

## ON-LINE MODELLING OF FLEXURAL DYNAMICS OF FLEXIBLE MANIPULATORS

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### 1. INTRODUCTION

This paper presents an investigation into the on-line modelling of flexible robot manipulators. The work is motivated by the requirement of adaptive controllers for such systems in an industrial environment. Among the flexible and rigid-body dynamics of the system, the former is considered, in an attempt to provide suitable command signal(s) within a feedback control mechanism for vibration suppression in the system. The system considered in this investigation incorporates a single-link motor-driven robot arm. This is shown in Figure 1, where  $\tau$ ,  $\theta$ ,  $\dot{\theta}$  and  $\alpha$  represent the torque input, hub angle, hub velocity and end-point acceleration respectively. This shares the same common constraints, with an industrial robot, such as the control torque can only be applied at the hub and only a finite number of sensors can be used at proper locations along the length of the arm.

There are several methods that can be used to model the system depending upon the type of analytical model required [1-4]. On the other hand, if a parametric model is required, a suitable technique of system identification can be used. In this investigation, linear parametric models of the system are developed using suitable parameter estimation techniques. In this manner, the type of model thus developed depends on the nature of the noise corrupting the system output. In case of uncorrelated white noise, a suitable parametric model would be an autoregressive with exogenous input (ARX) model. In case of uncorrelated coloured noise, however, a suitable parametric model would be an autoregressive moving average with exogenous input (ARMAX) model. Both these type of models are considered in this paper. The two proposed models are suitable for using statistical approaches in testing and validation. Moreover, they can

be validated using correlation tests. Furthermore, these are suitable for development of a number of adaptive control schemes.

## 2. DYNAMIC MODELLING

The transfer function of the system between torque input and hub angle, hub velocity and end-point acceleration can be obtained as the sum of flexural and rigid-body dynamics of the system [3]. For a full-order model these will contain an infinite number of terms corresponding to the infinite number of resonance modes of the system. It has been demonstrated that only the first few resonance modes dominantly characterise the flexible dynamics of the system. In this investigation, only the first two resonance modes of the system are considered. For the system under consideration, these are at approximately 12 Hz and 35 Hz [3]. In this manner, a reduced order model of the system, containing the first two modes of vibration, leads to a model order of 6 for the hub angle, 5 for hub velocity and 4 for end-point acceleration [3].

A linear parametric model of the system can be described as

$$A(z^{-1})y(k) = B(z^{-1})u(k) + C(z^{-1})\xi(k) \quad (1)$$

where,  $y(k)$  is output signal,  $u(k)$  is input signal,  $\xi(k)$  is zero mean uncorrelated white noise,  $A(z^{-1})$ ,  $B(z^{-1})$  and  $C(z^{-1})$  are polynomials described in the complex frequency  $z$  domain. The model in equation (1) is of the ARMAX type. For an ARX model  $C(z^{-1})=1$  in equation (1).

Since the development of on-line modelling techniques is the main interest of this work, a recursive algorithm is proposed. Among other advantages in using a recursive algorithm are computational efficiency and memory saving. In this manner, suitable parameter estimation algorithms that can be used to obtain the parameters of ARX and ARMAX models are the recursive least square (RLS) and the recursive extended least square (REL) algorithms respectively [5].

There are four alternative test inputs that are commonly used in recursive estimation. These are square wave, pseudo random binary sequence (PRBS), uniformly distributed noise and Gaussian distributed noise. The test signal used in this work is a PRBS. In the experiments

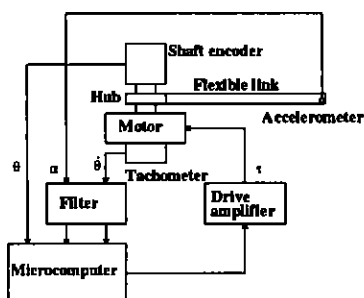


Figure 1: Schematic diagram of the flexible manipulator system.

conducted, the PRBS clock frequency was set to 50 Hz which covers the first two resonance modes of the system. The amplitude was set to  $\pm 4$  V which is equivalent to an input torque of  $\pm 0.176$  Nm. This will drive the system within its linear operating range. The sequence length of the PRBS was set to 127.

