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# VORTEX POWER FLOW ACTIVELY INDUCED IN A THIN RECTANGULAR PLATE

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## 1. INTRODUCTION

This paper deals with the active power flow control of a distributed-parameter planar structure, with the particular emphasis on a vortex power flow which has a potential to confine vibrational power, a cause of exciting structural modes, injected by a disturbance force. Without letting the vibrational power disperse into the structure, the control effect for suppressing the structural response can be achieved. This paper begins by deriving the necessary condition for producing a vortex power flow in the vicinity of a disturbance point force at an arbitrary exciting frequency. Then, with a view to monitoring the power flow occurring in the plate structure, a wave visualization system has been developed. With this system, it comes to possible to experimentally verify the wave propagation (the vortex power flow) taking place in the structure. In order to investigate the contribution of vibration modes to the vortex power flow formation, an energy stream function as well as a vorticity function is derived in a general form. With these functions, it is revealed quantitatively the generation mechanism of a vortex power flow induced in the vicinity of a disturbance force location. It also turns out that as many as 100 vibration modes are found to be necessary to form a vortex power flow pattern.

## 2. INDUCING A VORTEX POWER FLOW

It was reported that a vortex power flow occurs[1] when two vibration modes whose resonance frequencies are very close to each other are excited to similar levels. In this case, the location of the vortex power flow is to be determined by the interference of the two vibration modes. In other words, it is not possible to designate the vortex power flow location to occur in a structure. It would be desirable if the power flow injected into a structure by a disturbance force could be confined around the disturbance location, without letting it disperse into the stricture; then sufficient control effect for suppressing the structural vibration can be expected. With this in mind, this article begins with detailing the methodology to induce a vortex power flow around a disturbance point without scattering it around the structure.

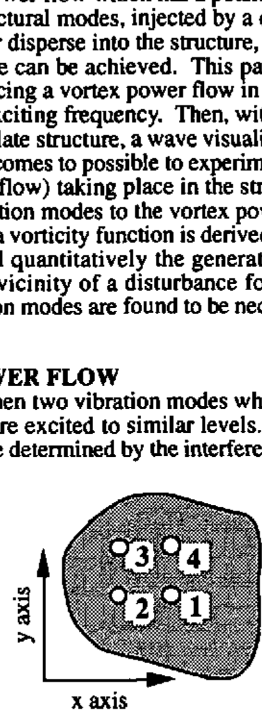


Fig. 1 Configuration of point forces

As shown in Fig. 1, let a set of four point forces be arranged in a quadrilateral fashion, with bottom left force labeled as force $f_1$, and the others number in counterclockwise fashion. Consider a simply supported, thin, lossless, steel rectangular plate of length $L_x$ and $L_y$ in the x and y directions, respectively, and of thickness h. The power flows $P_i$ injected at the location $i$ ($i = 1 \sim 4$) where point forces $f_i$ are acting, are given by

$$P_i = \frac{1}{2}\mathrm{Re.}[f_i \dot{w}^*(r_i)] \tag{1}$$

where $*$, $\dot{}$ and Re. denote the complex conjugate, time derivative and real part of the expression, respectively. The power flow, for instance, at location 1 is described by

$$P_1 = -\frac{\omega}{2}\{s_{12}\mathrm{Re}[jf_1 f_2^*] + s_{13}\mathrm{Re}[jf_1 f_2^*] + s_{14}\mathrm{Re}[jf_1 f_4^*]\} \tag{2}$$

where

$$s_{lm} = \sum_{i=1}^{N_m} \frac{4}{\rho_s h L_x L_y(\omega_i - \omega)} \sin\alpha_i x_l \sin\beta_i y_l \sin\alpha_i x_m \sin\beta_i y_m \ . \tag{3}$$

