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A SIMPLE ELECTRO-MECHANICAL MODEL OF THE DISTRIBUTED-MODE LOUDSPEAKER (DML)

Neil Harris

New Transducers Ltd (NXT), Huntingdon, UK

1. INTRODUCTION

New understanding of the relationships between the mechanical and acoustical properties of vibrating panels has resulted in the birth of a new class of acoustic radiator - the distributed-mode loudspeaker. NXT[1] have developed optimisation techniques, whereby stiff, light panels can be designed to have very uniform modal density, the pre-requisite for distributed-mode behaviour.

A DML is identified by the fact that its radiation is due to uniformly distributed free vibration in a stiff, light panel, and not to pistonic motion. A good approximation is to consider the panel as a randomly vibrating area, and use statistical quantities. The radiation intensity from such an area is shown in Morse & Ingard to depend on the square of the mean velocity, and hence constant velocity is required for a flat frequency response. In order to achieve this constant velocity with a constant force, the mechanical impedance must be resistive. A panel operating in bending waves meets this criterion. A mechanical model of the operation of the DML is presented, from which an equivalent circuit is developed. This mechanical circuit can be refined to include the effects of the mass, aperture of a finite sized coil, and the mass and compliance of the motor system.

2. OVERVIEW OF A DISTRIBUTED MODE LOUDSPEAKER (DML)

The radiation from a DML is due to uniformly distributed bending-waves in a stiff, light panel. Because bending waves are dispersive (the wave velocity is a function of frequency) [2], a good approximation is to consider the panel as a randomly vibrating area. The radiation intensity from such an area is shown in Morse & Ingard [3] to depend on the square of the mean velocity, and hence the requirement is for constant velocity. In order to achieve this constant velocity with a constant force, the mechanical impedance must be resistive. A panel operating in bending waves meets this criterion [4]. Expressions for bending wave velocity, wave-number and mechanical impedance are quoted below (see appendix for definition of symbols).

$$cp(\omega)=\sqrt{\omega\sqrt{B/\mu}} \qquad kp(\omega)=\sqrt{\omega\sqrt{\mu/B}} \qquad Zm=8\sqrt{B\mu} \tag{1}$$

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In order to develop a mathematical model of any physical system, it is necessary to make some assumptions. Because we are considering the DML to be randomly vibrating, the existing motion of the panel will be uncorrelated to any new input being applied, and therefore it looks like an infinite plate. Additionally, because the panel has low mechanical loss, we can suppose that all the energy supplied to the panel will eventually be dissipated by acoustic radiation. This set of assumptions has been shown to give useful results, and is correct in as much as the radiated pressure is proportional to the mean velocity in the panel. Thus, to calculate the radiated acoustic power, we need only to calculate the mechanical power delivered to the panel.

3. EQUIVALENT CIRCUIT OF A DISTRIBUTED MODE LOUDSPEAKER (DML)

Given that the DML is a resistance controlled device, and that we do not need to consider the acoustic radiation in detail, we can develop an equivalent circuit from Figure 1. This represents a simplified version of the "inertial magnet driver" application used by New Transducers Limited in their white paper [5].

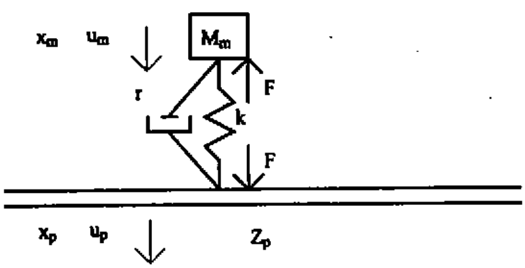

Figure 1. Mechanical elements and forces for a panel driven by a damped mass-spring oscillator.

The coupled equations of motion are given in (2) and (3) below.

$$M_m \frac{d^2 x_m}{dt^2} + r\left(\frac{dx_m}{dt} - \frac{dx_p}{dt}\right) + k\left(x_m - x_p\right) - F = 0 \qquad (2)$$

$$Z_p \frac{dx_m}{dt} + r\left(\frac{dx_p}{dt} - \frac{dx_m}{dt}\right) + k\left(x_p - x_m\right) + F = 0 \qquad (3)$$

If the force is assumed to be sinusoidal with angular frequency, ω, and using the same symbols to refer to the peak values of variables, then

