PHASE CONJUGATE ARRAY PERFORMANCE FOR ACTIVE SONAR

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1. ABSTRACT

The performance of phase conjugate arrays in shallow water environments is studied using a parabolic equation propagation model. Focussing due to phase conjugation is simulated and numerical measures of the achieved gains are defined. Array performance is then studied for horizontal and vertical arrays in three shallow water environments. Trends in array gains with number of elements are investigated. It is found that the gains from horizontal arrays can be comparable to gains from vertical arrays under some environmental conditions.

2. INTRODUCTION

Experiments have shown [1] that the process of phase conjugation can be used to focus underwater acoustic pulses both temporally and spatially using large vertical line arrays transmitting signals at frequencies around 400Hz. It would be useful if similar focussing could be achieved at higher frequencies and if focussing could be achieved using horizontal line arrays. As a precursor to the designing and building of such systems, numerical modelling of phase conjugate arrays has been carried out and is reported here.

Phase conjugate arrays could be exploited in a number of underwater applications and their application to active sonar is considered here. Numerical measures of the gains achievable through phase conjugation have been developed and used to investigate array design considerations, including the relative merits of vertical and horizontal line arrays of various lengths. The performance of phase conjugate arrays has been investigated in three representations of shallow water environments and trends with important environmental parameters have been investigated.

3. PHASE CONJUGATION

Phase conjugation is a method by which underwater acoustic signals can be adapted to focus in both space and time. If a pulse of sound is transmitted from an array, propagated through the ocean, reflected from an underwater target and propagated back to the array location, then the received echo will not be an exact replica of the transmitted pulse. Dispersion due to multipath effects will lengthen the echo, relative to the original transmission. An array capable of resolving multipaths can re-transmit a time-reversed version of the distorted echo so that sound travelling along the slowest paths is transmitted first. In this way it is possible to cause sound travelling via all paths to return to the array at the same time, thus removing the time dispersion of the first echo. As the various paths from source to target coincide only at the target position, acoustic signal strength will be greater at the target position than at other locations in the ocean. Thus, spatial focussing will also be achieved due to time reversal and re-transmission along multipaths. The process of time reversing a signal is equivalent in Fourier space to taking the complex conjugate of its spectrum and it is for this reason that the process is referred to as phase conjugation.

In order adequately to focus the acoustic field, a Phase Conjugate Array (PCA) must be capable of resolving all significant multipaths and this imposes restrictions on array design. The number and nature of multipaths varies with environment type and sonar configuration and to investigate the performance of PCAs, numerical simulation methods must be used. The simulations used in this study are now described.

4. NUMERICAL SIMULATION OF PCA PERFORMANCE

To simulate the performance of PCAs in a realistic environment, a realistic model of the propagation of underwater acoustic pulses is required. The method used in this study employed a wide angle parabolic equation solver, RAM [2], as the core of a frequency domain pulse-propagation model, SPUR [3]. This was used to model the propagation of sound pulses with frequencies around 2kHz.

The operation of PCAs can be split into a series of steps. First, an initial transmission of sound is made. In this study, the initial transmission made was a 0.1second burst of 2kHz sound emitted from the centre of the PCA under study. Propagation of this pulse was then modelled out to a range of 10km. At this range, three point scatterers were modelled for reasons that will be explained later in this paper. The propagation of sound from these three scatterers, back to all elements of the PCA was then modelled. The distorted echo was then phase conjugated and re-transmitted at each element location. The outward propagation, scattering and return propagation of these phase-conjugated signals was then modelled and the re-formed echo at the centre of the array was studied. The re-transmission of phase conjugated signals at the multiple array elements was made subject to a normalisation procedure that ensured equality in the total amount of acoustic energy transmitted during both the original and phase conjugated transmissions.

Pressure time series at the centre of an array are shown in Figure 1 with the echo after transmission from a point source shown in grey and the echo from the phase conjugated transmission shown in black. The echo after phase conjugation is higher in amplitude, indicating focussing due to phase conjugation. The peak is also sharper, i.e. it drops to one half of its peak pressure amplitude in a shorter time than does the broader echo resulting from point source transmission. The pressure signals were generated from SPUR, and consequently included the effects of realistic sound speed profile and seabed parameters.

