COMPARISON OF RECURSIVE BAYESIAN TRACKING ALGORITHMS FOR TRACKING SPERM WHALES

ML Hadley ISVR, University of Southampton, Southampton, UK
PR White ISVR, University of Southampton, Southampton, UK
T Lewis 'Song of the Whale' Team, International Fund for Animal Welfare, London, UK

1 INTRODUCTION

Passive acoustics have proven to be a useful tool in the study of sperm whales (*Physeter macrocephalus*) [1–4]. A number of methods have been proposed for the passive acoustic localisation of vocalising animals in general [5–7] and specifically for localisation of vocalising sperm whales using sea floor mounted sensors [8–10] and hydrophone arrays towed behind a vessel [11, 12].

Utilisation of towed hydrophone arrays is particularly common as deployment and transportation costs are relatively low. For bioacoustic applications, towed hydrophone arrays consist of a 100-400 meters cable with as few as two elements separated by a distance of up to 3 meters. The use of a line array leads bearing estimates between 0° and 180° with an ambiguity if the source lies to the left or the right. Estimates of the range of a target animal from these bearings only measurements can be obtained by plotting lines from the measured bearings to the animal and observing the range at which they intersect over time in the order of minutes. This process can be subject to considerable errors and requires manual intervention.

Estimating the position of a target using bearings only measurements is a classical tracking problem in passive sonar and heat signature tracking to which a number of Bayesian solutions have been proposed [13–18]. Such solutions are based on derivatives of the Kalman filter [19, 20] and particle filters [18, 21]. Further to this White and Hadley [8] utilised a particle filter to track a single vocalising sperm whale in a Cartesian based coordinate system using five sea floor mounted sensors.

It is proposed that Bayesian tracking algorithms can be utilised to automate the task of estimating the range of sperm whales from an observing vessel and provide automated range updates. Two Bayesian tracking algorithms are reviewed with both being applied to range and bearings tracking of simulated data with known statistics. These algorithms are then applied to tracking the range and bearing of a single sperm whale from an observing vessel.

2 TRACKING ALGORITHMS

Tracking algorithms are recursive Bayesian estimators where the current position of a target relative to an observer is estimated from the previous estimate and a related noise contaminated measurement. Tracking is a state estimation problem where the state of a tracked system (the target), $\mathbf{x}$, which here represents the animal's position, at time k can be defined as:

$$\mathbf{x}_k = \mathbf{f}\left(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}\right) + \mathbf{v}_{k-1} \tag{1}$$

where the function $\mathbf{f}(\ldots)$ models the process by which the system propagates from state $\mathbf{x}_{k-1}$ to state $\mathbf{x}_k$, which here is the animal's change in position. A noise term, $\mathbf{v}_k$ accounts for parameters not modelled by $\mathbf{f}(\ldots)$, such as the animal accelerating. A control vector, $\mathbf{u}_{k-1}$, accounts for the fact the observer maybe making measurements from a moving platform. The transformation by which the system state, $\mathbf{x}_k$, becomes a measurement, $\mathbf{z}_k$, a bearing in this case, can be modelled as:

$$\mathbf{z}_k = \mathbf{h}\left(\mathbf{x}_k\right) + \mathbf{w}_k \tag{2}$$

where $\mathbf{w}_k$ is a noise term accounting for processes not modelled by $\mathbf{h}(\ldots)$.

The task is to obtain the posterior distribution of $\mathbf{x}_k$ given $\mathbf{z}_{0:k}$ using Baye's rule [21]:

$$p(\mathbf{x}_k|\mathbf{z}_{0:k}) = \frac{p(\mathbf{x}_k|\mathbf{z}_k)\, p(\mathbf{x}_k|\mathbf{z}_{0:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} \tag{3}$$

It is then possible to use (3) to construct a minimum mean squared error (MMSE) estimate of the system state:

$$\mathbf{E}\{\mathbf{x}_k|\mathbf{z}_{0:k}\} = \int \mathbf{x}_k \cdot p(\mathbf{x}_k|\mathbf{z}_{0:k}) d\mathbf{x}_k \tag{4}$$

If the noise processes $\mathbf{v}_k$ and $\mathbf{w}_k$ are Gaussian and $\mathbf{f}(\ldots)$ and $\mathbf{h}(\ldots)$ are linear then (3) is optimally calculated by the Kalman filter [18, 22, 23]. However these restrictions aren't met in tracking applications where the noise process distributions can not be accurately represented by first and second order statistics and $\mathbf{f}(\ldots)$ and $\mathbf{h}(\ldots)$are non-linear. The unscented Kalman filter and particle filters are solutions that estimate (3) when the restrictions for the Kalman filter are not met. As discussed below, in section 3, $\mathbf{f}(\ldots)$ and $\mathbf{h}(\ldots)$ are both non-linear functions in this application which eliminates the Kalman filter as a tracking solution.

