THE LOUDSPEAKER PARAMETERS AND THEIR EVOLUTION

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1 INTRODUCTION

The quantities that have come to be dubbed the Thiele – Small parameters enable the response of a loudspeaker, electro-magnetic direct-radiating driver plus box, to be described and manipulated in the same manner as an electrical high-pass filter. The description of a vibrating mechanical system in terms of an electrical network originated with Maxfield and Harrison[1] in 1926. Electrical measurements of loudspeakers were described by McLachlan in 1934[2] and Beranek[3] offered a detailed description of a loudspeaker in terms of an electrical network in 1955.

During the 30's, 40's and 50's manufacturers and publishers offered copious graphs of impedance vs. frequency such as Fig 1[4] but no indication of their relationship to performance. In 1961, however, it became possible to define parameters of a driver from measurements of the impedance curves, interpreted initially from the simplified model of Fig 2, using the simple equipment of Fig 3. They could then be used to design a loudspeaker, driver plus box, as the analogue of an electrical high-pass filter with a desired transfer function[5].

Their initial publication in Australia evoked little interest, with a few notable and perceptive exceptions, but further energetic research by Small[6,7,8,9,10,11,12] at the University of Sydney and publication internationally led to their adoption universally from the 70's onwards.

This paper sets out to discuss a number of the topics, most of them useful but some not, that have arisen from the measurement and use of the parameters since those days, already quite a long time ago.

2 THE PARAMETERS

Five main parameters that characterise the performance of a driver, and another for the enclosure or box it is mounted in, are used to estimate the performance of a loudspeaker –

f_S the resonance frequency of the driver (Hz)

R_E the series resistance of the voice coil (ohms). It is usually taken previously as the d.c. resistance, but it properly includes the resistive component of "voice coil inductance", which varies with frequency but is small around f_S. It is different from the *rated* impedance, which is usually taken near the driver impedance in the region between 400 Hz and 1000Hz where the impedance is close to a minimum and substantially resistive.

Q_E the 'electrical' quality factor, the ratio of R_E to X_0 the reactance at resonance of the drivers motional impedance. It is a pure, dimensionless, number. Note that in estimating Q_E for the performance of a loudspeaker *system,* R_E must include *effectively* all the resistances in the voice coil circuit, of the coil itself, any passive crossover inductors, the connecting cables and the output impedance of the amplifier.

Q_M the 'mechanical' quality factor, another pure number, is the ratio of the shunt resistance of the driver's motional impedance to X_0 its reactance at resonance. In early publications, Thiele called this parameter Q_A for 'acoustical'.

The Q values, quality factors, affect the damping of the driver around its resonance. The higher the Q's the higher the frequency response will peak around resonance compared with the in-band response at higher frequencies. If the Q's are too low, the frequency response will sag around resonance. In applications where the impedance of the input device is large enough to increase Q_E significantly, as possibly with passive crossover networks or connecting cables, Q_E and Q_M affect the input impedance of the driver and its response in different ways and are therefore quoted separately. When Q_E is suitably adjusted and the driver then considered as fed from a low output impedance, the two Q's act together to control damping near resonance as --

Q_T, the "total" quality factor, in which Q_M and Q_E are combined in the manner of resistances in parallel,

$$Q_T = \frac{Q_E Q_M}{(Q_E + Q_M)} \qquad -\ (1)$$

When these parameters f_S Q_M Q_E and Q_T are measured under different conditions, their values change somewhat, and it is convenient during measurement and calculation, as distinct from design, to characterise each of them with a second sub-script, thus

f_{SA} is the value of f_S that is measured when the driver is unbaffled, "in **a**ir".

f_{SB}, slightly lower, is the value of f_S when the driver is mounted in a **b**ox, or on a baffle.

f_{SC} is the resonance frequency *of the system* comprising a driver mounted in a **c**losed box.
Similarly **Q_{MA} Q_{MB} Q_{MC}, Q_{EA} Q_{EB} Q_{EC}** and **Q_{TA} Q_{TB} Q_{TC}**.

Obviously the parameters to use in designing a loudspeaker system are **f_{SB} Q_{TB} Q_{EB}** and **Q_{TB}**, which are usually characterised in specifications as **f_S Q_{TS} Q_{ES}** and **Q_{TS}**.

V_{AS}, the volume of air equivalent to the acoustical compliance of the driver, may be specified in litres (i.e. cubic decimetres or milli-cubic-metres) or in Imperial units of cubic feet or cubic inches. This parameter controls the response through its ratio with

V_B, the volume of air in the box that the driver is mounted in.

The ratio, which may be written as **V_{AS}/V_B** but often as a ratio of compliances **C_{AS}/C_{AB}**, has a powerful effect on the design of a loudspeaker system. Because the volume (hence the compliance) of the box forms the denominator of the expression, the *bigger* the box the *smaller* is the figure for C_{AS}/C_{AB}. In his publications, Small has called this ratio **α**.

