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ON THE VIBRATION FIELD CORRELATION OF FLAT PLATE STRUCTURES

M W Bonilha & F J Fahy

Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton, SO17 1BJ, UK

1. INTRODUCTION

As a basis for a hybrid vibroacoustic model [2,7], an approximate representation of the bending wave field generated by the random vibration of plate-like components is presented in this work. The use of this approximate description is an artifice employed in an attempt to avoid the necessity of the precise determination of the structural modes by making use of the very condition that militates against deterministic computation. In other words, it is assumed that many high-order plate modes resonate in each frequency band of analysis, so that computational analysis by the currently available deterministic methods will produce unreliable results.

The representation here proposed is based on the computation of the plate spatial correlation characteristics. The correlation characteristics are described by a frequency-averaged parameter, the correlation coefficient of the plate response.

2. THEORETICAL REPRESENTATION

The bending vibration field on a flat plate under spatially uncorrelated random excitation is here represented by a free-wave model [1]. The displacements at points 1 and 2 close to the corner formed by the bottom and left edges of the rectangular plate are given by [1]

$$z_1(x,y,t) = \left(A^B_{ref}e^{-ik_y y_1} + A^B_{in}e^{ik_y y_1} + A^B_e e^{-\mu_y y_1}\right)\left(A^L_{ref}e^{-ik_x x_1} + A^L_{in}e^{ik_x x_1} + A^L_e e^{-\mu_x x_1}\right)\Phi(t), \quad (1)$$

$$z_2(x,y,t) = \left(A^B_{ref}e^{-ik_y y_2} + A^B_{in}e^{ik_y y_2} + A^B_e e^{-\mu_y y_2}\right)\left(A^L_{ref}e^{-ik_x x_2} + A^L_{in}e^{ik_x x_2} + A^L_e e^{-\mu_x x_2}\right)\Phi(t), \quad (2)$$

where $\Phi(t)$ represents the time dependence, the corner is chosen as the origin of the coordinate system, *B* refers to the bottom edge, *L* to the left edge, *ref* means reflected wave, *in* means incident wave, *e* means evanescent field component, $\mu_y = \sqrt{k_y^2 + 2k_x^2}$ and $\mu_x = \sqrt{k_x^2 + 2k_y^2}$, $k_x = k\cos\theta$ and $k_y = k\sin\theta$ are the wavenumbers of the wave that propagates with θ heading and bending wavenumber k.

Assuming that the vibration field comprises a large number of propagating waves with random phase relations, arriving from uniformly distributed directions we can write that the frequency-averaged correlation coefficient of the displacement (or acceleration) between two points in a vibration field is given by

$$\gamma_{12}(\mathbf{x}_1,\mathbf{x}_2,f_c)=\frac{\int_{k_1}^{k_2}\int_{0}^{\pi/2}\mathrm{Re}\left[\left\langle z_1(t)z_2^*(t)\right\rangle_t\right]d\theta dk}{\left[\int_{k_1}^{k_2}\int_{0}^{\pi/2}\left\langle z_1^2(t)\right\rangle_t d\theta dk\right]^{1/2}\left[\int_{k_1}^{k_2}\int_{0}^{\pi/2}\left\langle z_2^2(t)\right\rangle_t d\theta dk\right]^{1/2}}, \qquad (3)$$

where Re [ ] denotes the real part of a complex quantity, * represents a complex conjugate, $<>_t$ is the average in time, k_1 and k_2 are the bending wavenumbers at the lower and upper limits of the frequency band, and f_c is the centre frequency of the band. Substituting (1) and (2) in (3) and assuming that the amplitude of all incident waves is the same we obtain an expression for the correlation coefficient as a function of the reflection and evanescent field coefficients of the bottom and left boundaries, wavenumbers in x and y directions, and coordinates of points 1 and 2.

In order to compute the frequency-averaged value of the correlation coefficient the reflection and evanescent field coefficients need to be determined. For some boundary conditions these coefficients can be found in refs. [2,4] while a general expression for a boundary with an arbitrary normal or rotational stiffness are derived in refs. [1,2]. The integral in wave direction (θ) is evaluated using numerical integration while the integral in wavenumber space limited by k_1 and k_2 is estimated using an approximation suggested by Cook et al [3]. Numerical integration was employed to compute the correlation coefficient in most situations, however for the case of simply-supported edges a closed form solution was derived (eq. 6, ref. [7]).

For the case in which the left and bottom edges are clamped the reflection and evanescent field coefficients are

$$R_B=\frac{A_{ref}^B}{A_{in}^B}=-\frac{\mu_y+ik_y}{\mu_y-ik_y}, \qquad E_B=\frac{A_e^B}{A_{in}^B}=\frac{i2k_y}{\mu_y-ik_y},$$

$$R_L=\frac{A_{ref}^L}{A_{in}^L}=-\frac{\mu_x+ik_x}{\mu_x-ik_x}; \qquad \text{and} \qquad E_L=\frac{A_e^L}{A_{in}^L}=\frac{i2k_x}{\mu_x-ik_x}. \qquad (4)$$

where R denotes the reflection coefficient and E the evanescent field component coefficient.