2.1 UNSCENTED KALMAN FILTER

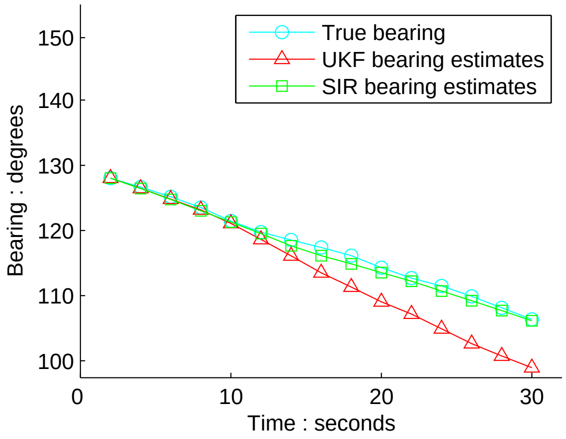
The unscented Kalman filter (UKF) applies the unscented transform [24] to the Kalman filter framework [19]. A set of $2N+1$ sigma points, $\mathcal{X}_{k-1}$, are strategically selected around the previous estimate $\hat{\mathbf{x}}_{k-1|k-1}$, to represent the system state. The sigma points lie at the mean and $\pm$ one standard deviation in each dimension of the N dimensions. The sigma points are then propagated through the system function (1) and a prediction of the next system state $\hat{\mathbf{x}}_{k|k-1}$ is made. Each sigma point is then transformed by the measurement function (2) to acquire a set of measurement sigma points, $\mathcal{Z}_{k|k-1}$, which predict the measurement vector. The points $\mathcal{Z}_{k|k-1}$ and the latest measurement $\mathbf{z}_k$ are used to calculate the adjustment needed to the prediction $\hat{\mathbf{x}}_{k|k-1}$ to give the final estimate $\hat{\mathbf{x}}_{k|k}$.

2.2 SAMPLING IMPORTANCE RESAMPLING PARTICLE FILTER

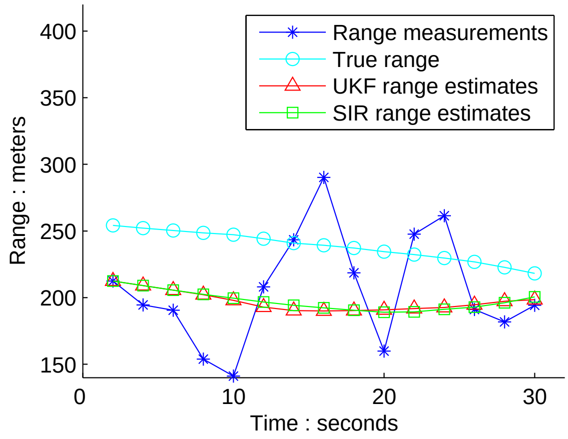
Particle filters differ from the UKF by replacing the small number of strategically selected sigma points for a large number of particles made up of randomly drawn samples with supporting weights. The Sampling Importance Resampling (SIR) particle filter [21,25] is the simplest form of functional particle filter. A set of particles $\mathbf{x}_k^{1:N}$ is drawn from the prior distribution $p\left(\mathbf{x}_k|\mathbf{x}_{k-1}^{1:N}\right)$ with weights calculated from the evidence term $\tilde{w}_k^i = p\left(\mathbf{z}_k|\mathbf{x}_k^{1:N}\right)$. After the weighting process has been completed the weights are normalised so they sum to one: $w_k^i = \frac{\tilde{w}_k^i}{\sum_{i=1}^N \tilde{w}_k^i}$. A MMSE estimate is then computed from the particles: $\hat{\mathbf{x}}_k = \sum_{i=1}^N w_k^i \mathbf{x}_k^i$. The final step is to resample the particles so that the less likely ones are replaced with duplicates of those that are more likely.

