Proceedings of the Institute of Acoustics

GEOMETRY CONSTRAINTS IN THE CONTROLLER DESIGN OF MULTIPLE SOURCE ACTIVE NOISE CONTROL SYSTEMS

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1. INTRODUCTION

Active noise control (ANC) is realised by artificially generating secondary (cancelling) source(s) of sound through detecting the primary (unwanted) noise and processing it by a suitable electronic controller, so that when the secondary wave is superimposed on the primary wave the two destructively interfere with one another and reduction of the unwanted sound occurs. Theoretical and practical investigations have shown that, generally, due to the broadband nature of the noise emitted by practical sources, the control process is required to realise suitable continuous frequency-dependent characteristics so that cancellation over a broad range of frequencies of the noise is achieved [1-4].

In practice, sources of noise can broadly be classified as compact or distributed. A compact source of noise is theoretically modelled as a point source with contours of equal pressure levels forming spherical surfaces around the source. A distributed source of noise, on the other hand, can be modelled as a set of point sources distributed around the surface of the source. In cancelling the noise of a compact source, a single detector is generally sufficient to obtain the required signal information needed to generate the cancelling signal. This leads to the realisation of a control structure incorporating a single input signal. However, in cancelling the noise of a distributed source, to obtain sufficient signal information, a muliple set of detectors will be required. This will lead to the realisation of a multi-input control structure. Similarly, at the output end, the performance requirements of the system as related to the physical extent of cancellation in the medium, will lead to the realisation of either single-output or multi-output control structures. Therefore, depending on the application a suitable control structure incorporating the required number of inputs and outputs can be employed. In either case, the dependence of the required controller characteristics on the geometric arrangement of system components can lead to design constraints under certain conditions.

This paper presents a coherent method of design of ANC systems for compact and distributed sources of noise in a three-dimensional non-dispersive propagation medium (free-field). An analysis of single-input single output (SISO), single-input multi-output (SIMO) and multi-input multi-output (MIMO) control structures is given. Conditions for the robust operation of such systems on the basis of optimum cancellation, in relation to controller design, are determined. These conditions are interpreted as constraints in the geometric compositions of the system. The controller design relations are developed in the frequency-domain. These can, equivalently, be thought of either in the complex frequency s-domain or the z-domain allowing the practical realisation of the corresponding controller in either the continuous-time or the discrete-time using analogue or digital techniques accordingly.

2. ACTIVE NOISE CONTROL STRUCTURE

A schematic diagram of a general ANC structure, namely the MIMO feedforward control structure (FFCS), is shown in Figure 1(a). A set of n (generally independent) primary point sources emit unwanted acoustic signals (noise) into the medium. These are detected by a set of n detectors, processed by the MIMO controller and fed to a set of k secondary sources. The secondary signals thus generated are superimposed upon the unwanted noise so that the noise level is reduced at a set of k observation points. The corresponding frequency-domain equivalent block diagram of Figure 1(a) is shown in Figure 1(b) where $\mathbf{E}$ $(n\times n)$, $\mathbf{F}$ $(k\times n)$, $\mathbf{G}$ $(n\times k)$ and $\mathbf{H}$ $(k\times k)$ represent transfer characteristics of acoustic paths between the primary sources and the detectors, the secondary sources and the detectors, the primary sources and the observers and the secondary sources and the observers respectively. $\mathbf{M}$ ($n\times n$ diagonal), $\mathbf{M}_O$ ($k\times k$ diagonal) and $\mathbf{L}$ ($k\times k$ diagonal) represent the transfer characteristics of the detectors, the observers and the secondary sources respectively. $\mathbf{C}$ $(n\times k)$ represents the transfer characteristics of the controller. $\mathbf{U}_D$ $(1\times n)$ and $\mathbf{U}_C$ $(1\times k)$ are the primary and secondary signals at the sources whereas $\mathbf{Y}_{OD}$ $(1\times k)$ and $\mathbf{Y}_{OC}$ $(1\times k)$ are the corresponding signals at the observation point respectively. $\mathbf{U}_M$ $(1\times n)$ and $\mathbf{Y}_O$ $(1\times k)$ represent the detected and observed signals respectively. As seen in Figure 1(a), each detector gives a combined measure of the primary and secondary waves that reach the corresponding detection point. The secondary waves thus reaching the detectors form closed feedback loops that can cause the system to become unstable. Therefore, a stability analysis of the system to lead to a robust design is necessary at a design stage [5].

