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# Non-contact transportation using ultrasonic wave levitation

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#### **ABSTRACT**

This paper deals with a non-contact transportation system using an ultrasound wave levitation phenomenon. By driving a flexible beam at an ultrasonic frequency, the ultrasound wave is generated, which has a potential to produce a power to levitate and transport a target object. This paper begins by clarifying the levitation mechanism the ultrasound wave provides. Active wave control is then introduced for oscillating the beam, thereby giving rise to the traveling wave, the amplitude and the phase of which may then be uniquely resolved. The relevance between these variables under control and the transportation mechanism is investigated. Because of a strong nonlinearity in the ultrasound field, the pressure distribution and the flow velocity need to be numerically analyzed. It is shown that the transportation force is produced by imbalanced pressure amplitudes which slightly tilt the object with the result that the transportation force is generated. It is also shown that the transportation force is under control, hence meticulous handling for a transportation system being viable.

#### 1 INTRODUCTION

Comparing with the conventional levitation system using static electricity, gas ejection or magnetic force, an ultrasonic levitation method has the following benefits: (1) It does not attract dusts; (2) Any kind of material may be levitated; (3) Driving mechanism is simple. If the transferring function is added to the ultrasound levitation system, the levitation and transferring method may be applicable for industry such as semiconductor community.

Basically two kinds of methods are conceivable for providing the transportation mechanism to the levitation system. The first one is to transfer the levitation system *per se* by employing the additional conveyer equipment. This method, however, entails the double mechanism for levitation and transfer, thereby causing complicity and cost, hence unviable. The other method which is the subject of the paper is to make use of the travelling wave generated by a distributed structure. When a flexible beam is excited by ultrasound frequency such that travelling wave is generated, the ultrasound corresponding to the travelling wave is radiated. By controlling the travelling wave using a certain control strategy, the ultrasound pressure levitates as well as transfers a target object.

The travelling wave-based levitation and transferring method has been reported<sup>1-5</sup> in the past, demonstrating experimentally the levitation and transportation of a target object. According to the proposal, a flexible beam at one end is excited to generate ultrasound while the other end of the beam is supported with some passive boundary to absorb the incoming travelling wave. Thus the mechanism based upon passive control may levitate as well as transport a target object, however, it lacks meticulous control for precisely transfer and position the object. Furthermore, the levitation mechanism and the transportation mechanism have yet to be clarified.

Taking into consideration the aforementioned points, this paper places its purpose on presenting an ultrasound levitation and transferring system bolstered by active wave control, and clarifying the levitation mechanism and transportation mechanism. This paper begins by clarifying the levitation mechanism the ultrasound wave provides. Active wave control<sup>6</sup> is then introduced for oscillating the beam, thereby giving rise to the traveling wave, the amplitude and

the phase of which may then be uniquely resolved. The relevance between these variables under control and the transportation mechanism is investigated. Because of a strong nonlinearity in the ultrasound field, the pressure distribution and the flow velocity needs to be numerically analyzed. It is shown that the transportation force is produced by imbalanced pressure amplitudes which slightly tilt the object with the result that the transportation force is generated. It is also shown that the transportation force is under control, hence meticulous handling for a transportation system being viable.

#### 2. WAVE ANALYSIS OF A FLEXIBLE BEAM

Consider a flexible sliding supported beam with active control force acting at both ends. First it needs to deal with the beam without external forces. The equation of motion of a flexible beam is then given by

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0$$
 (1)

where w, E, I,  $\rho$  and A are flexural displacement, Young's modulus, second moment of inertia, density and the cross section area of a beam. Furthermore, the flexural deflection of a beam is written as

$$w(x,t) = w(x)e^{j\omega t}$$
 (2)

Equation 1 is then given by

$$\frac{d^4w(x)}{dx^4} - k^4w(x) = 0$$
 (3)

where

$$k^4 = \frac{\rho A \omega^2}{EI} \tag{4}$$

Then the solution to Eq. 1 yields

$$-w(x) = c_1 e^{-jkx} + c_2 e^{-kx} + c_3 e^{jkx} + c_4 e^{kx}$$
(5)