Waves that propagate in singly- or doubly-curved shells have wavespeed that depend on the frequency and wave direction. Well above the ring frequencies of such shells the wavenumber approach that of an equivalent flat plate. The dispersion relation for a doubly-curved shell is given by [5]

$$k^4 = \frac{m(2\pi f)^2}{D} - \frac{Eh}{Dr_x^2}\left(X\cos^2\theta + \sin^2\theta\right)^2, \quad (5)$$

where $X=r_x/r_y$, h is the shell thickness, E is the Young's modulus, and r_x and r_y are the principal radii of curvature of the surface. The above dispersion relation can be substituted in equation (3) for the computation of correlation coefficients on doubly-curved shells.

3. EXPERIMENTAL MEASUREMENTS

The correlation coefficients of acceleration on randomly excited plates were measured using the procedure described in ref. [6]. Each plate was mechanically excited by means of a broad-band point random force and the auto and cross-spectrum between each pair of points acquired with lightweight accelerometers. In each set of measurements, one accelerometer was held at one extreme of the line (marked 1 in fig. 1) and the other displaced from it along the line at equally spaced intervals. The frequency range of interest was 0-5000 Hz and the results were integrated in 1/3 octave bands. In each band, the correlation coefficient of a pair of points was obtained by normalising the cross-spectrum by the product of the square of the auto-spectra.

4. EXPERIMENTAL RESULTS AND DISCUSSION

Clamped Plate with an attached steel bar

One of the plates employed in the experimental investigation was made of 3.16 mm thick steel with dimensions of 0.876m x 0.576 m. The plate was clamped along the edges by a wooden frame. A hollow steel bar having a cross sectional area of (25x25) mm^2 and thickness of 3.16 mm was attached to the plate by means of five light screws permitting easily removal of the bar. The plate was point excited by an electrodynamic shaker and no damping treatment was applied over the plate. The location of the excitation point and measurement lines used are shown in fig. 1.

As shown in fig. 2, theoretical and experimental correlation coefficient results are similar along a line close to one of the plate's corners (line 6). This agreement was observed even at 315 Hz 1/3 octave band when only 3 vibration modes are expected to be excited and were rather unchanged by the removal of the stiffener. The reflection and evanescent field component coefficients presented in eq. (4) are used in the computation of the theoretical correlation coefficient presented in fig. 2.

On the other hand, for points more than one wavelength far from the edges, the theoretical results only approach the experimental ones in frequency bands in which more than eight modes resonate. This situation is illustrated by the results presented in fig. 3 for lines 1 and 2 measured with the steel bar attached to the plate. The theoretical results presented in fig. 3 use the reflection, transmission and evanescent field component coefficients for a general spring attachment based on the equivalent translational and rotational stiffness of the steel bar [1,2]. Due to the fact that the excitation point is placed on the same side of the plate as line 1, reflection and evanescent field coefficients are used for line 1 and transmission and evanescent field coefficients are used for line 2. The correlation coefficients measured on lines 1 and 2 change considerably with the presence of the steel bar and when the steel bar is not attached to the plate they approach that of a diffuse bending wave field [6] above the 1000 Hz 1/3 octave band. The present theoretical approach predicts this variation reasonably well.

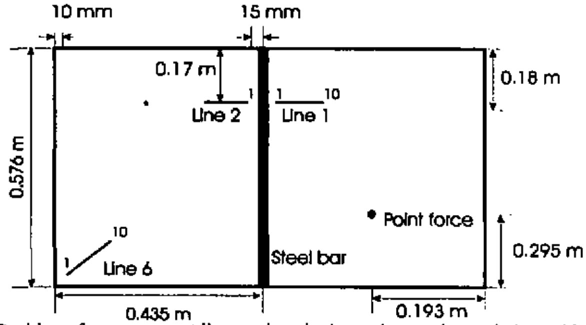

Fig. 1 - Position of measurement lines and excitation point on clamped plate with stiffener.

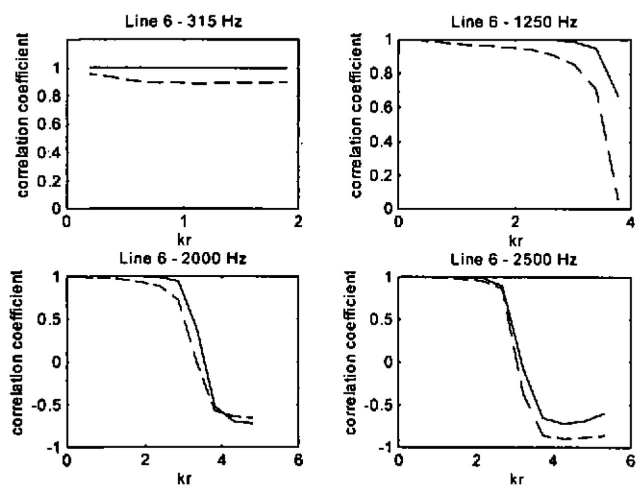

Fig. 2 - Frequency-averaged correlation coefficients along line 6 on plate without stiffener. 1/3 Octave bands. ——— theory (clamped edges); — — — experimental results.

