α_n normal absorption coefficient
α_∞ tortuosity
σ_a resistivity of the volume a of the equivalent cell in $Nm^{-4}s$
σ_d resistivity of the volume d of the equivalent cell in $Nm^{-4}s$
γ ratio of specific heats for air
μ dynamic viscosity of the air = 1.84×10^{-5} kg $m^{-1}s^{-1}$ (poiseville)
ρ_b bulk density of the foam in kgm^{-3}
ρ_o density of the air at normal conditions = 1.213 kg m^{-3}
ρ_s density of the solid skeleton in kg m^{-3}
ω radian frequency = $2\pi f$

INTRODUCTION

Acoustic attenuation and wave propagation in sound absorbing materials has been studied for roughly half a century. The early works assumed that the material was rigid, and sound energy was dissipated through viscous friction and heat conduction [1.2]. To describe more accurately acoustic propagation the additional complexity (physical properties) of material including motion of solid skeleton was added [3,5]. The additional loss of acoustical energy in solid skeleton was taken into consideration.

The physical structure of the polyurethane foam with open cells is described to a large extent by the porosity H. Porosity of the foam designed for sound absorption purpose is often greater than 95%. Other parameters which are strongly influencing sound absorption are resistivity σ, dynamic density of the air inside the foam $\rho(\omega)$ and dynamic bulk modulus $K(\omega)$. The single idealized cell of the foam according to Plateau's experimental data [6] could be described as a dodecahedron. Described structure presents a solidified liquid phase of the gas-liquid polyurethane mixture in a manufacturing process.

SOUND PROPAGATION IN FOAMS

The porous sound absorbing material including open cell foams could be characterized as a modified fluid when subject to acoustic one degree of freedom excitation. This approach requires assumption that only one dilatational wave is observed. This has been found to be reasonable assumption for various sound absorbents. In this theory of acoustical behavior of rigid porous media three equations of motion, continuity and state in one-dimensional form are:

$$\rho_0 (k_s/H)(\delta u/\delta t) + \sigma u = -(\delta p/\delta x) \quad (1)$$

$$H (\delta p/\delta t) + \rho(\delta u/\delta x) = 0 \quad (2)$$

$$c_0^2(\rho - \rho_0) = (p - p_0)\,\gamma \qquad (3)$$

In practice influence of solid structure vibrations developed by height acoustical pressure on sound absorption are pronounced only in high density foams ($\rho_b > 30$ kg m^{-3}).

MATHEMATICAL MODEL

A homogeneous and isotropic foam is assumed. The acoustic wavelengths are assumed to be much larger than cell greatest hydraulic radius R_a.

Presented model simplifies the shape of the equivalent cell but as a rule is possessing the same number of cells per characteristic length like in a real foam. Equivalent cell is cylindrical with two radiuses corresponding to two lengths. Radius R_a with corresponding length a models inside of the cylindrical volume of the cell and the second radius R_d with corresponding thickness d models volume surrounded by polymer ribs and forming opening between adjacent cells.

* Porosity H. By definition porosity is:

$$H = U/v = 1 - (\rho_b/\rho_s) \qquad (4)$$

involving number of cells and characteristic dimensions of the equivalent cell porosity is:

$$H = [N\pi(aR_a^2 + dR_d^2)/Ae] \qquad (5)$$

* Resistivity σ.

In the presented model resistivity of one string of the cells across thickness e can be calculated from:

$$\sigma_e = N_e(\sigma_a + \sigma_d) = N_e(8\mu/H)\,[\{(a+d)/a\}/R_a^2 + \{(a+d)/d\}/R_d^2] \qquad (6)$$

and resistivity of the foam is: $\sigma = [N_e\sigma_e^{-1}]^{-1}$ (7)

* Tortuosity α_∞: Tortuosity of the model of the foam is:

$$\alpha_\infty = [HA(a+d)/N\pi e]\,[\{a/(a+d)\}/R_a^2 + \{d/(a+d)\}/R_d^2] = \qquad (8)$$

$$[\{R_a^2(a/(a+d)\} + \{R_d^2(d/(a+d)\}]\,[\{a/(a+d)\}/R_a^2 + \{d/(a+d)\}/R_d^2] \qquad (9)$$

* Dynamic density of the air inside the foam $\rho(\omega)$:

$$\rho(\omega) = \alpha_\infty\,[\rho_a(\omega)\{a/(a+d)\}/R_a^2 + \rho_d(\omega)\{d/(a+d)\}/R_d^2]\,[\{a/(a+d)\}/R_a^2 + \{d/(a+d)\}/R_d^2]^{-1} \qquad (10)$$

where: (11, 12)

$\rho_a(\omega) = \rho_0\,[1+(c^2/3)] - i(\sigma_a H/\omega)$ and $\rho_d(\omega) = \rho_0[1+(c^2/3)] - i(\sigma_d H/\omega)$

in presented model parameter c in above equations is one.

* <u>Dynamic bulk modulus $K(\omega)$:</u>

$$K(\omega) = [(R_a^2)\, a/(a+d) + (R_d^2) d/(a+d)] * \{(R_a^2)\, \{a/(a+d)\}/K_a(\omega) + (R_d^2) * \{d/(a+d)\}/K_d(\omega)]^{-1} \quad (13)$$

where:

dynamic bulk modulus of the air in the volume a of equivalent cell is:

$$K_a(\omega) = 1/C_a(\omega) = \{(1/\gamma p_0)\,[\gamma-(\gamma-1)]\,(\rho_0)/\rho_a(N_{Pr}\omega)\}^{-1} \quad (14)$$

dynamic bulk modulus of the air is the volume d of equivalent cell is:

$$K_d(\omega) = 1/C_d(\omega) = \{(1/\gamma p_0)\,[\gamma-(\gamma-1)]\,(\rho_0)/\rho_d(N_{Pr}\omega)\}^{-1} \quad (15)$$

* <u>Acoustical impedance z is:</u>

$$Z(\omega) = (1/H)\,[\rho(\omega)\,K(\omega)]^{0.5} \quad (16)$$

* <u>Normal coefficient of acoustical absorption α_n</u>:

$$\alpha_n(\omega) = 1 - [(Z(\omega) - \rho_0 c_0)/(Z(\omega) + \rho_0 c_0)]^2 \quad (17)$$

CONCLUSIONS

The deviations of absorption data obtained from the presented model from experimental values are relatively high for both low (<500Hz) and high (>3 kHz) frequencies. Such deviations may be attributed to the assumption that the gaseous expansions in the foam are isothermal over the full range of audible frequencies. Also the complex geometry of foam cells is generally not available. Hence, the model was generalized similarly to the Attenborough and Biot-Allard approaches. In the case when the variations in cell size of the foam was large the deviation of calculated absorption from measurements increases.

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