$F(t) = Fe^{j\omega t}$ (and similarly for xm and xp)

$$-\omega^2 M_m x_m + j\omega r\left(x_m - x_p\right) + k\left(x_m - x_p\right) + F = 0 \tag{4}$$

$$j\omega Z_m x_p - j\omega r\left(x_m - x_p\right) - k\left(x_m - x_p\right) - F = 0 \tag{5}$$

or in matrix form, separating the stiffness, mass and resistance matrices

$$\left(\underline{\underline{K}} - \omega^2 \underline{\underline{M}} + j\omega \underline{\underline{R}}\right)\underline{x} - \underline{F} = 0 \quad \text{or} \quad \underline{x} = \left(\underline{\underline{K}} - \omega^2 \underline{\underline{M}} + j\omega \underline{\underline{R}}\right)^{-1} \underline{F} \tag{6}$$

where

$$\underline{\underline{M}} = \begin{pmatrix} M_m & 0 \\ 0 & 0 \end{pmatrix} \qquad \underline{\underline{K}} = \begin{pmatrix} k & -k \\ -k & k \end{pmatrix} \qquad \underline{\underline{R}} = \begin{pmatrix} r & -r \\ -r & r \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & Z_p \end{pmatrix} \tag{7a}$$

$$\underline{x} = \begin{pmatrix} x_m \\ x_p \end{pmatrix} \qquad \underline{F} = F\begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{7b}$$

So, the velocity in the panel is given by

$$Y_p = \frac{u_p}{F} = \frac{j\omega x_p}{F} \tag{8}$$

$$Y_p = j\omega\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} k - \omega^2 M_m + j\omega r & -k - j\omega r \\ -k - j\omega r & k + j\omega r + j\omega Z_p \end{pmatrix}^{-1}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{9}$$

$$Y_p = \frac{\omega^2 M_m}{\left(\omega^2 M_m\left(Z_p + r\right) - Z_p k\right) - j\omega\left(k M_m + r Z_p\right)} \tag{10}$$

By inspection, noting the velocity in the spring and damper is the difference between the velocities in the mass and panel, the equivalent circuit using the impedance analogue can be drawn as per Figure 2. It is a relatively straight forward task to verify that the ratio of panel velocity, u_p, to force, F, matches that given by the reciprocal of equation (10), i.e.

$$Zm_{eff} = Z_p\left(1 - \frac{k}{\omega^2 M_m}\right) + r - \frac{j}{\omega}\left(k + \frac{r Z_p}{M_m}\right) \tag{11}$$

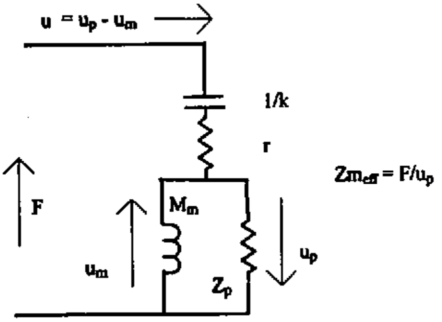

Figure 2. Impedance analogue model of DML panel.

4. PRACTICAL IMPLEMENTATION OF THE EQUIVALENT CIRCUIT FOR A MOVING COIL MOTOR

Figures 1 and 2 represent a DML panel driven by an idealised point source. If we consider the motor system to be moving coil, then M_m represents the mass of the magnet, cup and pole piece. The spring/damper represent a means of attachment of the motor to the panel. To account for the effect of the coil, we must add in series with Z_p a mechanical mass, M_c, equal to the mass of the voice coil. Additionally, the impedance Z_p is only real for a point source, and will generally be a complex quantity for a finite diameter voice coil. The reactive component, X_p, is small except at high frequencies.

Figure 3 shows a model of such a system, along with its equivalent circuit. For a complete electro-mechanical model, a gyrator and coil impedance should be added. There are computer programs on the market which facilitate such modelling, for example AkAbak [6].

The effective mechanical impedance relating up to F for Figure 3 is

$$Zm_{eff} = Z'_p\left(1-\frac{k}{\omega^2 M_m}\right)+r-\frac{j}{\omega}\left(k+\frac{rZ'_p}{M_m}\right) \tag{12}$$

where $Z'_p = R_p + jX_p + j\omega M_c$

At high frequencies, equation (12) simplifies considerably, becoming Z'p + r, i.e.

$$Zm_{eff} \approx \left(R_p + r\right) + j\left(X_p + \omega M_c\right) \approx R_p + j\omega M_c \tag{13}$$

which gives the high frequency limit for the DML as

$$f_{max} \approx \frac{R_p}{2\pi M_c} \tag{14}$$

A similar simplification gives the low frequency limit which, if we ignore k, is given by

$$Y_p \approx \frac{1}{R_p} + \frac{1}{j\omega M_m} \quad \text{so} \quad f_{min} \approx \frac{R_p}{2\pi M_m} \tag{15}$$

