behaviour starts to dominate over the damping-like behaviour is given approximately by ω_t defined in section 2, which can be written as $\omega_t = C_a / \rho_a$. Because of this behaviour the effective density can be approximated to:

$$\rho_{e(approx)} = \rho_a + \frac{C_a}{j\omega} \quad \textbf{(9a)}$$

or in non-dimensional form as:

$$\frac{\rho_{e(approx)}}{\rho_a} = 1 + \frac{1}{j\Omega} \quad \textbf{(9b)}$$

which is independent of the shape factor M, which means that Λ does not need to be determined. The normalised real and imaginary parts of the effective density using equation **(6)** and equation **(9b)** are plotted in **Figure 1** for the data given in **Table 1**, which is for a granular material made from glass beads [6]. This is an extreme example where M is much larger than that estimated by other researchers for other porous materials. It can be seen that mass-like term of the approximate expression, (the real part) underestimates that given by equation **(7a)** at low frequencies. However, below the normalised frequency Ω=1, viscous effects dominate and so the error in the estimate of the mass-like term will have a negligible effect. At high frequencies, when $\Omega \gg 1$, the actual expression for the mass-like term given tends toward the approximate value of ρ_a.

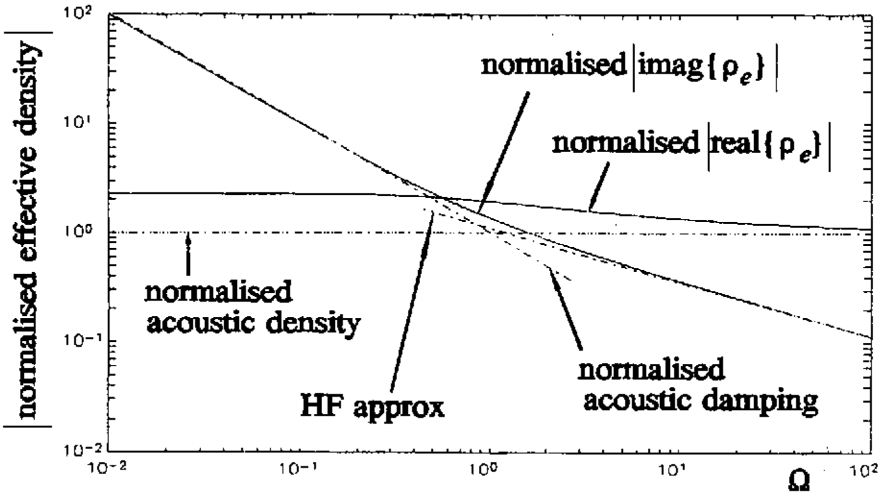

Figure 1 Effective density and normalised acoustic mass and acoustic damping

There is a negligible error between the approximate and actual low frequency value of the damping-like term. The high frequency approximation underestimates that predicted by equation **(7b)**, but again this does not have a large effect as the mass-like term dominates when Ω>1. The high frequency approximation for the imaginary part of the normalised effective density is given by $-0.5(M/\Omega)^{\frac{1}{2}}$, and this is also plotted in **figure 1**.

POROUS MATERIAL		
Porosity, ϕ	=	0.4
Viscous Permeability, k_o	=	1.5e-9 m^2
Thermal Permeability, k_o'	=	5e-9 m^2
Tortuosity, α_∞	=	1.37
Characteristic Viscous Dimension, Λ	=	31e-6 m
Characteristic Thermal Dimension, Λ'	=	320e-6 m
AIR		
Density @ 20°C, ρ_o	=	1.2 kgm^{-3}
Dynamic Viscosity @ 20°C, η	=	18.22e-6 Nsm^{-2}
Atmospheric Pressure	=	1.013e5 Nsm^{-2}
Prandtl Number	=	0.709
Ratio of Specific Heats, γ	=	1.4
CALCULATED VARIABLES		
M	=	5.07
M'	=	0.98
A	=	1.73
ω_t	=	2955 rad s^{-1}

Table I Porous Material Properties used in the Simulations (Taken from reference [6])

The normalised bulk modulus given in equation **(5)** can be written as:

$$\frac{B_e}{\gamma P} = k(1 + j\bar{\eta}) \qquad \textbf{(10)}$$

where k is the frequency dependent non dimensional stiffness of the fluid and $\bar{\eta}$ is the frequency dependent loss factor which is found by dividing the imaginary part of the bulk modulus by the real part. It can be seen that the bulk modulus behaves in a similar way to a viscoelastic material [13]. It has a non-dimensional stiffness of $1/\gamma$ at low frequencies, and 1 at high frequencies because of the assumption of isothermal and adiabatic behaviour at low and high frequencies respectively. The loss factor is small at low and high frequencies and peaks (a relatively small value <0.17) at a transition frequency. In this paper it is simply assumed that $B_e = P$.