Furthermore, the terms in Eqs. (2) and (3) satisfy the following relationships

$$\mathrm{Re}[jf_l f_m^*] = -\mathrm{Re}[jf_m f_l^*] \tag{4}$$

$$s_{lm} = s_{ml} \tag{5}$$

For the expressions in Eqs.(4) and (5), the Kirchhoff's law holds

$$P_1 + P_2 + P_3 + P_4 = 0 \tag{6}$$

Setting the power flow $P_i$ in terms of the force $f_i$ equal to the power flow $P_j$ in terms of the force $f_j$ from the diametrically opposed source, the necessary condition to produce a power flow vortex is

$$P_1 = P_3, \ P_2 = P_4 \ . \tag{7}$$

With Eq. (7) and the Kirchhoff's law described in Eq.(6), the following relationships between the power flows are obtained,

$$P_1 = -P_2, \ P_3 = -P_4 \ . \tag{8}$$

Eqs. (7) and (8) show that a source and sink of the power flow arise in turn, and thus this is the necessary condition to generate a vortex power flow. Consider the control force to produce the vortex power flow. Defining $f_1$ as the reference, which could also be regarded as a disturbance force, the other three forces used as control forces will be related to it by the relationship

$$f_i = \alpha_i e^{j\varphi_i} f_1 \quad (i = 2, 3, 4) \tag{9}$$

where $\alpha_i$ and $\varphi_i$ are the feedforward gain and phase of the $i$th control force, respectively. Note that there are four point forces acting on the plate, and that the number of the equations representing the necessary condition to induce the vortex power flow is two; thus the number of the unknown parameters must be also two. Therefore, for instance, by setting the phases to $\varphi_2 = \pi/2$, $\varphi_3 = \pi$, and $\varphi_4 = 3\pi/2$, the force amplitudes are found to be

$$\alpha_3 = \sqrt{\frac{s_{12}s_{14}}{s_{23}s_{34}}}\ \alpha_1 \tag{10}$$

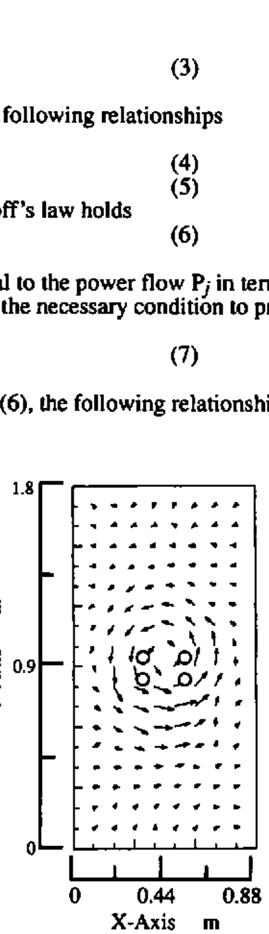


Fig. 2 Vibration intensity distrubution

$$\alpha_4 = \sqrt{\frac{s_{12}s_{23}}{s_{34}s_{14}}}\,\alpha_2 \ . \qquad (11)$$

Thus, above two equations describing the feedforward gains denote the necessary conditions for inducing a vortex power flow around the disturbance force location.

## 3. POWER FLOW CONFINEMENT

For the purpose of producing a vortex power flow at the center of the plate, four electro-dynamic shakers are installed at the center of the plate, with 10 cm and 20.5 cm placed apart in the x and y directions, respectively. Using the necessary conditions described by Eqs. (10) and (11), the four excitation forces are determined: all gains of the forces are set to 1 N in amplitude of a sinusoidal wave; each phase difference between the adjacent forces is set to 90 degrees; and the excitation frequency is set at 37 Hz, 99 % of the (1, 1) mode resonance frequency of the plate. Figure 2 shows the vibration intensity distribution obtained experimentally. As was to be expected, a vortex power flow is induced at the center of the plate, verifying the validity of the condition for inducing the vortex power flow.