### 3. EXPERIMENTAL RESULTS

In the experiments conducted both ARX and ARMAX models of the system between the torque input and hub angle, hub velocity and end-point acceleration were developed using the corresponding on-line parameter estimation algorithms accordingly. Figures 2 and 3 show the frequency responses of the models between the torque input and end-point acceleration thus obtained. It is noted that the first two resonance modes of the system were identified very closely. To show this further the estimated models and the actual system were excited with a bang-bang torque input and the outputs were compared. Figure 4 shows the corresponding outputs. It is noted that the outputs of the models are very close to the output of the actual system.

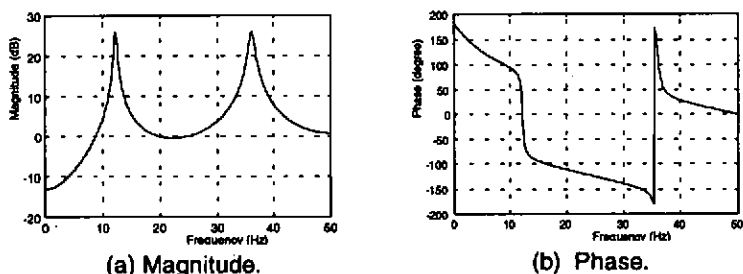


Figure 2: Frequency response of the end-point acceleration (ARX) model.

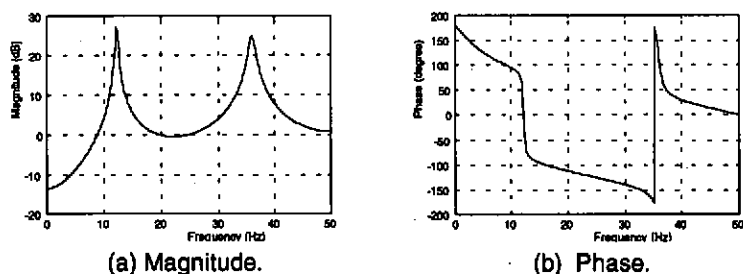


Figure 3: Frequency response of the end-point acceleration (ARMAX) model.

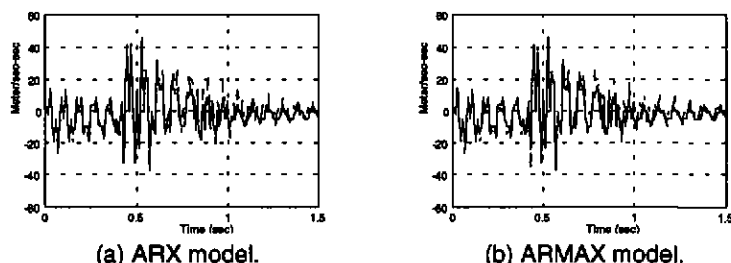


Figure 4: End-point acceleration response to bang-bang input of the model (broken-line) and the system (solid-line).

It was found through a process of validation of the models using correlation tests that the ARMAX model was better than the ARX model in characterising the system. This means that, in a system of the nature considered, there is a high probability for the noise corrupting the system output to be coloured rather than white noise.

#### 4. CONCLUSIONS

An investigation into the on-line identification of flexural dynamics of a flexible manipulator has been presented. It has been shown that the vibrational modes of the system can be completely identified using the RLS and REL algorithms. The noise that corrupts the system has a higher probability of being coloured rather than white noise. The on-line dynamic modelling of the system presented forms the basis of subsequent development of suitable adaptive control schemes for vibration suppression in flexible manipulator systems.

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