

# TIME-DOMAIN DOPPLER CORRECTION ALGORITHM FOR ORGANIC NOISE RANGING.

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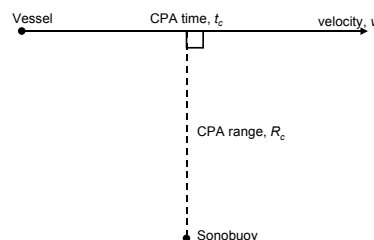
## 1. INTRODUCTION

The submarine's principal sensor for the detection and localisation of other vessels is its passive sonar. There are also a number of acoustic mines that employ passive acoustic techniques as their trigger mechanism. Thus most navies try to ensure that their vessels, especially those with an anti-submarine warfare or mine-hunting role, are as quiet as possible and do not exhibit any major vulnerabilities in their underwater acoustic signature. This includes measuring the strength (or sound pressure levels, SPLs) of the signature.

Most checks on the passive acoustic signatures of vessels are made on fixed-site noise ranges such as the Rona range in Scotland and the Atlantic Underwater Test and Evaluation Center (AUTEC) range in the Bahamas. These facilities are situated in relatively shallow waters, are sheltered from open ocean by land and are away from major shipping lanes. In consequence, the ambient noise on the ranges is generally quieter than that normally to be found in operating areas.

While the quiet conditions of such ranges are a major benefit, there are a number of major disadvantages with fixed ranges. They require a large capital investment unlikely to be affordable to emerging nations with small navies. Moreover even for more advanced nations with well-established navies, the cost and time of travel to these facilities is inconvenient especially if the navy has a global operation and re-measurement is required as the result of the emergence of a minor machinery fault.

For these reasons, there has been much interest in the last twenty years in organic sonobuoy ranging (OSR). In OSR the vessel itself deploys a calibrated, free-drifting sonobuoy, which transmits the acoustic signals back to the vessel, either by radio for surface vessels or by acoustic means for submarines. After deploying the buoy the vessel opens to some distance and then returns on a reciprocal straight leg course, analysing its own signature as it passes (see Figure 1).



**Figure 1 – OSR Plan View**

The advantages of such a system is that it does not incur high maintenance costs, for surface vessels it is relatively cheap to procure as an adaptation of sonobuoy technology, and it can be

deployed by the vessel whenever and wherever it is required. However, there are a number of complications with regard to organic noise ranging over that undertaken on a fixed range: tracking can be more difficult; one has potentially greater variability in propagation conditions; and the noise levels are generally greater in open ocean than on a range.

This paper is particularly concerned with eliminating the effects of Doppler smear on machinery tonals allowing measurements to be made at shorter ranges and higher speeds, overcoming some of the problems associated with the higher ambient noise levels. A method has been developed to correct in the time-domain for these effects. By the application of this algorithm the received signal is corrected for the differential motion between the vessel and sonobuoy back to the source signal (correction of propagation effects is applied separately). By this means, finer resolutions and longer integration times can be applied in spectral processing in order to make more accurate measurements.

## 2. DOPPLER-SMEAR

Unfortunately, measurement of tonal levels is very difficult at close ranges. The relative speed of the vessel to the sonobuoy causes the apparent frequency of the tonal to be shifted up or down according to whether the vessel is approaching or receding. Assuming that the speed of the vessel is much lower than the speed of sound, the apparent Doppler-shifted frequency of a stable tone as seen by the sonobuoy will be:

$$f(t) = f_0 \cdot \left( 1 - \frac{\frac{v^2}{c} \cdot (t - t_c)}{\sqrt{R_c^2 + v^2 \cdot (t - t_c)^2}} \right) \quad (1)$$

where  $f_0$  is the source frequency [Hz] of the tonal with the vessel at rest.

$v$  is the speed [m/s] of the vessel.

$c$  is the speed of sound [m/s] in water.

$t$  is time [s].

$t_c$  is the time [s] of the closest point of approach of the vessel to the sonobuoy.

$R_c$  is the range [m] of the closest point of approach of the vessel to the sonobuoy.

Unfortunately, near the closest point of approach of the vessel to the sonobuoy, where the signal-to-noise ratio of the signal is greatest, the apparent frequency will shift from high to low and the rate of change will be highest at the closest point of approach. In consequence the energy of any tonal will be smeared across a range of frequencies. In applying standard spectral techniques in order to isolate and improve the measurement signal-to-noise of the tone, one is thus limited in the frequency resolution that can be applied and still capture all the energy in the tonal.