The envelope of the absolute amplitude of waves associated with each of these responses are depicted in Fig. 3 in three dimensions for the cases before and after control. For the case before control, where only the disturbance force acts at location 1 (see Fig. 1), the typical (1, 1) modal pattern is acquired as shown at the upper picture of Fig. 3. The middle picture in Fig.3 illustrates the case after control, where the vortex power flow is generated at the center of the plate, while the lower one depicts the contour for this case. It is clear from the figure that the wave rotates around the center of the plate, with the displacement being zero at the center of the vortex. For the result of Fig. 5 the maximum displacement amplitude is 6.3 μm, 2.6% of the uncontrolled amplitude, 239 μm. Thus by confining the power flow around the disturbance force, causing the excitation of the vibration modes, such a significant control effect is considered to be obtained.

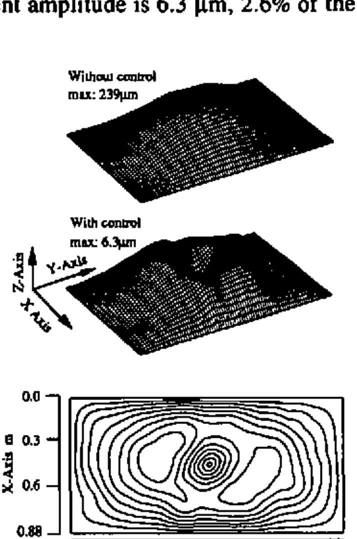

Fig. 3 Envelope of absolute wave amplitude and contour for the cases before and after control at 35 Hz

## 4. ENERGY STREAM FUNCTION AND VORTICITY FUNCTION

It was reported that a vortex power flow is induced[1] when two vibration modes whose resonance frequencies are very close to each other (like a degenerate mode) are excited simultaneously. In such a case, the vortex location is determined by the interference pattern of vibration modes called a vortex block[2]. Such a vortex is, as it were, a spontaneous vortex; while the vortex dealt with in this article is an actively induced one. Hence, it will be of interest to elucidate the generation mechanism of the actively induced vortex power flow. To this end, this paper derives a generic form of the energy stream function described by using $N_m$ modes. With the energy stream function, it will become possible to see how the power flow path is constructed as well as how the vortex configuration evolves. With a view to clarifying the characteristics of the vortex, this paper further derives the vorticity function described in a general form; it will become possible to see whether the vortex power flow actively generated is true or false. The energy stream function can be obtained as

$$\psi = \int I_{vx} dy - \int I_{vy} dx \tag{12}$$

where $I_{vx}$ and $I_{vy}$ are components of the vibration intensity vector in the x and y direction, respectively. Further expanding Eq. (12) leads to the energy stream function described by $N_m$ vibration modes

$$\psi = \frac{1}{2} \sum_{k=1}^{N_m} \sum_{l=1}^{N_m} \mathrm{Re.}[(\psi^x_{kl} + \psi^y_{kl})] \tag{13}$$

for the case when $k \neq l$

$$\psi^x_{kl} = \frac{1}{2} j\omega D w_k w_l^* [-(\alpha_k^2+\beta_k^2)\alpha_k \cos\alpha_k x \sin\alpha_l x (\frac{\sin(\beta_k-\beta_l)y}{\beta_k-\beta_l} - \frac{\sin(\beta_k+\beta_l)y}{\beta_k+\beta_l})$$
$$+(\alpha_k^2+\nu\beta_k^2)\alpha_l \sin\alpha_k x \cos\alpha_l x (\frac{\sin(\beta_k-\beta_l)y}{\beta_k-\beta_l} - \frac{\sin(\beta_k+\beta_l)y}{\beta_k+\beta_l}) - (1-\nu)\alpha_k\beta_k\beta_l \cos\alpha_k x \sin\alpha_l x$$
$$(\frac{\sin(\beta_k-\beta_l)y}{\beta_k-\beta_l} + \frac{\sin(\beta_k+\beta_l)y}{\beta_k+\beta_l})]$$

