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DEVELOPMENTS IN THE FFP METHOD FOR OUTDOOR SOUND PREDICTION

M J White (1) & Y L Li (2)

(1) U S Army Construction Engineering Research Laboratories, PO Box 9005, Champaign, IL 61826, USA, (2) Intel Corporation, 5000 W Chandler Blvd, Chandler, AZ 85226, USA

1. INTRODUCTION

Airborne sound waves experience a great deal of mechanisms that result in refraction, diffraction, reflection and scattering that may never be adequately realized by computation. It is sometimes conjectured after a long day in the field, that if only we knew precise measurements of all the variables: the temperature everywhere, the wind speed and direction and it variation with time, the ground impedance as a function of position, and of course the digitized terrain, and the height and density of vegetation, or snow, etc., then we would be well on our way to predict how sound waves should behave there. In fact though, that situation may be far from ideal, and it is likely that in any given situation only a small portion of that information is useful or relevant. At times, researchers have had an overwhelming success in comparing measured levels to simple predictive models such as the one in this paper. When this does occur, we happily suppose that the model accurately includes all of the important physical processes and purport that it "works." Almost as often, measurements disagree with predictions and we worry that something is missing from the measurements (see above) or from the model, thus our attempts to include them (see below).

In this paper, extensions to the Fast Field Program (FFP) introduced by Raspet, et al. [1], are outlined which have proven beneficial in applying it to a wider range of situations and in comparing it to measurements. Some other of our extensions to the FFP, to model scattering have gone without yet much success and this will be discussed briefly.

2. STARTING POINT

The conventional implementation of the FFP makes use of cylindrical symmetry about a single source point in a stratified atmosphere, such that the exact representation of pressure $p(\rho, z)$ is

$$p = \int_{-\infty}^{\infty} \hat{p} \mathrm{H}_0^{(1)}(k\rho) k \mathrm{d}k$$

with $\hat{p}(z)$ the height-dependent pressure, subject to impedance boundary conditions at the ground, radiation conditions at infinity, continuity of pressure and discontinuity of the velocity at the source. The time factor $\mathrm{e}^{-i\omega t}$ is assumed. In this equation, we see that $\hat{p}(z)$ does not depend on horizontal distance (ρ), but the one-dimensional second-order equation it solves depends on all other variables that describe the medium.

3. WIND AND TEMPERATURE

In the presence of wind, temperature and density stratification, the equation for the height-dependent pressure is

$$\varrho\mu^2 \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\varrho\mu^2}\frac{\mathrm{d}\hat{p}}{\mathrm{d}z}\right) + \left(\frac{\mu^2\omega^2}{c^2} - k^2\right)\hat{p} = -\delta\left(z - z_s\right)$$

with

$$\mu = 1 - \bar{w} \cdot \bar{k}/\omega$$

where the vector $\bar{k}$ is aligned with the horizontal direction between source and receiver, and $\bar{w}$ is horizontal wind flow, aligned in the direction of flow[2]. The quantities: density ϱ, sound speed c, $\bar{w}$,and hence μ, are continuously varying functions of height, but are not functions of horizontal position. In solving the differential equation for $\hat{p}$ above, it is usually sufficient to replace the continuously-varying profile with many thin homogeneous layers. This approximation results in simpler computations involving trigonometric functions of height. The disadvantage is that smoothly refracting profiles are replaced by sharp contrasts at the interfaces between layers. It remains a challenge to know *a priori* what maximum layer thickness will produce acceptable errors. It is yet another challenge to develop a generalized expression for $\hat{p}$ without resort to constant layers, but such an expression is desired when the number of layers exceeds the constraints of our computer.