3 APPLICATION TO SPERM WHALE TRACKING

Application of tracking filters to tracking the range and bearing of sperm whales requires definition of $\mathbf{x}_k$, $\mathbf{z}_k$, $\mathbf{f}(\ldots)$ and $\mathbf{h}(\ldots)$. In the literature the measurement $\mathbf{z}_k$ is typically a single bearing measurement [13–18]. These publications typically assume that the observer is able to out-manoeuvre the target and a complete and accurate knowledge of observer's own position and motion is available. If this is true the range of the target can be resolved from bearing measurements acquired by the observer. Unfortunately in bearings only sperm whale tracking this is not always true. The observing vessel's manoeuvres can be derived from GPS data, however the array cable length is typically of the order of hundreds of meters so is slow to respond to manoeuvres which makes out-manoeuvring the target animal extremely difficult. Range can be roughly measured using pairs of



(a) Bearings Tracking



(b) Range Tracking

Figure 1: True bearings and range, unfiltered range estimates generated using the sine rule, and bearings and range tracking results from the UKF and SIR particle filter for simulated data

bearing measurements and the sine rule. The tracking filter will then estimate a range with less error. The measurement vector can now be defined as:

$$\mathbf{z}_k = [\theta_k, r_k]^T \tag{5}$$

where θ_k is the measured bearing and r_k is the measured range. Although it's knowledge of the animal's range and bearing from the observer that is desired, it has been reported that tracking in a modified polar coordinate system, where the reciprocal of the range is tracked, leads to more stable filtering equations [13]. The system function, $\mathbf{f}(\ldots)$, requires the rate of change in range and the angular velocity of the target animal to be incorporated into the state vector to account for the animal's motion. The state vector can now be defined as:

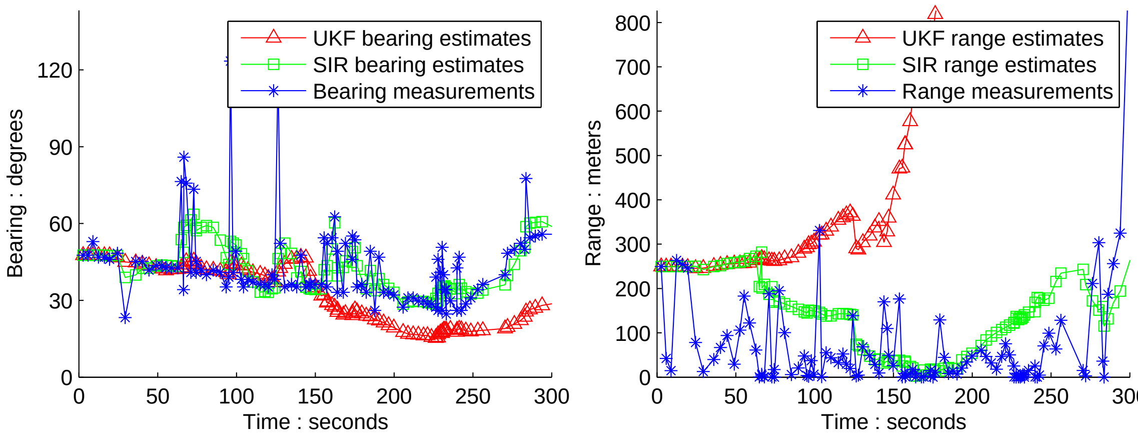
$$\mathbf{x}_k = \left[\frac{1}{r_k}, \frac{\dot{r}_k}{r_k}, \theta_k, \omega_k\right]^T \tag{6}$$

where r_k is the estimated range, $\dot{r}_k$ is the estimated rate at which the range is changing, θ_k is the estimated bearing and ω_k is the estimated angular velocity. The control vector $\mathbf{u}_k$ for the array can be estimated by delaying the observing vessel's GPS data.

3.1 SIMULATION DATA

It is extremely difficult to obtain accurate positioning information for dived sperm whales, therefore it is useful to test the proposed tracking filters on simulation data so that a ground truth is available for results comparison. Data is generated using the system function, $\mathbf{f}(\ldots)$, and measurement function, $\mathbf{h}(\ldots)$, utilised in the tracking filters. Gaussian noise samples are generated for $\mathbf{v}_k$, with variance $\sigma_v^2 = 0.28$ derived from figures reported by Whitehead [3] and Wahlberg [1], and $\mathbf{w}_k$ with variance $\sigma_w^2 = 0.005$.

Figure 1a) shows the true bearings of the target from the observer and the results from both tracking filters. It can be see that the UKF diverges from the true bearings whereas the particle filter closely tracks the true bearing. Figure 1b) shows the true range of the target, the range estimates that have been back calculated using pairs of bearing estimates and the sine rule - used as a range measurement input to the tracking filters - and the range estimation results from each filter. In keeping with how the system would be initialised in practice, the filters are initialised with the first range measurement. Both range estimates converge towards the true range from the initial range estimate.