From simple differentiation, the slope of the Doppler frequency at the closest point of approach is found to be:

$$s = \frac{f_0 \cdot v^2}{c \cdot R_c} \quad (2)$$

It can be seen that the slope, and thus the Doppler smear, increases the smaller the CPA range so that decreasing the range to improve signal-to-noise directly exasperates the Doppler smear problem. It can also be seen that the slope also increases with velocity. Thus, making

measurements at higher speeds, where the propulsion-related tones are likely to be louder is also problematical.

### 3. TIME-DELAY CORRECTION

The time-domain Doppler-correction algorithm is derived below assuming a straight line, steady course for the vessel past the sonobuoy. It assumes that the CPA range, CPA time and velocity of this pass have been derived by other means.

Assume an acoustic signal  $s'(\tau)$  is radiated from the vessel at time  $\tau$ . This signal  $s(t)$  is received by the sonobuoy at a time  $t$  given by:

$$t = \tau + \frac{R(\tau)}{c} \quad (3)$$

where  $R(\tau)$  is the range [m] of the vessel from the sonobuoy at time  $\tau$ .  
 $c$  is the speed of sound [m/s].

The separation distance at a time  $\tau$  is given by:

$$R(\tau) = \sqrt{R_c^2 + v^2(\tau - \tau_c)^2} \quad (4)$$

where  $R_c$  is the minimum range [m] of the vessel from the sonobuoy.  
 $v$  is the speed [m/s] of the vessel.  
 $\tau_c$  is the time [s] of the closest point of approach of the vessel to the sonobuoy.

Substitution of (4) into (3) gives:

$$t = \tau + \frac{1}{c} \sqrt{R_c^2 + v^2(\tau - \tau_c)^2} \quad (5)$$

Noting that we usually have the observed time of the closest point of approach rather than actual time:

$$\tau_c = t_c - \frac{R_c}{c} \quad (6)$$

Substitution of (6) into (5) gives:

$$t = \tau + \frac{1}{c} \sqrt{R_c^2 + v^2 \left( \tau - t_c + \frac{R_c}{c} \right)^2} \quad (7)$$

### 4. MODIFICATION FOR DISCRETE-TIME SIGNALS

Since the correction is to be undertaken computationally the received signal is not provided as a continuous function of time but as a discrete time-series:

$$s(n) \equiv s(t) \quad ; t \equiv t_0 + n \Delta t \quad (8)$$

where  $t_0$  is the time associated with the beginning of the received digital time-series  
 $n$  is the index of the received digital time series

$\Delta t$  is the sample period

$$\Delta t \equiv \frac{1}{f_s}$$

$f_s$  is the sample rate of the data

The object is to derive the equivalent discrete time-series as emitted by the source neglecting any correction for propagation loss:

$$s'(k) \equiv s'(\tau) \quad ; \tau \equiv \tau_0 + k.\Delta t \quad (9)$$

where  $\tau_0$  is the time associated with the beginning of the source digital time-series  
 $k$  is the index of the source digital time series

Substitution of (8) and (9) into (7) gives:

$$n = \left( \frac{\tau_0 - t_0}{\Delta t} \right) + k + \frac{\sqrt{R_c^2 + v^2 \left( \tau_0 + k.\Delta t - t_c + \frac{R_c}{c} \right)^2}}{c\Delta t} \quad (10)$$

Without loss of generality, it can be assumed that the re-sampled source time-series starts at the same time as the received time-series (i.e.  $t_0 = \tau_0$ ) so (10) can be reduced to:

$$n = k + \frac{\sqrt{R_c^2 + v^2 \left( t_0 + k.\Delta t - t_c + \frac{R_c}{c} \right)^2}}{c\Delta t} \quad (11)$$

The idea is then to create the equivalent source signal by taking for every sample  $s'(k)$  the value of the received signal  $s(n)$  where  $n$  is derived from the index  $k$  and the characteristics of the vessel pass using the above equation.

## 5. DISCRETE-TIME SIGNAL INTERPOLATION

In general the value  $n$  will be non-integer. One approach in re-sampling might simply be to truncate to the nearest integer  $n'$  and neglect the fractionally remainder,  $\Delta n$ :

$$n' \equiv \lfloor n \rfloor$$

$$\Delta n \equiv n - \lfloor n \rfloor \quad (12)$$

where  $\lfloor \cdot \rfloor$  denotes the floor operation i.e. biggest integer value less than the real value.