Using SPUR to generate pressure signals, it was possible to investigate the operation of PCAs but some numerical quantification of array performance was required before trends in array behaviour could be studied. Measures of performance were chosen for the four basic types of gain that can be obtained from PCAs.

A measure of the increase in the energy of the echo from the mid-water target was used to express PCAs' ability to focus on desired scatterers. This was termed the focussing gain, f_g, and is defined in equation (1),

$$f_g = 10\log_{10}\left[\frac{\int \left|p_{tc}(t)\right|^2 dt\Big|_{PCA}}{\int \left|p_{tc}(t)\right|^2 dt\Big|_{Pt.Source}}\right] \quad (1)$$

where $p_{tc}(t)$ is the echo from the mid-water target, measured at the centre of the array, the denominator is calculated after the original transmission and the numerator is calculated after the subsequent phase-conjugated transmission.

The ability of PCAs to focus sound away from the sea surface and the seabed is desirable because it provides the possibility of reducing sea surface and seabed reverberation. The problem of predicting time histories of reverberation was too complicated to be attempted in this study and instead, an indirect indication of PCAs' ability to reduce reverberation was required. It was for this reason that three point scatterers were considered. The mid-water scatterer represented the target onto which focussing was sought. One of the other targets was placed 0.1m from the sea surface and was used to measure the ability of the array to focus sound away from the sea surface. This was quantified via the surface gain, s_g, a measure of the decrease in the ratios of acoustic energy at the near-surface and mid-water targets between the original and phase conjugated transmissions. The mathematical definition of this is given in equation (2)

$$s_g = 10\log_{10}\left[\frac{\int |p_t(t)|^2 dt \Big|_{PCA}}{\int |p_s(t)|^2 dt \Big|_{PCA}}\right] - 10\log_{10}\left[\frac{\int |p_t(t)|^2 dt \Big|_{Pt.Source}}{\int |p_s(t)|^2 dt \Big|_{Pt.Source}}\right] \qquad (2)$$

Where $p_s(t)$ is the pressure signal at the near-surface target and $p_t(t)$ is the pressure signal at the mid-water target. Similar arguments and definitions were used to define the bottom gain, b_g, which measured the arrays' ability to focus sound away from the seabed and the third point scatterer was located at the sediment/water interface.

The final measure of PCA performance, t_c, quantified the time compression achieved from phase conjugation. This was simply the logarithmic ratio of the time widths of the signal at the centre of the array measured before and after phase conjugation. The width of a pulse was taken as the separation between the first and last times that the pulse reached half its maximum intensity amplitude. The mathematical definition of the time compression was therefore

$$t_c = 10\log_{10}\left[\frac{t_{1/2}\Big|_{PCA}}{t_{1/2}\Big|_{Pt.Source}}\right] \qquad (3)$$

Array performance was studied in three environments to investigate trends in array behaviour with changing environment. All three environments had a water depth of 91m. The baseline environment had a sound speed profile typical of summer conditions and a seabed that was poorly reflective of acoustic energy. The winter environment had the same seabed as the baseline case but had a sound speed profile representative of a well-mixed, isothermal water column. The third environment used had a more reflective seabed but the same summer profile as in the baseline environment. Environmental parameters are summarised in Table 1.

For the environments described in Table 1 the measures of performance fg, sg, bg and tc were used to investigate the performance of vertical and horizontal 91m-long arrays with a variety of numbers of elements. For horizontal arrays, two configurations were studied, broadside and endfire. That is, the array axis was oriented perpendicular and parallel to the plane joining the array centre and the targets. The length of horizontal arrays was set to 91m for consistency with the vertical line arrays as these were limited to a maximum length equal to the water depth.
The results of this procedure are described below.

5. ARRAY PERFORMANCE IN BASELINE ENVIRONMENT

PCA measures of performance were calculated for Vertical Line Arrays (VLAs) covering the entire extent of the water column. A maximum of 31, equally spaced elements were used but calculations were also made for smaller numbers of elements. All VLAs covered the whole water column and had elements equally spaced about the central element. The numbers of elements used were 1, 3, 5, 9, 17 and 31. Similar lengths and array spacings were used to study horizontal line arrays in endfire (HLAE) and broadside (HLAB) configurations.