To non-dimensionalise the results the wavenumber and impedance of the porous material are normalised to the wavenumber k_o and characteristic impedance Z_o of the fluid in an infinite space respectively. The non-dimensionalised wavenumber and impedance are therefore given by:

$$\hat{k}_c = (\alpha_\infty \gamma)^{\frac{1}{2}} \left(\frac{\rho_e}{\rho_o}\right)^{\frac{1}{2}}, \qquad \hat{Z}_c = \left(\frac{\alpha_\infty}{\gamma}\right)^{\frac{1}{2}} \left(\frac{\rho_e}{\rho_o}\right)^{\frac{1}{2}} \qquad \textbf{(11a,b)}$$

By combining equation **(6)** with equations **(11a,b)** explicit expressions for the wavenumber and the impedance can be determined. However, these expressions are complicated and it is difficult to obtain physical insight. To proceed with the analysis, the approximate effective density from equation

(9b) is substituted into equations **(11a,b)**. The non-dimensionalised wavenumber is given by:

$$\hat{k}_c = \left(\frac{\alpha_\infty \gamma}{2\Omega}\right)^{\frac{1}{2}} \left\{ \left((\Omega^2+1)^{\frac{1}{2}} + \Omega \right)^{\frac{1}{2}} - j\left((\Omega^2+1)^{\frac{1}{2}} - \Omega \right)^{\frac{1}{2}} \right\} \qquad \textbf{(12)}$$

At low frequencies when $\Omega \ll 1$ equation **(12)** reduces to:

$$\hat{k}_c = \left(\frac{\alpha_\infty \gamma}{2\Omega}\right)^{\frac{1}{2}} (1 - j) \qquad \textbf{(13)}$$

which shows that the real and imaginary parts of the wavenumber are equal but opposite in sign. This result is similar to that derived by Attenborough [12]. It can be seen that the viscous effects cause the wave to be dispersive the wavenumber being proportional to the square root of frequency (the wavenumber of air in free-space is proportional frequency). This means that the wave speed of the fluid in the porous material is frequency dependent; it increases with the square root of frequency. Because the imaginary part of the wavenumber is large the propagating acoustic wave in the porous medium is heavily attenuated.

At high frequencies when $\Omega \gg 1$ then equation **(12)** reduces to:

$$\hat{k}_c = (\alpha_\infty \gamma)^{\frac{1}{2}} \left(1 - \frac{j}{2\Omega}\right) \qquad \textbf{(14)}$$

which has a real component that is much larger than the imaginary part. The real part is very close to unity and hence the wavenumber in the free fluid is similar to the wavenumber of the fluid contained within the porous material. As mentioned above, if the term containing M is included, then the imaginary part would be larger and hence the acoustic damping in the porous material would be greater. Equation **(14)** gives a lower bound for the imaginary part.

The non-dimensional impedance becomes:

$$\hat{z}_c = \left(\frac{\alpha_\infty}{2\gamma\Omega}\right)^{\frac{1}{2}} \left\{ \left((\Omega^2+1)^{\frac{1}{2}} + \Omega \right)^{\frac{1}{2}} - j\left((\Omega^2+1)^{\frac{1}{2}} - \Omega \right)^{\frac{1}{2}} \right\} \qquad \textbf{(15)}$$

The low frequency approximation to this is:

$$\hat{z}_c = \left(\frac{\alpha_\infty}{2\gamma\Omega}\right)^{\frac{1}{2}} (1 - j) \qquad \textbf{(16)}$$

and the high frequency approximation is given by:

$$\hat{z}_c = \left(\frac{\alpha_\infty}{\gamma}\right)^{\frac{1}{2}} \left(1 - \frac{j}{2\Omega}\right) \qquad \textbf{(17)}$$

The real and imaginary parts of the non-dimensional wavenumber and characteristic impedance are plotted in **Figures 2** and **3** using equations **(12)** and **(15)** respectivelyand equations **(11a,b)**. The data for the simulations is taken from **Table 1**. It can be seen that the discrepancies between the actual and approximate expressions over the entire non-dimensional frequency range is relatively small and would probably be good enough for practical noise control purposes. The approximate expressions have the major advantage of simplicity. They allow non-dimensional analysis to be undertaken, which shows very clearly two distinct frequency regimes; a low frequency regime when the viscous effects are much greater than the mass effects and high frequencies when the mass effects are much greater than the viscous effects. It is convenient, therefore, to non-dimensionalise frequency to the frequency when the mass effects are equal to the viscous effects. In the low frequency regime the real part of the impedance is positive and the imaginary part is negative as shown in equation **(16)**. This means that the normal incidence acoustic response of a porous material of infinite extent behaves as a stiffness in parallel with a damper.