4. WKB-TYPE SOLUTION

In the special case where the refracting index increases or decreases monotonically with height, the modified WKB method yields a solution for $\hat{p}$, in terms of Airy functions [3]. For downward refraction,

$$p = 2\pi \mathrm{e}^{i\pi/6} \mathrm{Ai}\left(\alpha_>\right) \tilde{T} / \sqrt{\alpha'_> \alpha'_<}$$

and for upward refraction

$$p = 2\pi \mathrm{e}^{i\pi/6} \mathrm{Ai}\left(\alpha_> \mathrm{e}^{i2\pi/3}\right) \tilde{T} / \tilde{R} \sqrt{\alpha'_> \alpha'_<}$$

with

$$\tilde{T} = \mathrm{Ai}\left(\alpha_{<} e^{i2\pi/3}\right) + \tilde{R}\mathrm{Ai}\left(\alpha_{<}\right)$$

where

$$\tilde{R} = \frac{\mathrm{Ai}'\left(\alpha_0 e^{i2\pi/3}\right)\alpha_0' e^{i2\pi/3} + ik_0\tilde{\beta}\mathrm{Ai}\left(\alpha_0 e^{i2\pi/3}\right)}{\mathrm{Ai}'\left(\alpha_0\right)\alpha_0' + ik_0\tilde{\beta}\mathrm{Ai}\left(\alpha_0\right)}$$

with

$$\tilde{\beta} = \beta + i\alpha_0''/2k_0\alpha_0'$$

and β is the normalized admittance of a local reaction boundary. The primes denote the derivative with respect to the argument. The argument to the function $\alpha(z)$ has been suppressed and the notation $z_{>} = \max(z, z_s)$, $z_{<} = \min(z, z_s)$ and $z_0 = 0$ is used to simplify the equations. Also, $k_0 = \omega/c_0$. The scale function $\alpha(z)$ is evaluated from the modified WKB technique and is simply,

$$\alpha = \mp\left(\int_{\min(z,z_t)}^{\max(z,z_t)} \sqrt{\pm k_{\mathrm{eq}}^2}\,\mathrm{d}z\right)^{2/3}$$

with k_{eq}^2 equal to the coefficient of $\hat{p}$ in the differential equation for $\hat{p}$, and z_t is the location of the turning point, where $k_{\mathrm{eq}}^2 = 0$, and the upper/lower signs are taken to produce a positive argument within the radical. In the very special case that the squared refraction index is linear in height, the formulas above reduce to duplicate the exact solutions found by several workers.

The chief benefit of the new formula for $\hat{p}$ is a savings in computer time. The computational cost of the Airy function solution in cases we have tested is often between 5 and 15 times the expenditure per homogenous layer. In other words, if 20 layers are needed to correctly represent the smooth profile, the Airy function solution would be quicker. Higher frequency cases (> 1 kHz) sometimes require hundreds of layers. The disadvantages of the Airy function solution are that it is approximate (except in the special case mentioned), it does need a special function routine (the Airy function and its derivative) and that the form we give must be monotonic decreasing or increasing. This last requirement could be overcome by permitting the profile to vary monotonically within layers, which we plan to implement.

5. "FAST" INTEGRATION

The height-dependent pressure has at least one simple pole in the vicinity of $k = k_0$, and the nature of $\hat{p}$ is such that its magnitude is large in the neighborhood of these poles and decreases elsewhere, so that the integral in Eq. (1) can be truncated in its upper limit at a finite location, $k_{\max}$. There is some difficulty in specifying $k_{\max}$ such that it does not terminate the integral prematurely, but the relation

$$k_{\max}^2 = k_0^2 + \left[7.5 \times 10^{-3} f + \left(2.5 - 6.25 \times 10^{-4} f\right) / \left|z - z_s\right|\right]^2$$

has worked well, below 1 kHz. Note that $k_{\max}$ diverges as the source and receiver heights approach one another, making the integration task more difficult. Separating source and receiver in height by a small amount may be sufficient to avoid the problem and achieve a practical result. For large values of $k\rho$, the Hankel function in Eq. (1) is replaced by the first term in its asymptotic series and leads to

$$p = \frac{1-i}{2\sqrt{\pi\rho}} \int_0^{k_{\max}} \left(\hat{p}\sqrt{k}\right) e^{ik\rho} dk$$