$$\psi^y_{kl} = \frac{1}{2} j\omega D w_k w_l^* [-(\alpha_k^2+\beta_k^2)\beta_k \cos\beta_k y \sin\beta_l y (\frac{\sin(\alpha_k-\alpha_l)x}{\alpha_k-\alpha_l} - \frac{\sin(\alpha_k+\alpha_l)x}{\alpha_k+\alpha_l})$$
$$+(\nu\alpha_k^2+\beta_k^2)\beta_l \sin\beta_k y \cos\beta_l y (\frac{\sin(\alpha_k-\alpha_l)x}{\alpha_k-\alpha_l} - \frac{\sin(\alpha_k+\alpha_l)x}{\alpha_k+\alpha_l}) - (1-\nu)\alpha_k\alpha_l\beta_k \cos\beta_k y \sin\beta_l y$$
$$(\frac{\sin(\alpha_k-\alpha_l)x}{\alpha_k-\alpha_l} + \frac{\sin(\alpha_k+\alpha_l)x}{\alpha_k+\alpha_l})] \quad , \tag{14}$$

where D and ν mean the bending stiffness and a Poisson's ratio of the panel, respectively.

A vorticity function can also be defined in the vibration intensity field as

$$\zeta = -\frac{1}{2} (\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}) \, . \tag{15}$$

In the same way as discussed in the energy stream function, the vorticity function in Eq. (15) can also be described in a general form, however, due to the page limit, this paper rather omits the further expansion.

## 5. MODAL ENERGY

Figure 4 illustrates the modal energy of each vibration mode of the plate with and without the vortex power flow at the center of the panel. The vertical axis of the figure indicates modal indices arranged from the lowest resonance frequency of the panel to the highest, while the horizontal axis shows the associated modal energy of the panel. As the exciting frequency of the primary force is close enough to the (1, 1) mode resonance frequency, 99 % of its frequency, the modal energy of the (1, 1) mode is conspicuous. For the case with a vortex power flow induced at the center of the plate, however, the modal energy of the (1, 1) mode is perfectly suppressed.

Observe that modal energies of the (1, 3), (2, 2), (1, 5) modes etc., are also completely suppressed. On the contrary, the modal energies of the (1, 2) , (2, 1), (1, 4) modes and so forth increase as a result of inducing the vortex power flow at the center of the plate.

By utilizing the vibration modes selected from the greatest contributor to the total energy, the energy stream function can be evaluated. Figures 5 and 6 show how the energy stream function and its corresponding contour evolve with the number of vibration modes picked up from the greatest contributor to the total energy. As is clear from these figures, with only two vibration modes, a foundation of the vortex power flow is already fabricated at the center of the panel. The shape of the contour is, however, elliptic with a longer axis in the y direction. By increasing the number of the contributing modes, the contour changes its form from an ellipse to a circle which is close to the vibration intensity distribution pattern as shown in Fig. 2. Gavric et al.[3] reported that at least 1000 vibration modes are needed to convergence of the vibration intensity distribution. Regarding the actively induced vortex power flow, however, 100 vibration modes are needed to form a precise power flow pattern. It should be noted that the modal energy of the (1, 2) mode, the greatest contributor to the total energy, is 1000 times as large as that of the tenth one, the (2, 7) mode; $3.44 \cdot 10^{-8}$ Nm for the (1, 2) mode, while $3.41 \cdot 10^{-11}$ Nm for the (2, 7) mode. In other words, the tenth vibration mode contrib-