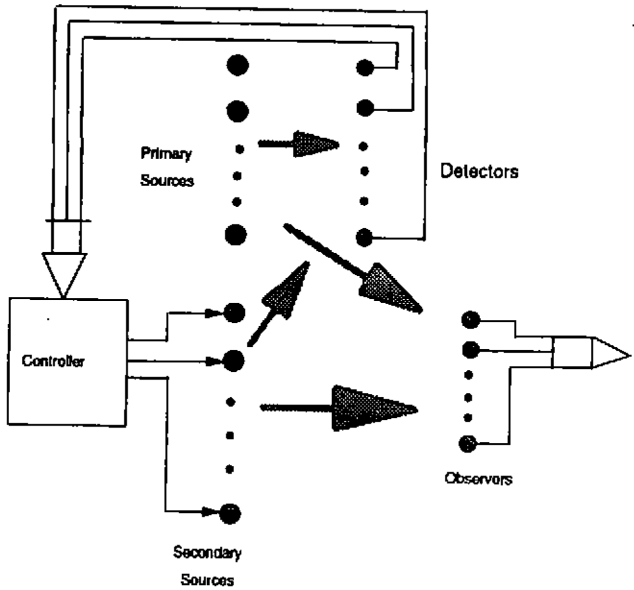
The objective in Figure 1 is to reduce the level of noise to zero at the observation points. This corresponds to the minimum variance design criterion in a stochastic environment. This requires the observed primary and secondary signals at each observation point to be equal in magnitudes and have a phase difference of 180°. Synthesising the controller within the block diagram in Figure 1(b) on the basis of this objective yields the required controller transfer function as

$$\mathbf{C} = \mathbf{M}^{-1}\Delta^{-1}\mathbf{G}\mathbf{H}^{-1}\mathbf{L}^{-1} \tag{1}$$

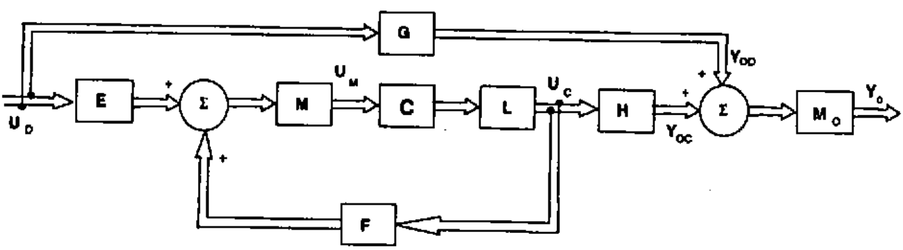
where Δ is an $n\times n$ matrix given by

$$\Delta = \mathbf{G}\mathbf{H}^{-1}\mathbf{F} - \mathbf{E} \tag{2}$$

Equation (1) represents the required controller design relation for optimum cancellation of noise at the observation points.



(a) Schematic diagram.



(b) Block diagram.

Figure 1: Active noise control structucr.

3. CONTROLLER DESIGN CONSTRAINTS

It follows from equation (1) that for given detectors and secondary sources with necessary electronic components, the controller characteristics required for optimum cancellation is dependent on the characteristics of the acoustic paths from the primary and secondary sources to the detection and observation points. In particular, if the set of detection and observation points are such that the determinant of Δ in equation (1) becomes zero then the critical situation of infinite gain controller (IGC) requirement arises. The locus of such points in the medium (as a practical limitation in the design of the controller) is therefore of crucial interest. Under such a situation, equation (2) can be written, for periodic waves, as

$$|\Delta(j\omega)| = |\mathbf{G}(j\omega)\mathbf{H}^{-1}(j\omega)\mathbf{F}(j\omega) - \mathbf{E}(j\omega)| = 0 \qquad (3)$$

where $\mathbf{E}(j\omega)$, $\mathbf{F}(j\omega)$, $\mathbf{G}(j\omega)$ and $\mathbf{H}(j\omega)$ represent the frequency responses of the corresponding acoustic paths in Figure 1, j is the unit imaginary number and ω is the radian frequency. These, for the propagation medium under consideration are given by

$$\begin{aligned} \mathbf{E}(j\omega) &= \{\mathbf{e}_{iq}(j\omega)\} = \left\{\frac{A}{r_{eiq}} e^{-j\frac{\omega}{c} r_{eiq}}\right\}, & \mathbf{F}(j\omega) &= \{\mathbf{f}_q(j\omega)\}^T = \left\{\frac{A}{r_{fq}} e^{-j\frac{\omega}{c} r_{fq}}\right\}^T, & i &= 1,2,\ldots,n \\ & & & & p &= 1,2,\ldots,n \qquad (4) \\ \mathbf{G}(j\omega) &= \{\mathbf{g}_{iq}(j\omega)\} = \left\{\frac{A}{r_{giq}} e^{-j\frac{\omega}{c} r_{giq}}\right\}, & \mathbf{H}(j\omega) &= \{\mathbf{h}_{pq}(j\omega)\} = \left\{\frac{A}{r_{hpq}} e^{-j\frac{\omega}{c} r_{hpq}}\right\}, & q &= 1,2,\ldots,k \end{aligned}$$