Measures of performance calculated for these arrays are shown in Figure 2. The graph in the top left of the figure shows how focussing gain changes with number of elements. The data for the VLA are shown on the curve marked with the crosses. The data show how fg increases as the number of elements increases. This indicates that the array performance is improved by increasing the number of elements. The gain of the array is roughly proportional to the logarithm of the number of elements.

The graph in the lower left corner of the figure shows how the surface gain, sg, changes with the number of elements in the array. The gain is roughly equal to 3dB for all VLAs with fewer than 9 elements. For arrays with more elements than 9, the surface gain increases. This indicates that the sparser arrays are unable to focus sound in the vertical at the target range. The denser arrays are capable of focussing sound away from the near-surface target because of their improved ability to resolve multipaths. This ability is a consequence of the arrays' ability to discriminate between sound travelling at different angles.

The graph in the top right hand corner of the figure shows bottom gain data and it is clear that these gains are very much lower than the surface gains. This is because the dominant paths from the array to the mid-water target were bottom interacting. That is, the paths underwent repeated bottom reflections during propagation from the centre of the VLA to the mid-water target. For this reason, any sound transmitted by the PCA into paths associated with propagation to the target would also be associated with bottom interaction.

Increases in number of elements beyond 7 led to bottom gain decreasing, i.e. the array became worse at focussing sound away from the seabed. In this situation, the array has been "seduced" by the bottom target and focused on it, rather than the in-water target. This is a feature of PCA performance [4] as they focus sound at the location of the largest echo. If this echo is associated with the near-bottom target then the PCA will focus sound near to the seabed and bg will become large and negative.

The negative values of time compression shown in the lower right hand corner of the figure indicate that the echo received at the centre of the VLA was wider than the echo arising from the simple point source transmission. This is because, in the presence of multipaths, the signal transmitted by the PCA is of greater duration than the original 0.1 second pulse. The echo returning from the target will only be time compressed if phase conjugation is working well. The negative time compressions shown in the figure indicate that this is not the case for the VLAs with the lower numbers of elements. However, it should be noted that as the number of elements in the array increased and array performance improved, the time compression increased.

The results for horizontal line arrays in endfire (HLAEs) are shown in Figure 2 by the lines marked with asterisks. It should be noted that the results for the one-element array are not the same as for the one-element VLA because the central elements of the two arrays are not in the same location. The HLAE is at the mid-water depth and its end element is located at the same location as the centre-point of the VLA. There is consequently a 40.5m offset between the elements used for the single element calculations for the VLA and HLAE cases.

Gains for the HLAE are greater than for the VLA for all measures of performance. Indicating that the horizontal array configuration is better than the vertical configuration for the situation studied. There is no guarantee that this will remain the case for other deployment depths of the HLAE.

Figure 2 shows that focussing gains for horizontal line arrays in broadside configuration (HLABs), shown by the data marked by diamonds, are linearly proportional to the logarithm of the number of elements in the array. This is because the array is perpendicular to the line joining the centre of the array to the target and the environment is horizontally stratified. The propagation paths from elements to the target are therefore very similar, the only difference arising from very small changes in the element-target distance along the array. As the array is 91m long and the target is 10,000m distant, these changes are very small indeed and the echoes received at each of the elements are therefore effectively the same. The focussing gain therefore arises from the coherent summation of all echoes that takes place at the centre of the array after phase conjugation.

Figure 2 shows that all other measures of array performance are independent of the number of elements in the HLAB. This is because the array is in broadside aspect and is consequently incapable of focussing sound in the vertical plane including the target. Thus, although the acoustic energy density of the echo from the mid-water target increased, as shown by the increases in f_g, the energy of the echoes from the surface and seabed targets increased by the same amount. This resulted in the values of s_g and b_g remaining unchanged. The bottom gain for the HLAB was negative, indicating that the seabed target, not the desired mid-water target, dominated the total pressure signal received at the HLAB. For this reason, the horizontal line arrays in broadside aspect were not considered further in this study.