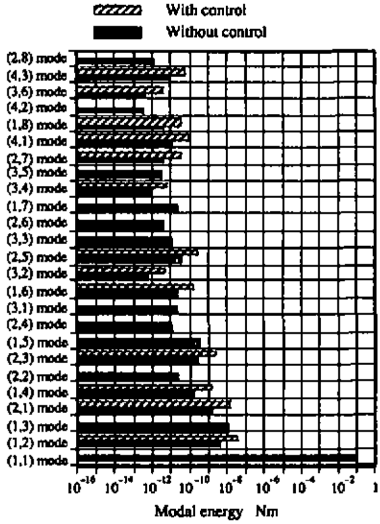


Fig. 4 Modal energy vs. vibration modes

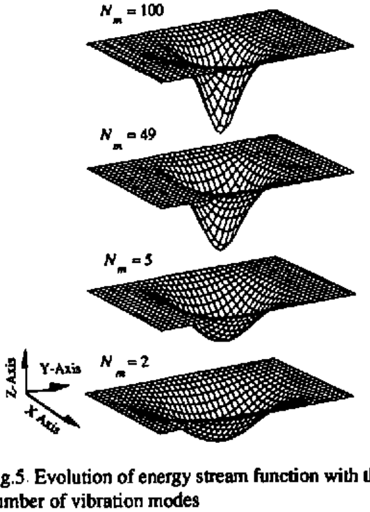


Fig.5 Evolution of energy stream function with the number of vibration modes

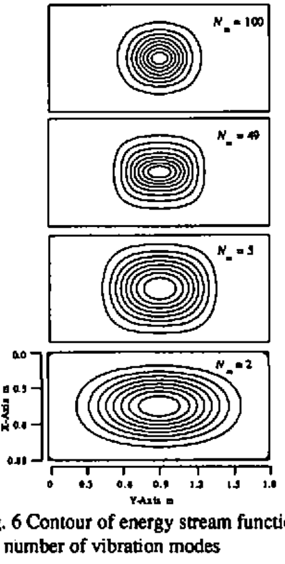


Fig. 6 Contour of energy stream functions with the number of vibration modes

utes merely 0.1 % to the total energy, not to mention the 100th modal contribution being just little. However, even such small contributors to the total enegy are needed to form the precise vortex power flow configuration.

The vorticity function can be likewise obtained by picking up the vibration modes in order of the contribution rate to the total energy. Figure 7 shows the three dimensional picture of the vorticity function, demonstrating its evolution with the number of vibration modes. It can be seen from these figures that only two vibration modes are sufficient to construct an approximate form of the vorticity function at the center of the panel. In the same way as discussed in the energy stream function, the contour of the vorticity function with two vibration modes shows an elliptical form, changing its form from an ellipse to a circle as the number of the modes increases.

Recall the degenerate modal case[2] where sub-vortices exist inside a major vortex. If this is the case, the vorticity of each sub-vortex is non-zero; thereby the sub-vortex is a real vortex; however, the vorticity of the major vortex turns out to be zero; hence it is not a real vortex but a derivative of the sub-vortices. On the contrary, the actively induced vortex generated at the center of the plate has the vorticity, and hence it is real.

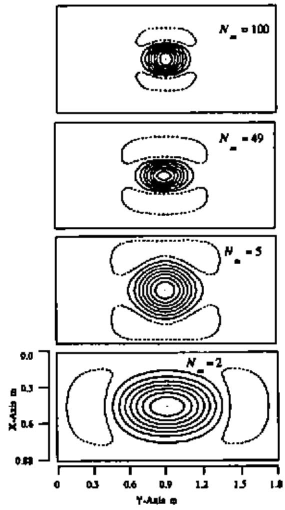


Fig. 7 Contour of vorticity function (Solid line; positive, dotted line negative value)

## 6. CONCLUSION

It was succeeded experimentally in generating a vortex power flow at the center of a simply supported rectangular panel. As the result, the power flow causing the excitation of vibration modes was confined in a specific area, and so the vibration level was significantly suppressed; the maximum amplitude of the panel without power flow control was 239 μm, that was suppressed to only 6.3 μm by the power flow control. With the development of the wave visualization system, monitoring of the wave propagation in a planar structure became possible. Furthermore, an energy stream function as well as a vorticity function was derived in a general form. It was also revealed that the structural modes with even small contribution to the total energy of a structure are needed to construct a power flow path; for the vortex case actively induced at the center of the panel, 100 vibration modes turned out to be essential to form a circular vortex power flow path which agrees with the experimental power flow. Finally, the characteristics of a vortex power flow was investigated, showing the vortex is a combination vortex consisting of a forcing and free vortices.

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