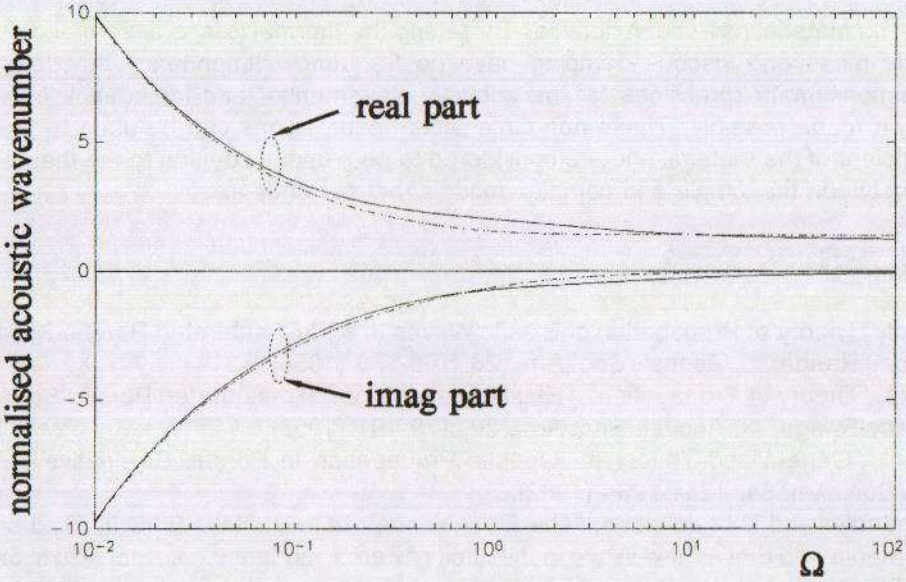

Figure 2 Real and imaginary parts of the normalised acoustic wavenumber ---- equation **(12)**, ___ equation **(11a)**

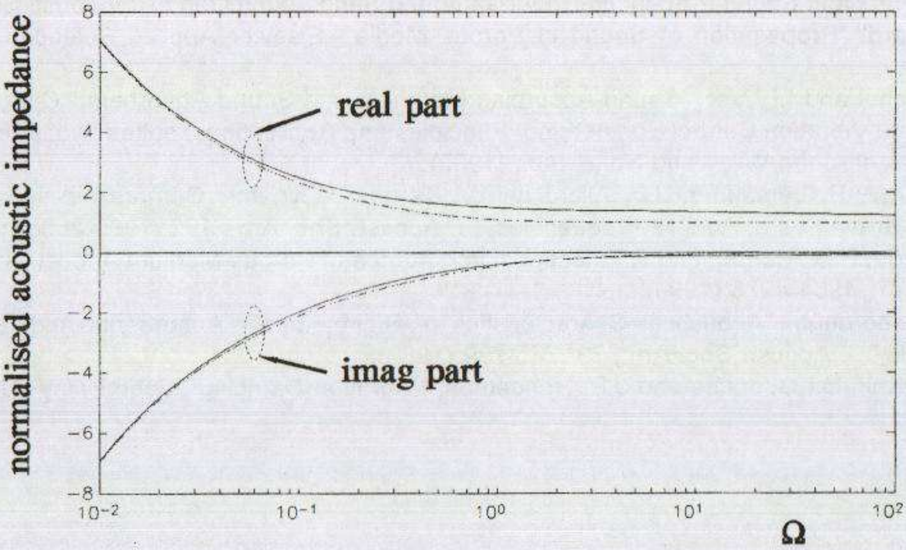

Figure 3 Real and imaginary parts of the normalised characteristic acoustic impedance ------equation **(15)**, ___ equation **(11b)**

5. CONCLUSIONS

In this paper the acoustic behaviour of rigid-framed porous materials has been studied. Non-dimensional expressions for the acoustic wavenumber and acoustic characteristic impedance of a rigid-framed porous material have been derived. By setting the thermal losses to zero and assuming that the acoustic mass and viscous damping have no frequency dependency the derivation of simplified non-dimensional expressions for the acoustic wavenumber and impedance of a porous material has been made possible. These non-dimensional expressions can be used to predict the absorption coefficient of the material and are considered to be a useful addition to the theory as they bridge the gap between the simple and complex models that currently exist.

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