Passenger Car

In ref. [6] measurements of vibration field correlation carried out at different parts of a car bodyshell are reported. The measurement lines used for the roof are shown in fig. 4 and the experimental results obtained are plotted as a function of accelerometers spacing in fig. 5. It is clear from fig. 5 that the correlation coefficient varies with the wave direction. This variation is caused by the curvature of the roof. The dispersion relation presented in eq. (5) was used to predict the variation of the roof bending wavenumber with wave direction. The theoretical results presented in fig. 5 model the roof as a clamped doubly-curved shell with ring frequencies around 150 Hz and 300 Hz. As shown, the variation of the correlation coefficient with the line orientation is reasonably well predicted and the agreement between experimental and theoretical results is also acceptable considering that the roof has a variable curvature.

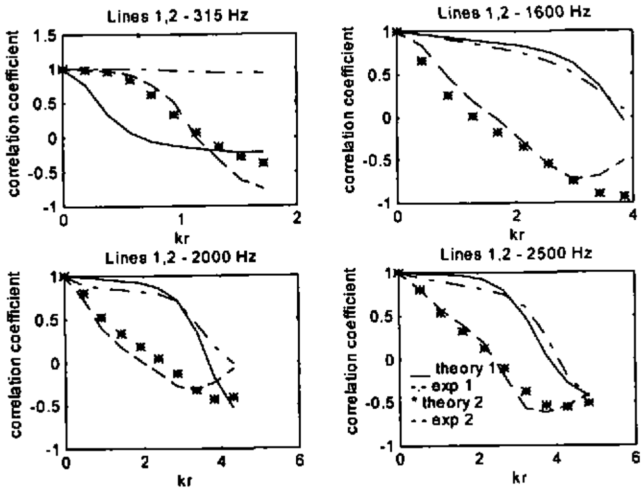

Fig. 3 - Frequency-averaged correlation coefficients along lines 1 and 2 for plate with stiffener. 1/3 Octave bands. Line 1: ——— theory; – · – · – exper. Line 2: * * theory; — — exper.

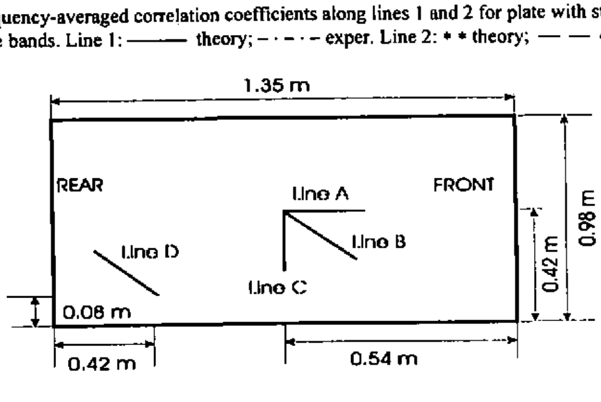

fig. 4 - Sketch of roof with measurement lines

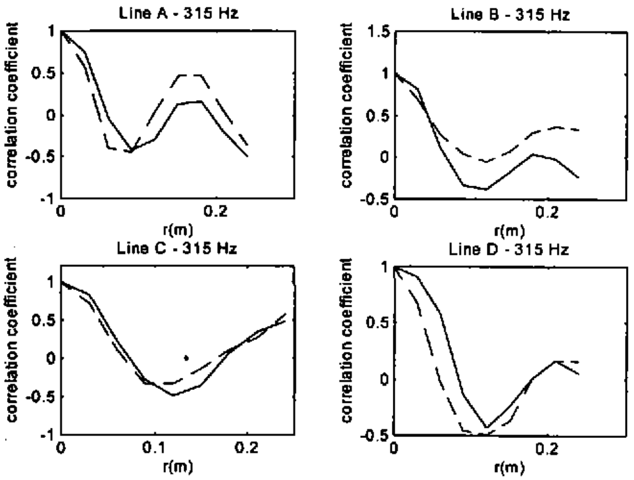

Fig. 5 - Frequency-averaged correlation coefficients on passenger car roof. 1/3 Octave bands. ——— theory (clamped edges); — — — experimental results.

5. CONCLUSIONS

An approximate description of a modally-dense bending vibration field is here presented. This model can be used as one component of the hybrid probabilistic-deterministic model described in refs. [2,7] to predict the sound pressure levels generated by a flat plate that radiates inside an enclosed cavity or to estimate the frequency-averaged external radiation efficiency of flat plates [2]. Corrections are here suggested to model different boundary conditions, stiffeners and single or double curvature.

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