6. ENVIRONMENTAL EFFECTS ON ARRAY PERFORMANCE

The performance of PCAs was studied in the three environments and results for the vertical configuration are shown in Figure 3. The figure shows that gains in the three environments are different by up to 10dB. Highest values of f_g, s_g and b_g are obtained for the winter conditions and this is because the sound speed profile in the winter environment is isothermal over the whole water column. The baseline and reflective environments have two isothermal layers separated by a thermocline. The increased vertical correlation of the environment in the winter case results in increased acoustic vertical correlation and hence in improved array performance.

Values of f_g, s_g and b_g obtained for the reflective environment tended to be higher than for the baseline environment. This is because the more reflective seabed supported more multipaths from source to target, resulting in greater gains from phase conjugation. This is also shown by the time compressions shown in the bottom right-hand corner of Figure 3. The reflective environment is shown to result in the greatest time compressions and this is because of the greater time spreading in the highly-multipath reflective environment for the original point-source transmission.

Figure 4 shows gains achieved for horizontal PCAs in endfire configuration. The greatest gains are observed in all four cases for the baseline environment, except for surface gain where the reflective and baseline environments give broadly similar gains. These trends of array performance with environment type are different from the VLAs where the best performance was observed for the winter environment. This is because the greater gains in the winter environment for VLAs arose from increased correlation in the vertical but this could only be exploited by vertical arrays.

The poorer performance of the HLAEs in the winter environment can be explained as follows. The change from summer to winter conditions has associated with it an increase in the effective acoustic duct depths because of the absence of the thermocline in the winter profile. This thermocline splits the water column into two ducts and consequently reduces the depth of the duct in which dominant propagation modes are concentrated. An increase in effective duct depth, He, will decrease the horizontal wavenumber difference between adjacent propagating normal modes. This can be shown analytically for the simple case of an isovelocity waveguide with sound speed c where the horizontal and vertical wavenumbers are related to the depth of the water column via

$$\gamma_n = \frac{(2n-1)\pi}{H_e}$$
$$\xi_n = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \gamma_n^{\,2}} \qquad (4)$$

The upper equation relates the vertical wavenumber γ_n of the nth mode to the water depth and the lower equation relates the vertical wavenumber to the horizontal wavenumber ξ_n of the mode. The relation implies that the wavenumber difference between the nth mode and its immediate neighbour is, for small n, given by

$$\left|\xi_n - \xi_{n+1}\right| \approx \frac{2\pi nc}{fH_e^{\,2}} \qquad (5)$$

This relation shows how increasing the depth of an acoustic duct decreases the spacing between adjacent modes in the duct. While (5) holds only for an isovelocity waveguide, the same trend of modal wavenumber spacing with increasing duct depth holds in the presence of depth dependent sound speed. This is shown in Figure 5 where the percentage difference in horizontal wavenumbers between adjacent modes is plotted in the baseline and winter environments. Modal wavenumbers were calculated for the first ten propagating modes using the SUPERSNAP [5] computer program with a source frequency of 2kHz. The modal horizontal wavenumbers are clearly more closely spaced in the winter environment.

Thus, changing from summer to winter conditions decreases the wavenumber spacing between adjacent modes. The ability of an array to focus sound relies on it being able to preferentially couple sound into modes, i.e. it must be able to distinguish between modes. The minimum wavenumber difference, $\delta\xi_{min}$, that an array of length L is capable of resolving is given by the approximate relation

$$\delta\xi_{\min} \approx \frac{2\pi}{L} \qquad (6)$$

For a fixed array length therefore, a decrease in the spacing of modal wavenumbers will reduce the array's ability to distinguish between modes. It is for this reason that the performance of HLAEs is made worse by the change from summer to winter conditions.

Comparison of Figures 2 and 3 shows that, although the gains are greater for the horizontal arrays in the baseline environment, gains in the other environments are generally greater for the vertical array. This is most pronounced in the winter environment.

7. DISCUSSION

Previous studies [1] of the use of phase conjugate arrays in underwater acoustics have concentrated on vertical arrays. However, the comparability of the gains obtained using horizontal and vertical arrays suggests that the use of phase conjugation with horizontal arrays in endfire configuration is worthy of further investigation. This is especially so in view of the greater ease of deployability and operation of horizontal arrays in comparison to vertical arrays.