Differentiating the deflection of the beam with respect to x produces the slope  $\theta_x$ , bending moment  $m_x$  and sheer force  $f_x$ . Integrating these variables into a vector, we have

$$\mathbf{z}(x) = col(-w_x \quad \theta_x \quad \frac{m_x}{EI} \quad \frac{f_x}{EI}) \tag{6}$$

The state vector may further be expressed as

$$\mathbf{z}(x) = \mathbf{B}(x)\mathbf{c} \tag{7}$$

where

$$\mathbf{B}(x) = \begin{pmatrix} e^{-jkx} & e^{-kx} & e^{jkx} & e^{kx} \\ -jke^{-jkx} & -ke^{-kx} & jke^{jkx} & ke^{kx} \\ -k^2e^{-jkx} & k^2e^{-kx} & -k^2e^{jkx} & k^2e^{kx} \\ jk^3e^{-jkx} & -k^3e^{-kx} & -jk^3e^{jkx} & k^3e^{kx} \end{pmatrix}$$
(8)

$$\mathbf{c} = col(c_1 \quad c_2 \quad c_3 \quad c_4) \tag{9}$$

The matrix **B** moreover expands to

$$\mathbf{B}(x) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -jk & -k & jk & k \\ -k^2 & k^2 & -k^2 & k^2 \\ jk^3 & -k^3 & -jk^3 & k^3 \end{pmatrix} \begin{pmatrix} e^{-jkx} & 0 & 0 & 0 \\ 0 & e^{-kx} & 0 & 0 \\ 0 & 0 & e^{jkx} & 0 \\ 0 & 0 & 0 & e^{kx} \end{pmatrix}$$
$$= \mathbf{KD}(x)$$
(10)

For brevity, setting  $\mathbf{z}(0) = \mathbf{z}_0$ ,  $\mathbf{z}(x) = \mathbf{z}_x$  so that

$$\mathbf{z}_0 = \mathbf{K}\mathbf{D}(0)\mathbf{c} = \mathbf{K}\mathbf{c} \tag{11}$$

$$\mathbf{z}_{x} = \mathbf{K}\mathbf{D}(x)\mathbf{c} \tag{12}$$

Using Eq.11, Eq.12 becomes

$$\mathbf{z}_{x} = \mathbf{K}\mathbf{D}(x) \mathbf{K}^{-1}\mathbf{z}_{0} \equiv \mathbf{T}(x)\mathbf{z}_{0}$$
(13)

where the transfer matrix T is defined as

$$\mathbf{T}(x) = \begin{pmatrix} t_1 & t_4 & t_3 & t_2 \\ k^4 t_2 & t_1 & t_4 & t_3 \\ k^4 t_3 & k^4 t_2 & t_1 & t_4 \\ k^4 t_4 & k^4 t_3 & k^4 t_2 & t_1 \end{pmatrix}$$
(14)

and where

$$t_{1} = \left(e^{-jkx} + e^{-kx} + e^{jkx} + e^{kx}\right) / 4$$

$$t_{2} = \left(-je^{-jkx} - e^{-kx} + je^{jkx} + e^{kx}\right) / 4k^{3}$$

$$t_{3} = \left(-e^{-jkx} + e^{-kx} - e^{jkx} + e^{kx}\right) / 4k^{2}$$

$$t_{4} = \left(je^{-jkx} - e^{-kx} - je^{jkx} + e^{kx}\right) / 4k$$
(15)

Once the initial state vector is obtained, the state vector at any position may then be determined. Furthermore, taking into consideration the boundary condition; sliding support, the initial state vector is written as

$$\mathbf{z}_{0} = \begin{pmatrix} -w_{0} \\ 0 \\ m_{0} / EI \\ f_{1} / EI \end{pmatrix} \tag{16}$$

where  $f_1$  is the control force acting at the left end of the beam. Next, it needs to obtain the initial displacement  $w_0$  and bending moment  $m_0 \neq EI$ . From Eq.13, the state vector at I is given by