Alternatively, if we ignore Mm for a moment, it is evident that

$$f_{min} \approx \frac{k}{2\pi R_p} \tag{16}$$

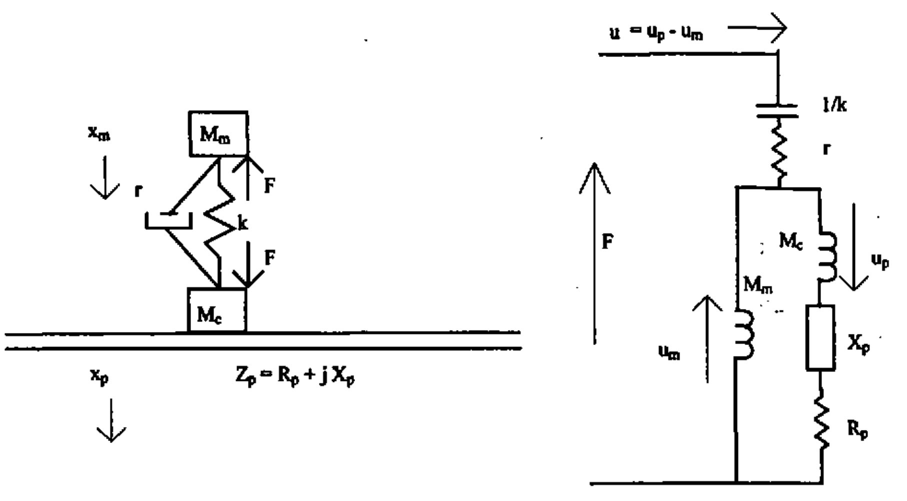

Figure 3. Impedance analogue model of DML panel with moving coil motor system.

5. CONCLUSIONS

An electro-mechanical model has been presented which will enable acoustic engineers to use existing software programs to investigate the application of DML technology to their acoustic problems. Given that a stiff, light panel can be designed to have optimal modal distribution and low loss, it has been shown that in order to model the acoustic pressure or acoustic power, it is sufficient to calculate the mean velocity in the panel.

The bandwidth of the DML is seen from equations (14) to (16) to depend only on the ratio of magnet mass, coil mass, and suspension stiffness to the panel mechanical impedance. The panel properties affect the sensitivity and frequency limits only via the mechanical impedance. It is possible to design a *single* DML to be substantially flat in pressure *and* power response over a very wide bandwidth without any electrical filters, something which is impossible to achieve with conventional loudspeaker technology.

6. APPENDIX - Glossary of symbols

Panel parameters

E = Young's modulus, Pa
ρ = mass density, kg/m^3
ν = Poisson's ratio
h = thickness of panel, m
B = bending rigidity of panel, Nm
μ = mass per unit area of panel, kg/m^2
$Z_p = R_p + j.X_p$ = mechanical impedance of panel, kg/s

Note that for an isotropic panel $B = \frac{1}{12}\frac{E}{(1-\nu^2)}h^3$ and $\mu = h\rho$

Acoustic parameters

ω = angular frequency, rad/s
c = speed of sound in air, m/s
k = acoustic wave number = ω/c, m^{-1}
cp = speed of bending-wave in panel (in-plane), m/s
kp = structural wave number = ω/c, m^{-1}

Mechanical parameters

x, x_p, x_m = out-of-plane displacements, m
u, u_p, u_m = out-of-plane velocities, m/s
F = force, N

General

bar over variable = rms value, e.g. $\bar{u}$
single underline = vector, e.g. $\underline{x}$
double underline = matrix, e.g. $\underline{\underline{M}}$

7. REFERENCES

[1] NXT is a registered trademark of New Transducers Ltd., a subsidiary of Verity Group PLC.
[2] P. M. Morse, 'Vibration and Sound', pp 115-116. McGraw Hill
[3] Morse and Ingard, 'Theoretical Acoustics', Section 7.4. McGraw Hill
[4] Morse and Ingard, 'Theoretical Acoustics', Section 5.3.19. McGraw Hill
[5] 'NXT White Paper', (C) New Transducers Ltd., 1996
[6] AkAbak 2.0, Panzer & Partner; Steinstrasse 15, D-81667, Munich, Germany.