The fundamental process of phase conjugation relies on the presence of multipaths and the array's ability to resolve these multipaths. If an array is deployed in environmental conditions where there is only one path from source to target then gains from phase conjugation will be small. It is this fact that is the basis of the predicted variations in array performance with environmental conditions. Similarly, if an array is incapable of resolving the multipaths present in the ocean, then it will not be able to exploit them to achieve gains via phase conjugation. This is the reason for the variations in gains with numbers of elements in the array.

The requirements for successful phase conjugation are therefore

- the presence of multipaths and
- the ability of an array to resolve those multipaths.

The requirements for array and environment are therefore inextricably linked and should be considered together when the performance of phase conjugate arrays is studied.

8. CONCLUSIONS

Acoustic gains achievable through phase conjugation in active sonar in shallow water environments were predicted using a frequency domain parabolic equation propagation model. Numerical measures of these gains were produced and calculated for vertical and horizontal arrays in summer and winter conditions with highly- and poorly-reflecting seabeds. Gains of up to 20dB were observed as a result of phase conjugation. Gains were shown to depend on sound speed profile and seabed reflectivity. For a summer environment, horizontal arrays were shown to give higher gains than vertical arrays of the same length and with the same number of elements. This trend was reversed in a winter environment.

9. REFERENCES

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		Baseline Environment		Winter Environment		Reflect Environment	
Depth (m)	Seawater Sound Speed (ms^{-1})	0	1494.3	0	1500.0	0	1494.3
Depth (m)	Seawater Sound Speed (ms^{-1})	19	1494.6	---	---	19	1494.6
Depth (m)	Seawater Sound Speed (ms^{-1})	50	1490.8	---	---	50	1490.8
Depth (m)	Seawater Sound Speed (ms^{-1})	91	1491.5	91	1501.7	91	1491.5
Seabed Sound Speed (ms^{-1})		1800		1800		1800	
Seabed Density ($g\ cm^{-3}$)		2.0		2.0		2.0	
Seabed Attenuation (dB per wavelength)		2.88		2.88		0.288	

Table 1 Environmental parameters used.

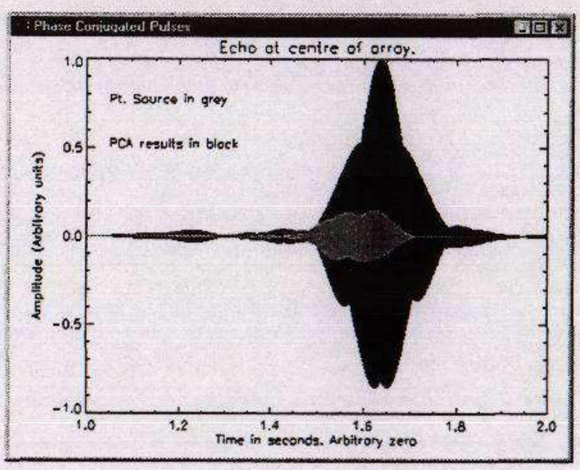

Figure 1 Echo at array centre after point source and PCA transmission

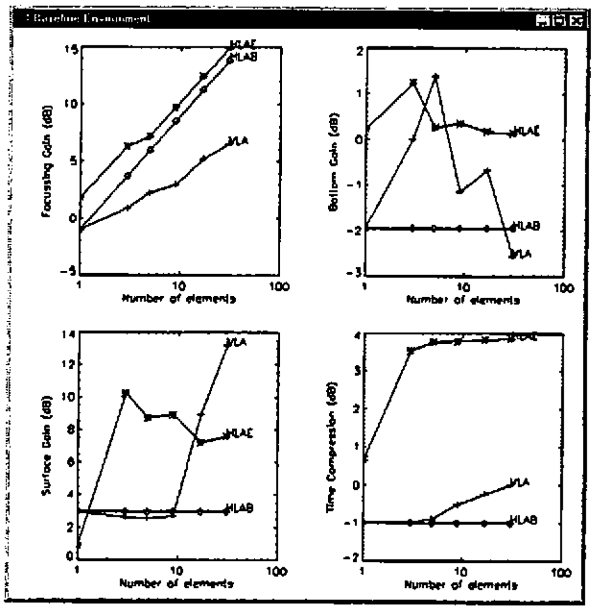

Figure 2 Measures of performance for 91m arrays in baseline environment.