where A is a constant and r_{eiq}, r_{fq}, r_{giq} and r_{hpq} are the distances of the acoustic paths with transfer characteristics $\mathbf{e}_{iq}(j\omega)$, $\mathbf{f}_q(j\omega)$, $\mathbf{g}_{iq}(j\omega)$ and $\mathbf{h}_{pq}(j\omega)$ respectively. Note that equation (3) is given in terms of the characteristics of the acoustic paths in the system. This implies that the IGC requirement is a geometry-related problem in an ANC system. Therefore, an analysis of equation (3) will lead to the identification of loci of (detection and observation) points in the medium for which the IGC requirement holds. To obtain the solution of equation (3) a SISO system is considered first. The results obtained are then used and extended to the SIMO and MIMO ANC systems.

3.1. Single-Input Single-Output System

Let the ANC system in Figure 1 incorporate a single primary source ($n = 1$) and a single secondary source ($k = 1$). Thus, substituting for $\mathbf{E}(j\omega)$, $\mathbf{F}(j\omega)$, $\mathbf{G}(j\omega)$ and $\mathbf{H}(j\omega)$ from equation (4) into equation (3) with the associated distances denoted as r_e, r_f, r_g and r_h and simplifying yields

$$\left(\frac{r_e}{r_f}\right)e^{-j(r_f-r_e)\frac{\omega}{c}} = \left(\frac{r_g}{r_h}\right)e^{-j(r_h-r_g)\frac{\omega}{c}}$$

This equation is true if and only if the amplitudes as well as the exponents (phases) on either side of the equation are equal. Equating the amplitudes and the phases, accordingly, yields

$$\frac{r_e}{r_f} = \frac{r_g}{r_h} = a \quad , \quad r_f - r_e = r_h - r_g \qquad (5)$$

where a is a positive real number representing the distance ratio. Equation (5) defines the locus of points for which $|\Delta(j\omega)| = 0$ and the controller is required to have an infinitely large gain. Note that this equation is in terms of the distances r_e, r_f, r_g and r_h only. Therefore, the critical situation of the IGC requirement is determined only by the locations of the detector and observer relative to the primary and secondary sources.

Manipulating equation (5) further yields the locus of IGC requirement as

$$\frac{r_e}{r_f} = \frac{r_g}{r_h} = 1 \quad \text{for} \quad a = 1 \qquad (6)$$

$$\left[\frac{r_e}{r_f} = \frac{r_g}{r_h} = a \quad \text{and} \quad \frac{r_e}{r_g} = 1\right] \quad \text{or} \quad \left[\frac{r_e}{r_g} = \frac{r_f}{r_h} = 1\right] \quad \text{for} \quad a \neq 1 \qquad (7)$$

Equations (6) and (7) give the locus of detection and observation points in the medium relative to the primary and secondary sources for which the IGC requirement holds. Analysing these in a three-dimensional space coordinate system (assuming the primary and secondary sources are located at points P and S respectively) will reveal that, in particular, equation (6) defines the locus as a plane surface that perpendicularly bisects the line joining points P and S. Equation (7), on the other hand, defines the locus as a circle (IGC circle) with centre along the line PS and on a plane that is parallel with that defined by equation (6). The radius of the IGC circle is given by the distance between the detector and the line PS [6].

Note in Figure 1(a) that if the observer and the detector coincide with one another then the feedback control structure (FBCS) is obtained. In such a process $|\Delta| = 0$ will result which will always require a controller with an infinitely large gain. With a practically acceptable compromise between system performance and controller gain, and careful consideration of system stability, reasonable amounts of cancellation of noise can be achieved with this structure.