(a) Bearings Tracking

(b) Range Tracking

Figure 2: Unfiltered bearings estimates and range estimates generated using the sine rule, and bearings and range tracking results from the UKF and SIR particle filter for sperm whale data

3.2 ACOUSTIC SPERM WHALE DATA

Figure 2 shows the bearing and range tracking results for 5 minutes of acoustic sperm whale data. Although no ground truth is available the performance of the two tracking algorithms can be evaluated on how they follow the general trend of the noisy measurements data without diverging. Despite positive early results (>100 seconds), Figure 2a) shows the bearing tracking results from the UKF diverge from the range measurements. The bearings tracking results from the SIR filter appear more sensitive to small variations in bearing, however the SIR filter does not diverge from the general trend of the bearing measurements.

Figure 2b) shows the range tracking results in comparison to the range measurements. Again the UKF shows good early tracking results until it starts to diverge between 50 and 100 seconds before becoming unstable at 125 seconds. The SIR filter range estimates seem to better follow the general trend in the range measurements and remain stable.

3.3 RESULTS ANALYSIS

The characteristics of how the UKF and SIR particle filter function are reflected in the way each response to erroneous measurements. Figure 2a) shows a bearing measurement of 24° at 30 seconds. The UKF maintains the general trend in bearing change and estimates the bearing as 45° when the SIR estimate is 39°. The UKF deviates less from the general trend in bearings until 100 seconds where upon it departs from this trend. At 150 seconds the variance of the noise on the bearing measurements increases, i.e. the signal to noise ratio (SNR) decreases, for a period of 50 seconds. During this period the bearing measurement noise increases and the range measurements derived from the bearings become zero. This causes the UKF to become unstable and diverge, however the SIR filter remains stable and is able recover to continue tracking when the bearing measurement noise decreases again. This is because the SIR filter is better able to model higher order statistics in the posterior distribution and therefore is better able to adapt to changes in the measurement noise. Performance gains may be made from improved modelling of the system and measurements noise distributions, rather than using a simple Gaussian approximation. Such improvements could include limits on animal swim speed or modelling the measurement noise with a Cauchy distribution.

4 CONCLUSION AND DISCUSSION

The results presented in section 3.2 demonstrate the applicability of tracking filters to tracking marine mammals using towed hydrophone arrays. Tracking performance is affected by the SNR of the measurements made which is dependent on hardware, software and environmental factors. False sperm whale click detections result in erroneous bearing measurements being made leading to increased measurement noise variance. This can be a consequence of an algorithmic failure or the result of other noise sources in the water. In the work presented the range measurements are made from pairs of bearing measurements, therefore any erroneous bearing measurements also result in noise on the range measurements. Both these problems may be reduced by increasing the number of array elements. This would allow more robust bearing and range measurements and beam-forming which could further improve the SNR. Heading sensors on array elements would allow more accurate results from the system function model (1) because $\mathbf{u}_k$ would be more accurately known.

Particle filters have also been proven to be applicable to tracking multiple targets [26] which is very difficult with the UKF. The ability of the particle filter to model any distribution and readily available access to the modelled distribution provides a potential solution to the left-right ambiguity problem associated with passive acoustic tracking with towed hydrophone arrays. Adaptation to tracking with different configurations of hydrophones is relatively simple as it would simply require modification of measurement function. These factors as well as the results presented here make the SIR particle filter an appropriate method for tracking sperm whales from towed hydrophone arrays.

ACKNOWLEDGEMENTS

The data for this work was provided by the International Fund for Animal Welfare's Song of the Whale team.

REFERENCES

1. M. Wahlberg. The acoustic behaviour of diving sperm whales observed with a hydrophone array. *Journal of Experimental Marine Biology and Ecology*, 281(1-2): pp. 53 – 62, 2002.

2. W. Zimmer, P. Tyack, M. Johnson, and P. Madsen. Three-dimensional beam pattern of regular sperm whale clicks confirms bent-horn hypothesis. *Journal of the Acoustical Society of America*, 117(3): pp. 1473 – 85, 2005.

3. H. Whitehead. *Sperm Whales Social Evolution in the Ocean*. The Univeristy of Chicago Press, 2003.

4. T. Lewis, D. Gillespie, C. Lacey, J. Matthews, M. Danbolt, R. Leaper, R. McLanaghan, and A. Moscrop. Sperm whale abundance estimates from acoustic surveys of the ionian sea and straits of sicily in 2003. *Journal of the Marine Biological Association of the United Kingdom*, 87(1): pp. 353 – 357, 2007.