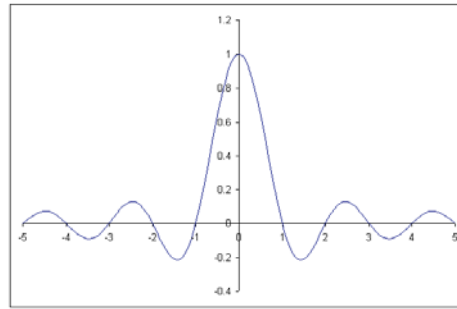
Unfortunately this produces spurious results both in terms of levels and extraneous spectral features. It is therefore necessary to take the fractional remainder into account. The problem is then to compute the level of the signal at an arbitrary time from the set of samples at discrete times. Since the original signal is bandlimited to the Nyquist frequency, Shannon's sampling theory gives that a signal  $x(t)$  at an arbitrary time  $t$  can be uniquely reconstructed from the samples  $x(m)$  via:

$$x(t) \equiv \sum_{m=-\infty}^{\infty} x(m) \cdot h_s(t - m \cdot \Delta t) \quad (13)$$

where  $h_s()$  is the sinc function

$$h_s(t) \equiv \frac{\sin(\pi t / \Delta t)}{\pi t / \Delta t}$$

The form of the sinc function is shown in Figure 2. It is noted that the peak amplitude of the function is one for  $t=0$  and that zero-crossings occur at all non-zero multipliers of the sampling period  $\Delta t$ . An interpretation of this formulation is that the sinc function is shifted so its peak is at the time  $t$ , and the signal magnitude at this time is then the summation of the original samples weighted by the value of the sinc function at the location of each sample.

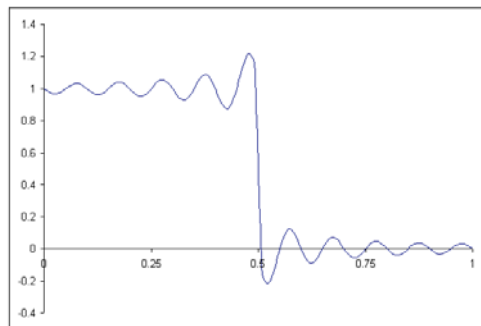


**Figure 2 – The Sinc Function**

Direct application of this formulation is not practical as it includes a summation over all samples. However it can be seen from Figure 2 the sinc function weights exponentially decay so a reasonable approximation is to truncate the summation in (13) at some point where the contribution of the remaining samples is considered not to be significant:

$$x(t) \cong \sum_{m=-M}^M x(m) \cdot h_s(t - m \cdot \Delta t) \quad (14)$$

From FIR filter design, it is known that this simple truncation results in a ripple in the response of the summation known as Gibb's phenomena (i.e. some frequencies are accentuated, while others are reduced) as seen in Figure 3.



**Figure 3 – Example of Gibb's Phenomena**

## 6. SINC FUNCTION WINDOWING

In this application the ripple in the stopband is of no concern as the original data is bandlimited and therefore there is no signal in this region to be aliased down. However the passband ripple is important as it will affect the measurement results at these frequencies. One approach to reducing the effects of the ripple would be to increase the number of coefficients retained in the expansion. However, this does not fully eliminate the ripple but merely concentrates its effect at the higher frequencies.

Gibb's phenomena occurs because of the discontinuity at  $m=M$ . The approach in spectral windowing is to apply a truncation function that has smoother characteristics at the extremes:

$$x(t) \cong \sum_{m=-M}^M x(m) \cdot h'_s(t - m \cdot \Delta t) \quad (15)$$

$$\text{where } h'_s(t) \equiv h_w(t) \cdot h_s(t)$$

There are many window functions that have been developed over the years for this purpose. In the course of this development it has become apparent that, for a fixed length filter, there is a trade-off between the ripple and the transition band roll-off. A window function that has been found to be of great use in these types of applications is the Kaiser-Bessel window function as it incorporates this trade-off as a parameter:

$$h_w(t) = \frac{I_0\left(\beta \cdot \sqrt{1 - \left(\frac{t}{\Delta t}\right)^2}\right)}{I_0(\beta)} \quad (16)$$

where  $I_0()$  is the zeroth-order modified Bessel function of the first kind.

There are many standard approximations for the modified Bessel function (e.g. [1]) enabling the window function to be derived.

Kaiser [2] has found an empirical estimate of  $\beta$  for a given ripple size to be :

$$\beta = \begin{cases} 0.1102 \cdot (A - 87) & A > 50 \\ 0.5842 \cdot (A - 21)^{0.4} + 0.07886 \cdot (A - 21) & 21 \leq A \leq 50 \\ 0.0 & A < 21 \end{cases} \quad (17)$$

where  $A \equiv -20 \cdot \log_{10}[\min(\delta_p, \delta_s)]$

$\delta_s$  is the ripple in fractional terms in the stopband

$\delta_p$  is the ripple in fractional terms in the passband

Noting that we are only interested in the passband ripple and imposing a requirement for an accuracy of 0.1 dB across the measurement band, this gives a value for the Kaiser-Bessel window function parameter  $\beta$  of 3.24. In addition, Kaiser has developed empirical estimates of the number of coefficients to be retained,  $N=2M+1$ :