3.2. Single-Input Multi-Output System

Let the ANC system in Figure 1 incorporate a single primary source ($n = 1$) and k secondary sources. Thus,

$$\begin{aligned} \mathbf{C} = \{\mathbf{c}_j\}, \quad \mathbf{E} = [\mathbf{e}], \quad \mathbf{F} = \{\mathbf{f}_j\}^T, \quad & i = 1,2,\ldots,k \\ \mathbf{G} = \{\mathbf{g}_j\}, \quad \mathbf{H} = \{\mathbf{h}_{ij}\}, \quad & j = 1,2,\ldots,k \end{aligned} \tag{8}$$

Substituting for $\mathbf{E}(j\omega)$, $\mathbf{F}(j\omega)$, $\mathbf{G}(j\omega)$ and $\mathbf{H}(j\omega)$ from equation (8) into equation (3), using equation (4) with the associated distances denoted as r_e, r_{fi}, r_{gi} and r_{him} respectively and simplifying yields

$$\left.\begin{aligned} &\frac{r_e}{r_{fi}} = \frac{r_{g1}}{r_{hi1}} = \frac{r_{g2}}{r_{hi2}} = \ldots = \frac{r_{gk}}{r_{hik}} = a_i \\ &r_{fi} - r_e = r_{hi1} - r_{g1} = r_{hi2} - r_{g2} = \ldots = r_{hik} - r_{gk} \end{aligned}\right\} \quad ; \quad i = 1,2,\ldots,k \tag{9}$$

and

$$\begin{aligned} &\frac{r_{fi}}{r_{fm}} = \frac{r_{hi1}}{r_{hm1}} = \frac{r_{hi2}}{r_{hm2}} = \ldots = \frac{r_{hik}}{r_{hmk}} = a_{im} \quad ; \quad i = 1,2,\ldots,k \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ; \quad m = 1,2,\ldots,k \\ &r_{fm} - r_{fi} = r_{hm1} - r_{hi1} = r_{hm2} - r_{hi2} = \ldots = r_{hmk} - r_{hik} \quad ; \quad i \neq m \end{aligned} \tag{10}$$

where a_i and a_{im} are positive real numbers representing distance ratios. Equations (9) and (10) define the loci of detection and observation points for which $|\Delta(j\omega)| = 0$ and the controller, for optimum cancellation of noise at the observation points, is required to have infinitely large gain in each secondary path. In particular, equation (9) describes the locus of detection and observation points relative to the location of the primary source and secondary source i ($i = 1,2,\ldots,k$), i.e. k pairs of sources, whereas equation (10) describes the locus of detection and observation points relative to secondary sources i ($i = 1,2,\ldots,k$) and m ($m = 1,2,\ldots,k$), i.e. $\sum_{i=1}^{k-1} i$ pairs of sources. Thus, using the results obtained in the previous sub-section, assuming the two sources in question are located at points X and Y, the following two situations lead to the IGC requirement

(a) When the detector and all observers are equidistant from points X and Y. This defines the locus of detection and observation points as a plane (the IGC plane) that perpendicularly bisects the line XY.

(b) When the distance ratios from point X to the detector and observer m $(m = 1,2,\ldots,k)$ and to each pair of observation points as well as the distance ratios from point Y to the detector and observer m $(m = 1,2,\ldots,k)$ and to each pair of observation points are each equal to unity. This defines the locus of detection and observation points as a circle (the IGC circle), with centre along a straight line passing through points X and Y, on a plane that is at right angles with this line.

3.3. Multi-Input Multi-Output System

Let the ANC system in Figure 1 incorporate n primary sources and k secondary sources. Thus, the transfer characteristics $\mathbf{E}(j\omega)$, $\mathbf{F}(j\omega)$, $\mathbf{G}(j\omega)$ and $\mathbf{H}(j\omega)$ are defined as in equation (4). Substituting for these functions from equation (4) into equation (3) and simplifying yields

$$\frac{r_{em1}}{r_{ei1}} = \frac{r_{em2}}{r_{ei2}} = \ldots = \frac{r_{emn}}{r_{ein}} = \frac{r_{gm1}}{r_{gi1}} = \frac{r_{gm2}}{r_{gi2}} = \ldots = \frac{r_{gmk}}{r_{gik}} = a_p$$

$$r_{ei1} - r_{em1} = r_{ei2} - r_{em2} = \ldots = r_{ein} - r_{emn} = r_{gi1} - r_{gm1} = r_{gi2} - r_{gm2} = \ldots = r_{gik} - r_{gmk} \qquad (11)$$

$$i = 1,2,\ldots,n \quad ; \quad m = 1,2,\ldots,n \quad ; \quad i \neq m$$

$$\frac{r_{em1}}{r_{fi1}} = \frac{r_{em2}}{r_{fi2}} = \ldots = \frac{r_{emn}}{r_{fin}} = \frac{r_{gm1}}{r_{hi1}} = \frac{r_{gm2}}{r_{hi2}} = \ldots = \frac{r_{gmk}}{r_{hik}} = a_{ps}$$

$$r_{fi1} - r_{em1} = r_{fi2} - r_{em2} = \ldots = r_{fin} - r_{emn} = r_{hi1} - r_{gm1} = r_{hi2} - r_{gm2} = \ldots = r_{hik} - r_{gmk} \qquad (12)$$