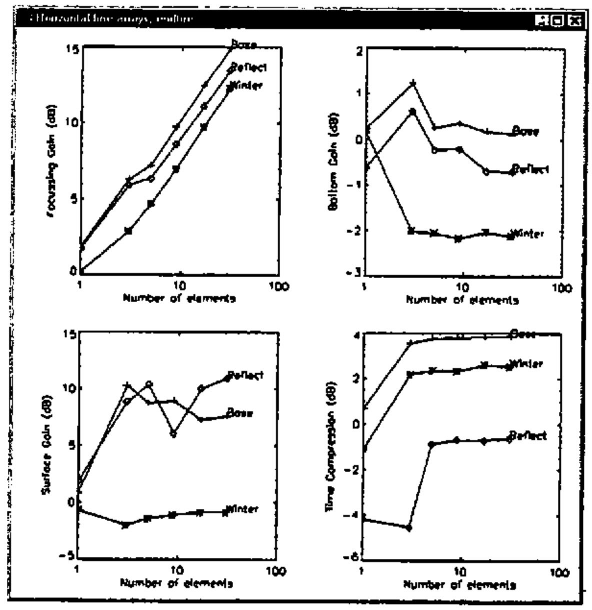

Figure 3 Measures of performance for HLAEs in three environments.

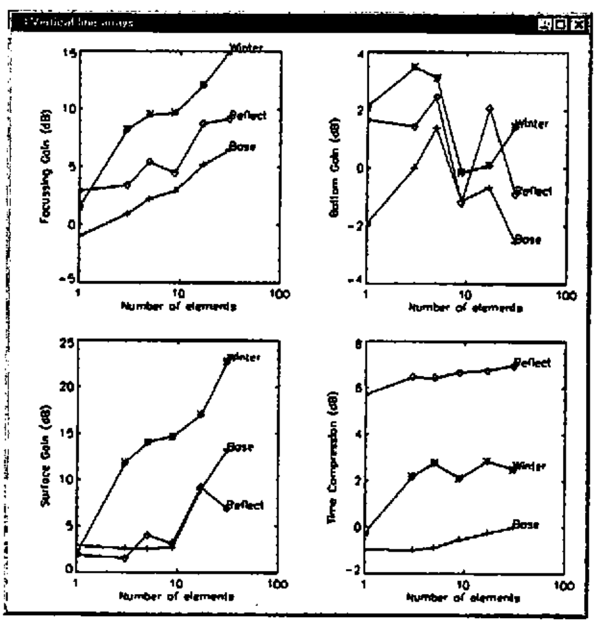

Figure 4 Measures of performance for VLAs in three environments.

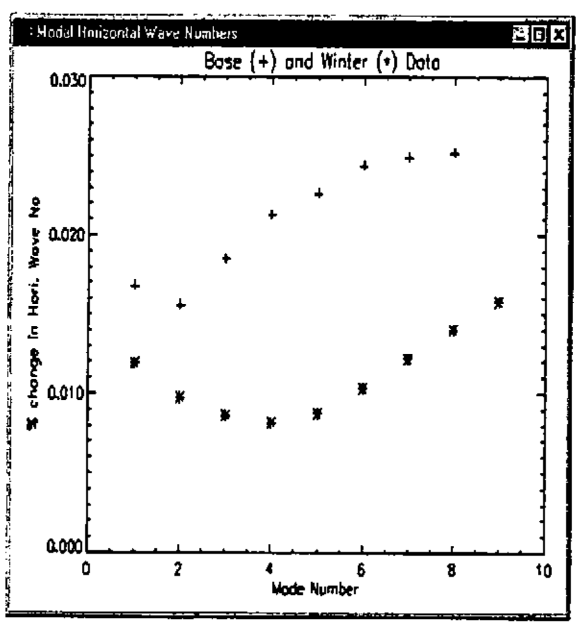

Figure 5 Percentage differences in modal horizontal wavenumbers in baseline and winter cases