$$\mathbf{z}_{l} = \mathbf{T}(l)\mathbf{z}_{0} \tag{17}$$

Therefore, the initial state vector  $\mathbf{z}_0$  is obtained as

$$\mathbf{z}_{0} = \begin{pmatrix} \frac{-f_{1}k^{4}t_{2}t_{3} + f_{1}t_{1}t_{4} - f_{c}t_{4}}{EIk^{4}(k^{4}t_{2}^{2} - t_{4}^{2})} \\ 0 \\ \frac{-f_{1}t_{1}t_{2} + f_{1}t_{3}t_{4} + f_{c}t_{2}}{EI(k^{4}t_{2}^{2} - t_{4}^{2})} \\ f_{1} / EI \end{pmatrix}$$

$$(18)$$

where  $f_c$  is the control force acting at the right end of the beam.

Using the wave number matrix K, the state vector may be transformed to the wave vector

$$\mathbf{z}(x) = \mathbf{K}\mathbf{w}(x) \tag{19}$$

where

$$\mathbf{z}(x) = \mathbf{K}\mathbf{w}(x) \tag{20}$$

The wave vector is written as

$$\mathbf{w}(l) = \mathbf{K}^{-1}\mathbf{T}(l)\mathbf{K} \quad \mathbf{w}(0)$$

$$= \mathbf{D}(l)\mathbf{w}(0)$$
(21)

where  $w_1$  and  $w_3$  are forward wave and backward wave, while  $w_2$  and  $w_4$  are near field waves at x = 0, x = I, respectively.

Define the control forces  $f_1$  and  $f_c$  as

$$\begin{cases} f_1 = Ae^{j\omega t} \\ f_c = f_1 e^{j\varphi} \end{cases}$$
 (22)

In the numerical analysis, the excitation frequency and the amplitude A are set to 20 kHz and 60 N, respectively while the phase difference  $\phi$  is varied.

Figures 1 and 2 show the envelope of the beam deflection driven under the condition,  $\phi$  = 0° and  $\phi$  = 90°, respectively. Apparently, as a result of changing the phase difference  $\phi$ , the waveform of the progressive wave varies. At  $\phi$  = 0°. The standing wave is dominant over the whole region of the beam. When the phase difference is assigned on the control force, the progressive wave emerges and finally becomes dominant at  $\phi$  = 105°. When the negative phase difference is given, the backward wave is then produced albeit the waveform is the same as that with the positive phase difference. At  $\phi$  = 0° and  $\phi$  = 180°, the standing wave is generated and hence dominant over the beam so that even if the phase difference is varied the prominent progressive wave is not generated.

Figure 3 shows the plots of the maximum deflection around the center of the beam where influence of the near-field is little versus the phase difference. Comparing with Fig.4 depicting the standing wave ratio  $w_1 / w_3$  at each phase difference, it is worth investigating the relation between the progressive wave and the phase difference. When the backward wave becomes dominant over the whole region of the beam; that is  $w_1 = 0$ , the standing wave ratio  $w_1 / w_3$  is 0 whereas when the progressive wave becomes dominant;  $w_3 = 0$ , the standing wave ratio  $w_1 / w_3$  becomes  $\infty$ . When the forward wave and backward wave are completely balanced;  $w_1 = w_3$ , the standing wave ratio becomes 1.

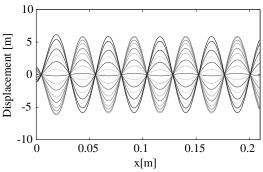


Figure 1 Envelope of the beam at 20 kHz (0 degree)

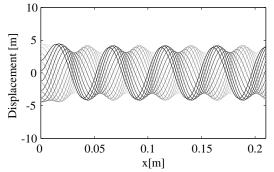


Figure 2 Envelope of the beam at 20 kHz (90 degrees)