which usually must be solved numerically. The FFP technique is encompassed in recognizing that the integral transform is a Fourier transform of $F = \hat{p}\sqrt{k}$and that provided $\Delta\rho\Delta k = 2\pi/N$, the transform can be accomplished by the Fast Fourier Transform, thus the designation "Fast" Field Program. In the above, the spacing between adjacent receivers is $\Delta\rho = (\rho_{\max} - \rho_{\min})/(N-1)$, and the spacing between adjacent values of k is $\Delta k = (k_{\max} - k_{\min})/N$ and N is the number of receivers and the number of evaluations of F (the point $k_{\min}$ is excluded here). Combining these last three relations, we see that the receiver spacing depends solely on the lower and upper limits of integration $\Delta\rho = 2\pi/(k_{\max} - k_{\min})$, which is probably not desired.

The method we advocate in place of the straightforward use of the FFT for performing the integral is iterative refinement of the chirp-z transform. It allows one to independently specify $\Delta\rho$, so that placement of the receiver array does not depend on the details of mathematically solving for the field. In this case,$\Delta\rho\Delta k = 2\pi q/N$, where q is an arbitrary positive real number. The chirp-z algorithm requires two sequences X_n and Y_n, each of length $2N$,

$$Y_{(m+2N)\%2N} = e^{-im^2\pi q/n}, m = -N+1, N-1$$

$$X_m = \begin{cases} F_m e^{ik_m\rho_{\min}} Y_m^{-1}, & m = 0, N-1 \\ 0, & m = N, 2N-1 \end{cases}$$

where F_m are evaluated at $k_m = k_{\min} + (m+1)\Delta k$. The transform is accomplished by

$$\tilde{F}_m = \frac{\Delta k}{2N} e^{ik_m\Delta\rho} Y_m^{-1} \mathrm{FFT}^{-1}\left[\mathrm{FFT}\left(X_n\right)\mathrm{FFT}\left(Y_n\right)\right]$$

where FFT denotes the sum

$$\mathrm{FFT}\left(X_n\right) = \sum_{\ell=0}^{N-1} X_n e^{i2\pi n\ell/N}$$

as performed by the FFT algorithm. Care must be taken to evaluate the exponents in the sequences X_n and Y_n in high precision, to avoid possible errors in their phases due to rounding. If the sequence $\tilde{F}_m$ has not yet converged, perhaps because Δk is not yet small enough to resolve the variation in F, then $\tilde{F}_m$ can be refined by repeating the steps to evaluate X_m with k_m shifted left by $\Delta k/2$ and summing one-half of the old and new sequences $\tilde{F}_m$. The receiver locations are unaffected by the refinement, and similar passes of refinement can be performed until convergence is obtained.

6. SCATTERING

The computation of the diffracted field scattered by some object such as a wall of hill, or from a realistic uneven ground surface or from inhomogeneities (turbulence) in the air is of high interest, and is a challenge with the FFP. We have succeeded in using the FFP technique to evaluate the field propagating past a discontinuous change in the boundary impedance on a flat ground. In this case, the height dependent pressures in medium 1 (containing the source) and medium 2 are connected by a vertical screen, on the surface of which are computed

$$p = 2\int_0^\infty dz' \int_{-\infty}^\infty G_1(\bar{r}_s, \bar{r}')\frac{\partial G_2(\bar{r}', \bar{r})}{\partial x'}dy'$$

where

$$G_1(\bar{r}_s, \bar{r}') = \frac{1}{2\pi}\int_0^\infty \hat{p}_1 J_0(k\left|\bar{r}' - \bar{r}_s\right|)k dk$$

$$G_2(\bar{r}', \bar{r}) = \frac{1}{2\pi}\int_0^\infty \hat{p}_2 \frac{\partial J_0(k\left|\bar{r} - \bar{r}'\right|)}{\partial x'}k dk$$

for the situation described in Ref. [5]. In that case, the source and receiver heights are 1.5 and 1.8 m, the frequency is 160 Hz, and the transition between soft and hard surfaces (surface flow resistivities 2×10^5 Nsm^{-4} and 2×10^7 Nsm^{-4}) is considered. Additional calculations are provided for upward and downward refraction (sound speed gradients of 0.1 s^{-1} and -0.1 s^{-1}. Results of the calculations are displayed relative to the sound level in a free field in Figs. (1) & (2). The results shown are what one might expect: higher signal levels in downward refracting