5. J. Spiesberger. Locating animals from their sounds and tomography of the atmosphere: Experimental demonstration. *Journal of the Acoustical Society of America*, 106(2): pp. 837 – 46, 08 1999.

6. J. Spiesberger. Hyperbolic location errors due to insufficient numbers of receivers. *Journal of the Acoustical Society of America*, 109(6): pp. 3076 – 9, 2001.

7. D. H. Cato. Simple methods of estimating source levels and locations of marine animal sounds. *Journal of the Acoustical Society of America*, 104(3 I): pp. 1667 – 1678, 1998.

8. P. White and M. Hadley. Introduction to particle filters for tracking applications in the passive acoustic monitoring of cetaceans. *Canadian Acoustics*, 36(1): pp. 146 – 152, March 2008.

9. P. White, T. Leighton, D. Finfer, C. Powles, and O. Baumann. Localisation of sperm whales using bottom-mounted sensors. *Applied Acoustics*, 67(11-12): pp. 1074 – 90, 2006.

10. E.-M. Nosal and L. N. Frazer. Track of a sperm whale from delays between direct and surface-reflected clicks. *Applied Acoustics*, 67: pp. 1187–1201, 2006.

11. A. Thode. Three-dimensional passive acoustic tracking of sperm whales (physeter macrocephalus) in ray-refracting environments. *Journal of the Acoustical Society of America*, 118(6): pp. 3575 – 3584, 2005.

12. A. Thode. Tracking sperm whale (physeter macrocephalus) dive profiles using a towed passive acoustic array. *Journal of the Acoustical Society of America*, 116(1): pp. 245 – 53, July 2004.

13. V. J. Aidala and S. E. Hammel. Utilization of modified polar coordinates for bearings-only tracking. *IEEE Transactions on Automatic Control*, AC-28(3): pp. 283 – 294, 1983.

14. A. Farina. Target tracking with bearings-only measurements. *Signal Processing*, 78(1): pp. 61 – 78, 1999.

15. C. Hue, J.-P. Le Cadre, and P. Perez. Sequential monte carlo methods for multiple target tracking and data fusion. *IEEE Transactions on Signal Processing*, 50(2): pp. 309–325, February 2002.

16. R. Karlsson and F. Gustafsson. Range estimation using angle-only target tracking with particle filters. *Proceedings of the American Control Conference*, 5: pp. 3743 – 3748, 2001.

17. Y. Jiaxiang, X. Deyun, and Y. Xiuting. Square root unscented particle filter with application to angle-only tracking. *Proceedings of the World Congress on Intelligent Control and Automation (WCICA)*, 1: pp. 1548 – 1552, 2006.

18. Ristic, Arulampalam, and Gordon. *Beyond the Kalman Filter - Particle Filters for Tracking Applications*. Artech House, 2004.

19. E. Wan and R. Van Der Merwe. The unscented kalman filter for nonlinear estimation. *Proceedings of the IEEE 2000 Adaptive Systems for Signal Processing, Communications, and Control Symposium*, : pp. 153 – 8, 2000.

20. R. Van der Merwe and E. Wan. The square-root unscented kalman filter for state and parameter-estimation. *2001 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings*, vol.6: pp. 3461 – 3464, 2001.

21. M. Arulampalam, S. Maskell, N. Gordon, and T. Clapp. A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking. *IEEE Transactions on Signal Processing*, 50(2): pp. 174 – 88, February 2002.

22. R. E. Kalman. A new approach to linear filtering and prediction problems. *Transactions of the AMSE - Journal of Basic Engineering*, 82: pp. 35 – 45, 1960.

23. S. Bozic. *Digital and Kalman Filtering*. Arnold, 1979.

24. S. Julier, J. Uhlmann, and H. Durrant-Whyte. A new method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Transactions on Automatic Control*, 45(3): pp. 477 – 82, 2000.

25. N. Gordon, D. Salmond, and A. Smith. Novel approach to nonlinear/non-gaussian bayesian state estimation. *IEE Proceedings F (Radar and Signal Processing)*, 140(2): pp. 107 – 13, April 1993.

26. C. Hue, J.-P. Le Cadre, and P. Perez. Tracking multiple objects with particle filtering. *IEEE Transactions on Aerospace and Electronic Systems*, 38(3): pp. 791 – 812, 2002.