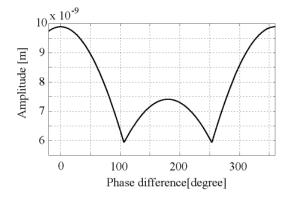


Figure 3 Amplitude of each phase difference at 20 kHz

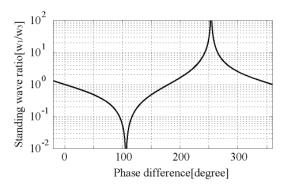


Figure 4 Standing wave ratio of each phase difference at 20 kHz

Clearly from Fig.2, when the beam is excited by two control forces in phase or out of phase, the standing wave emerges. Note that the standing wave ratio in this case is 1; however, there exists 30% in difference between the two standing wave amplitudes. As the phase difference  $\varphi$  increases from 0°, the standing wave ratio decreases while the backward wave contribution augments. As a result, the maximum amplitude of the beam lessens. If this is the case, backward wave and forward wave coexist in the same region of the beam, with the result that the envelope becomes not smooth but rough. At  $\varphi$  = 105°, the standing wave ratio becomes 0 and the maximum displacement amplitude at the center of the beam becomes minimum.

In this case, backward wave thoroughly dominates and the standing wave disappears. Moreover, the envelope of the beam deflection becomes smooth. Note that  $\phi$  providing the standing wave ratio being 0 varies when the driving frequency changes. When the phase  $\phi$  further increases, the standing wave ratio also increases. What it means is that the ratio of the forward wave to the backward wave increase, and hence the predominance of the backward wave over the forward wave continues till the phase becomes  $180^{\circ}$ . At  $\phi = 180^{\circ}$ , the standing wave ratio becomes 1 and the standing wave ratio again hits 1 and hence standing wave soundly governs the beam. Thus, the phase  $\phi$  is explicitly related to the progressive wave so that active wave control in terms of the phase leads to the control of progressive wave.

In summary, the backward emerges when  $\phi$  is between 0° and 105°, whereas the forward wave appears when  $\phi$  is between 0° and - 105°. The maximum amplitude of the beam deflection at the center of the beam is determined as contribution rate of the standing wave and the progressive wave.

#### 3. NUMERICAL ANALYSIS OF ULTRASOUND FIELD

#### A. Ultrasonic field model

Now that the progressive wave is controlled via active wave control, it is worth considering the relation between active wave control and ultrasonic levitation force. To do this, it needs to establish a model for analyzing the sound pressure radiated from a flexural wave of the beam. In overviewing the ultrasonic analysis, there are basically two approaches. First is the nonlinear acoustic approach 11,12 from the viewpoint of Langevin radiation pressure or Rayleigh radiation pressure, and the other via fluid dynamics 7-10 in terms of cylindrical squeeze film.

According to the squeeze film theory, the following items are clarified; (1) the temporal average of the sound pressure inside the squeeze film is larger than that of atmospheric pressure, (2) squeeze film forming depends upon the squeeze number determined by fluid viscosity and excitation frequency, and (3) the squeeze film shape is independent of the amplitude of an exciting surface.

Taking into consideration the facts observed in the preceding experiment, i.e., (1) the levitation gap is less than 1 mm, (2) it needs to consider the viscosity because of the transfer of an object, and (3) a squeeze film forming is not dependent on the amplitude of the exciting surface, this paper deals with the fluid dynamical approach.

The governing equation of an ultrasonic sound field is then given by the following Navier Stokes equation and the continuous equation,

$$\nabla v = 0 \tag{23}$$

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\Delta v \tag{24}$$

where v, p,  $\rho$ ,  $\mu$ ,  $\nabla$  and  $\nabla$  are fluid velocity, pressure, density, viscosity of the fluid, gradiant and the Laplacian. Regarding the boundary condition of the beam at both ends, the state variables of the beam in Eq.25 are assigned.