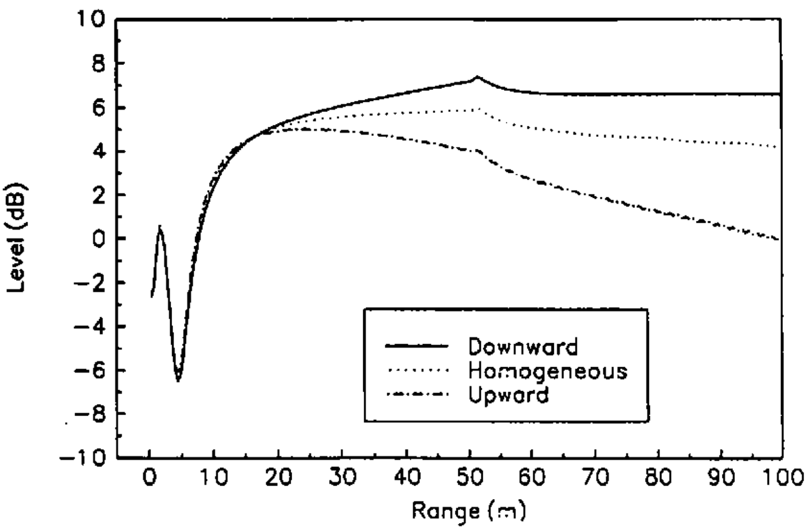

Fig. 1 Sound propagation from a hard to a soft surface.

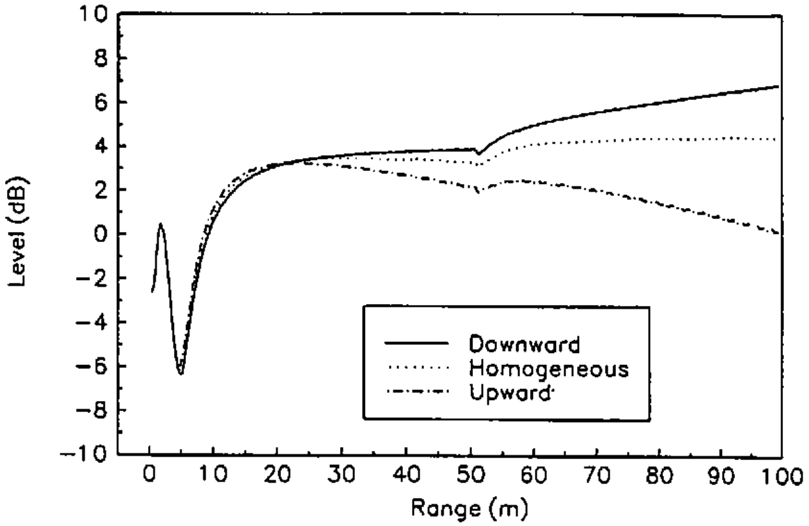

Fig. 2 Sound propagation from a soft to a hard surface.

profiles and over harder surfaces. In principle, the differences between the two media could be arbitrary, and a number of screens could be used to separate many scattering objects. One may logically extend this model to cover a wide range of conditions, perhaps as ambitious as that of our Introduction. In practice, we find that the above formulation is difficult to implement and tedious to evaluate. Possibly the strongest argument against further pursuit of this use of the FFP is its inaccuracy. When results of the pressure calculations are themselves used as sources as in this formulation, both the magnitudes and phases must be highly accurate for acceptable accuracy of the final result.

7. REFERENCES

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