$$N = \begin{cases} \frac{A - 7.95}{2.285 \cdot \Delta\Omega} & A \geq 21 \\ \frac{5.794}{\Delta\Omega} & A < 21 \end{cases} \quad (18)$$

$$\text{where } \Delta\Omega \equiv \frac{|f_{stop} - f_{pass}|}{f_s}$$

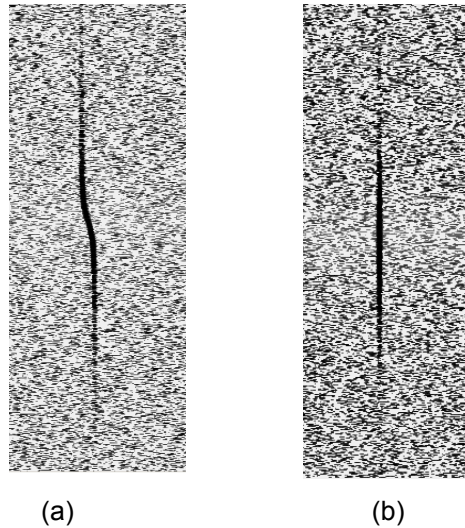
$f_{stop}$  is the frequency of the stopband

$f_{pass}$  is the frequency of the passband

For a transition band of approximately 5% of the bandwidth this gives that around 135 coefficients (i.e.  $M=67$ ) are needed to be retained.

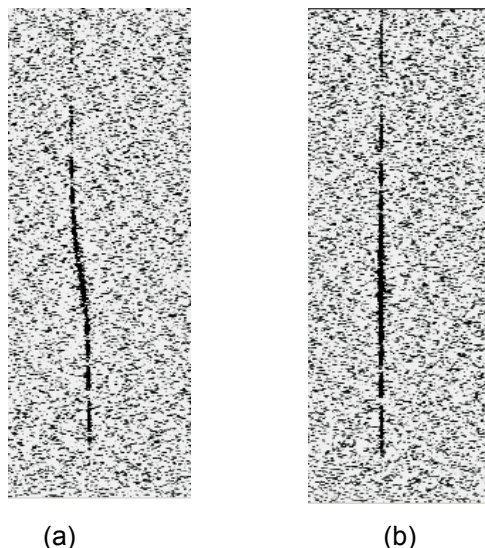
## 7. EXAMPLES

The time-domain Doppler correction has been applied to both synthetic data and to at-sea collected data.



**Figure 4 – Synthetic Data Example**

The synthetic signal consisted of simple sinusoids whose frequencies were modulated for Doppler effects corresponding to a contact moving at 10 m/s, at a closet point of approach of 500m to the a sonobuoy. Source levels were assumed static at 135dB ref. 1 $\mu$ Pa @1m and received levels amplitude modulated as a function of time to represent spherical spreading from the contact. Simple white noise was added to represent the ambient noise continuum at a level of 60 dB ref 1 $\mu$ Pa. Figure 4 shows the lofargrams of a synthetic 537 Hz tone before (a) and after (b) Doppler correction has been applied.



**Figure 5 – Real Data Example**

The at-sea data was collected from a sound source, generating a series of tones, being towed past a moored sonobuoy. Acoustic analysis indicated that it passed the sonobuoy at a speed of 6.7 m/s at a closest point of approach of 492 m. Figure 5 show the lofargrams of a 896 Hz tone from the source before (a) and after Doppler correction (b).

## 8. SUMMARY

An algorithm has been developed to provide correction for the Doppler shift in the underwater acoustic signature of a moving vessel in the time-domain. This technique should allow accurate measurements of the signature to be made of a vessel by organic sonobuoy ranging even at small closest point of approach and fast speeds. The effectiveness of the algorithm against both simulated data and real data has been shown.

Recently an algorithm has come to our attention that employs tabulation and linear interpolation to speed calculation of the windowed sinc function [3]. While it hasn't yet proved possible to incorporate this algorithm, it has the potential to speed up the calculations by an order of magnitude.

## REFERENCES

- [1] Press W.H. et al., "Numerical Recipes in C – The Art Of Scientific Computing", Second Edition, Cambridge University Press, 1992.
- [2] Haykin S., "Modern Filters", Macmillan Coll. Div., 1989.
- [3] Smith J.D., Gossett P., "A Flexible Sampling Rate Conversion Method", Proceedings of the International Conference on Acoustics, Speech, and Signal Processing, San Diego, Vol. 2, pp. 19.4.1 – 19.4.2, New York, IEEE Press. 1984.