$$\begin{cases} v_{a} = v_{b} = Re \left[ \frac{\partial w(x)e^{j\omega t}}{\partial t} \right] = Re \left[ j\omega \left( -t_{1}w_{0} + t_{3} \frac{M_{0}}{EI} + t_{2} \frac{f_{1}}{EI} \right) e^{j\omega t} \right] \\ = Re \left[ j\omega \left( t_{1} \frac{-f_{1}k^{4}t_{2}t_{3} + f_{1}t_{1}t_{4} - f_{c}t_{4}}{EIk^{4}(k^{4}t_{2}^{2} - t_{4}^{2})} + t_{3} \frac{-f_{1}t_{1}t_{2} + f_{1}t_{3}t_{4} + f_{c}t_{2}}{EI(k^{4}t_{2}^{2} - t_{4}^{2})} + t_{2} \frac{f_{1}}{EI} \right) e^{j\omega t} \right] \\ p_{a1} = p_{a0} - \rho_{a} \frac{\partial \phi_{a}}{\partial t} = p_{a0} - \rho_{a} \frac{d\phi}{\partial t dy} dy = p_{a0} - \rho_{a} \frac{v_{b}}{\partial t} dy \\ = p_{a0} + \rho_{a} Re \left[ \omega^{2} \left( t_{1} \frac{-f_{1}k^{4}t_{2}t_{3} + f_{1}t_{1}t_{4} - f_{c}t_{4}}{EIk^{4}(k^{4}t_{2}^{2} - t_{4}^{2})} + t_{3} \frac{-f_{1}t_{1}t_{2} + f_{1}t_{3}t_{4} + f_{c}t_{2}}{EI(k^{4}t_{2}^{2} - t_{4}^{2})} + t_{2} \frac{f_{1}}{EI} \right) e^{j\omega t} \right] dy \end{cases}$$

where  $p_{a1}$  and  $p_{a2}$  are the pressures before and after control, and  $\phi$  is the velocity potential. As with the particle velocity normal to the beam, the surface velocity of the beam is utilized. Another important point on the boundary condition is the treatment of a progressive wave. It is true that the wave propagates as a flexural wave along the beam so that it is likely to introduce the horizontal velocity component of the beam. However, every tiny element continuously connected moves vertically and hence the horizontal movement of the element is null. In the numerical analysis, the vertical velocity of the beam is matched with the fluid velocity at the very contact point, which is introduced as the extraneous force of incompressible Navier-Stokes equation.

## B. Analytical results

Using a model shown in Fig.5, both Eqs. 23 and 24 are numerically analyzed via a HSMAC method, computational fluid dynamics algorithm. To do this, solving procedure continues until the error in the continuous equation becomes less than 0.001. Taking into consideration the sound wavelength of 48.4 mm at 20 kHz, the object length is set at 72.6 mm, 1.5 times the wavelength.

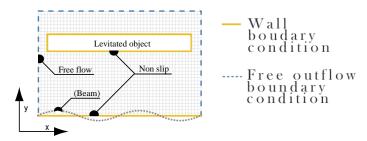
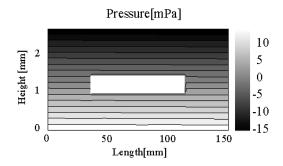


Figure 5 Numerical calculation model

Figure 6 shows the temporally averaged pressure distribution in the fluid field with the atmospheric pressure being set at 0. It is seen that the pressure beneath the object is larger than that above the object, hence levitation pressure confirmed. Illustrated in Fig. 7 is the temporal average of fluid velocity below the levitated object expressed as vectors. Comparing with the pressure in the vertical direction, the pressure induced by the flexural bending wave is noticeable. Note that the fluid in the vicinity of the object moves in the same direction with that of the vibrating beam, however the directions become opposite where the observation location is far from the beam and maximum near the levitated object.

Figures 8 and 9 illustrates the time averaged pressure under the object, depicted with the phase difference  $\phi$  varying. Note that the pressure in Fig. 8 is governed by the forward wave while the pressure in Fig.9 dominated by backward wave. In both cases, the pressure amplitudes at two bumps differ due to the phase difference  $\phi$ , thereby generating the pressure

gradient.



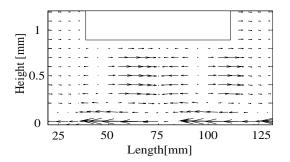
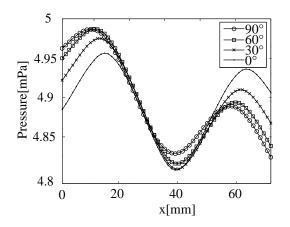


Figure 6 Time averaged pressure distribution around the levitated object at 90 deg.

Figure 7 Time averaged velocity around the the levitated object at 90 deg.