$$\frac{r_{fm1}}{r_{fi1}} = \frac{r_{fm2}}{r_{fi2}} = \ldots = \frac{r_{fmn}}{r_{fin}} = \frac{r_{hm1}}{r_{hi1}} = \frac{r_{hm2}}{r_{hi2}} = \ldots = \frac{r_{hmk}}{r_{hik}} = a_s$$

$$r_{fi1} - r_{fm1} = r_{fi2} - r_{fm2} = \ldots = r_{fin} - r_{fmn} = r_{hi1} - r_{hm1} = r_{hi2} - r_{hm2} = \ldots = r_{hik} - r_{hmk} \qquad (13)$$

$$i = 1,2,\ldots,k \quad ; \quad m = 1,2,\ldots,k \quad ; \quad i \neq m$$

where a_p, a_{ps} and a_s are positive real numbers representing distance ratios. Equations (11), (12) and (13) define loci of detection and observation points relative to the locations of the sources in the medium for which the IGC requirement holds. In particular, equation (11) defines the locus of such points relative to the locations of primary sources i and m, equation (12) defines the locus with respect to the locations of primary source m and secondary source i and equation (13) defines the locus with respect to the locations of secondary sources i and m. Thus, the locus of detection and observation points leading to the IGC requirement is, in this case, defined in relation to the locations of all possible pairs of sources, each pair considered at a time. In this manner, for a system with n primary sources and k secondary sources a total of $\sum_{i=1}^{k-1} i + \sum_{m=1}^{n-1} m + nk$ pairs of sources can be identified. Among these, the primary sources considered with one another lead to $\sum_{m=1}^{n-1} m$ pairs, the secondary sources considered with one another lead to $\sum_{i=1}^{k-1} i$ pairs and the remaining nk pairs are formed by considering each primary source with the secondary sources. It follows from the analysis presented in the previous sub-sections that, in each case, assuming the two sources in question are located at points X and Y, the following two situations lead to the IGC requirement

(a) When all the detectors and observers are equidistant from points X and Y. This defines the locus of detection and observation points as a plane (the IGC plane) that perpendicularly bisects the line XY.

(b) When the distance ratios from point X to each pair of detection points, to detector i $(i = 1,2,\ldots,n)$ and observer m $(m = 1,2,\ldots,k)$ and to each pair of observation points as well as the distance ratios from point Y to each pair of detection points, to detector i

each equal to unity. This defines the locus of detection and observation points as a circle (the IGC circle), with centre along a line passing through points X and Y, on a plane that is at right angles with this line.

4. CONCLUSION

An analysis and design procedure for ANC systems in a three-dimensional non-dispersive propagation medium on the basis of optimum cancellation of noise has been presented. The relation between the transfer characteristics of the required controller and the geometrical arrangement of system components has been studied and conditions interpreted as geometrical constraints in the design of ANC systems have been derived and analysed.

The dependence of controller characteristics on the characteristics of the acoustic paths in the system, arising from geometrical arrangement of system components, can sometimes lead to practical difficulties in the design of the controller and to instability problems in the system. In particular, there are specific arrangements of system components, identified as loci of detection and observation points relative to the sources, which lead to the critical situation of infinite-gain controller requirement. In an ANC system with n primary sources and k secondary sources, the locus is defined in relation to the location of all possible pairs of sources, each pair considered at a time. In this manner, assuming the two sources in question are located at points X and Y, the following two situations lead to the IGC requirement

(1) When all the detectors and observers are equidistant from points X and Y. This defines the locus of detection and observation points as a plane that perpendicularly bisects the line XY.

(2) When the distance ratios from point X to each pair of detection points, to detector i $(i=1,2,\ldots,n)$ and observer m $(m=1,2,\ldots,k)$ and to each pair of observation points as well as the distance ratios from point Y to each pair of detection points, to detector i $(i=1,2,\ldots,n)$ and observer m $(m=1,2,\ldots,k)$ and to each pair of observation points are each equal to unity. This defines the locus of detection and observation points as a circle, with centre along a straight line passing through points X and Y, on a plane that is at right angles with this line.

In a FBCS, where both the detection and observation points coincide with one another, the situation leading to the IGC plane corresponds to the detection point(s) being on the IGC plane. In a FFCS, however, this corresponds to the situation when the detection as well as the observation points are on the IGC plane. With the situation leading to the IGC circle, on the other hand, a FBCS always satisfies the requirement. In a FFCS, however, it is possible to minimise the region of space occupied by the IGC circle by a proper geometrical arrangement of

5. REFERENCES

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