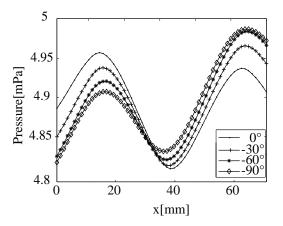


Figure 8 Time averaged pressure under the levitaged object.

Figure 9 Time averaged pressure under the levitated object

Consider the transferring force acting on the levitated object. For this purpose, the following three items needs to be discussed.

- (a) Force acting on both sides of the levitated object
- (b) Force due to the fluid viscosity acting around the levitated object
- (c) Force contributing to the transfer due to the tilting of the levitated object

Regarding the force acting on both sides of the object, the transferring force due to the pressure difference and the effective area may be given by

$$F_p = F_{p,right} - F_{p,left} = (P_{right} - P_{left})A_p$$
(26)

Consider the viscous stress acting on the object due to the velocity gradient beneath the object is written as

$$\tau = \mu \frac{du}{dy} \tag{27}$$

Therefore, the transferring force due to the fluid viscosity acting on the undersurface of the object is given by

$$F_{v} = \tau A_{h} = \mu \frac{du}{dy} A_{h} \tag{28}$$

To calculate the force in the above, the finite difference method is introduced

$$\frac{du}{dy} \doteq \frac{-3u_i + 4u_{i+1} - u_{i+2}}{2\Delta y} \tag{29}$$

Consider the transferring force due to the tilting of the levitated object. Table 1 presents the list of three forces contributing to the transferring force, the direction of which toward the rightward is defined as positive. Furthermore, the transferring force induced by the tilting of the object is shown for the two cases; maximum gradient angle is 1° and 0. 1°. Apparently from the table, of three forces the force due to the tilting of the object is the largest, followed by the force due to the pressure difference on both sides of the object and then the force due to the fluid viscosity. Note that three forces varies in correspond to the phase difference  $\varphi$ . In the case of  $\varphi$ =90°, the ratio of the force amplitude in terms of (b):(a):(c) is 1:264:2900(0.1°) or 29000(1°). In this case the transferring force directs to rightward albeit the progressive wave on the beam travels leftward so that the direction of the movement of the levitated object and the travelling wave is opposite. The transferring force increases in proportion to the phase difference, and when the phase inverses the transferring force also inverses.

Transportation	Phase difference [ $ heta$ ]						
force [N]	90	60	30	0	-30	-60	-90
Viscous force	1. 55E-11	1.22E-11	6.48E-12	-1.68E-13	-6.82E-12	-1.24E-11	-1.58E-11
Pressure difference	3. 07E-09	2.48E-09	1. 22E-09	-4.37E-10	-2.03E-09	-3.11E-09	-3.44E-09
Angle[°]	(1)	(0.6)	(0.3)	(0)	(-0.3)	(-0.6)	(-1)
Component of levitated force	4. 50E-07	2.70E-07	1.35E-07	0	-1.35E-07	-2.70E-07	-4.50E-07
Angle[°]	(0.1)	(0.06)	(0.03)	(0)	(-0.03)	(-0.06)	(-0.1)
Component of levitated force	4. 50E-08	2.70E-08	1.35E-08	0	-1.35E-08	-2.70E-08	-4. 50E-08

Table 1 Comparison of transportation forces

## 4. CONCLUSION

For the purpose of clarifying the transferring mechanism of a levitated object via ultrasonic sound, the theoretical analysis and numerical analysis were conducted. The issues presented are summarized in the following.

- (1) Relation between the phase difference, a control parameter of active wave control, and the progressive wave generated along the beam is clarified.
- (2) Relation between progressive wave and the associated transferring force is elucidated. Furthermore, three factors contributing to the transferring force; viscous force, pressure difference acting on both sides of the object and pressure gradient caused by flexural wave are discussed. As a result, the contribution rate of the pressure gradient to generating the transferring force is found to be conspicuously high.
- (3) Progressive wave produced by active wave control generates the pressure gradient between the levitated object and the travelling wave along the beam, thereby slightly tilting the object and thus providing the transferring force.
- (4) It is found that the travelling direction of the progressive wave and the moving direction of the object is opposite.

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