These six parameters of the driver (or five when Q_E and Q_M are combined into Q_T) , along with C_{AS}/C_{AB}, control the response that is obtained from a loudspeaker driver/box combination, and to them must be added, whenever the box is vented (ported),

f_B, the box resonance, the frequency where the acoustical mass of air in the vent resonates with the compliance of the air in the box. It is most easily appreciated, for example, as the frequency of the note that results from blowing across the top of a bottle, which is of course another variant of the Helmholtz resonator. Small has called its ratio with the driver resonance, f_B/f_S = ***h.***
These last two parameters were found from a second set of measurements, with the driver mounted in a vented box of known volume.
Three other driver parameters are of lesser importance, though still significant in some designs –

Q_L, the "leakage" Q, has a small effect in vented box designs. Its effect on the frequency response can be remedied by using a somewhat larger box and a slightly increased Q_T. It is referred to the parameters of the driver.

Q_B, the "box" Q is similar, but not identical, in which the resistive losses are referred to the box.

L_E, the voice coil inductance, in series with R_E, can be important in passive crossovers. It includes a "semi-inductance" whose reactance component varies with the square root of frequency and is accompanied by a series resistance of equal magnitude. Hence both the resistance and reactance components of the voice coil "inductance" vary with frequency, and any specification of L_E or its resistance component R_L applies only at one frequency.

3 USE OF THE PARAMETERS

Once the values of the parameters of a driver are known they can be applied to calculating the transfer function of a system that incorporates it. The most obviously desirable transfer functions are the maximally-flat Butterworths, whose amplitude responses remain as flat as possible within their pass-band and then change rapidly to a stop- band where they attenuate the response at a rate of 6n dB per octave, where n is the order of the filter, 2 for a closed-box loudspeaker and 4 with a vented box.

If a response of other than Butterworth shape is desired it could be produced, but each response requires its own distinctive collection, or group or constellation of parameters, that came to be known as an "alignment". If a suitable alignment was not available, a straightforward calculation would let the designer know what to expect, how much it deviated from his ideal.

In the 60's an engineer was known by his slide rule. For greater precision, tables of 4 figure logarithms were needed, but when from the late 60's computing calculators and computers with stored programs became generally available, even lengthy calculations became a simple matter and the calculation of the parameters presented no great difficulty.

4 AUXILIARY ELECTRONIC FILTERING

The advent of the parameters made it possible not only to manipulate designs of loudspeakers for optimum performance, but also to convolve their native transfer functions with other factors to modify their performance. They could be equalized, using a biquadratic active filter with a 2nd order numerator to cancel an undesirable 2nd order factor in the loudspeaker response and substitute a desirable 2nd order factor for its denominator[13]. Alternatively, the 2nd order transfer function of a closed box loudspeaker, or the two 2nd order factors of a vented-box loudspeaker function, could be coaxed into the 2nd order factors of a desirable response of higher order, e.g. 3rd or 4th order for a closed box, 5th or 6th order for a vented box. The remaining factor is then provided by an active 1st order (simply a choice of coupling capacitor) or 2nd order filter (generally a single Sallen and Key element) ahead of the amplifier. When th order of the final transfer response is 4 or greater, there is also flexibility in choosing which factor or factors are to be realised in the loudspeaker and which in the auxiliary filter.

Auxiliary filters of this kind need no greater complexity than 2nd order. Little improvement is gained from higher orders. With such modest auxiliary filtering at the lowest frequencies where the driver can no longer radiate useful acoustic power, response can be maintained to the low frequency limit with a minimum of stress on the driver and its amplifier.

Benson[14] showed that connecting a capacitor in series with a loudspeaker not only reduces the cone excursion at very low frequencies, it also extends the low frequency response. The resonance of a driver plus closed box, where the impedance goes through a maximum and a pure resistance, usually sets the low frequency limit of the system. At lower frequencies the impedance falls, presenting a reactive component that is inductive. With a capacitance connected in series, resonance occurs, that increases the current into the driver and so extends the low frequency response. At the same time it attenuates the power fed to the driver at even lower frequencies. However it does require a driver with a rather high value of Q_T, around 1, the exact value depending on the relative values of Q_M and Q_E.

It also requires a fairly high value of series capacitance, of the order of 1000 μF when the driver resistance is low, around 4 ohms. And it must be remembered that, in this case as in every case, each response requires its own unique alignment of parameters.

Unfortunately, addition of series capacitance does not help a vented box, whose native response generally starts to fall at frequencies below f_B its box resonance. The driver impedance goes through a minimum at f_B and, at frequencies immediately below, presents a capacitive reactance component. Thus a series capacitance produces no resonance and no increase in transfer of electrical power.

5 MEASUREMENT PROCEDURES

The parameters are often measured nowadays using computer programs that take a large number of impedance readings and then calculate parameter values by fitting the readings to a model. For a reader that uses such a procedure, the following comments may be of little interest, though some fine details still remain in the choice of method.

However, for readers who still take readings more laboriously by hand, and those who wish to understand in the hope of estimating them, several aspects of measurement procedure must be emphasized or explained.

The first explanation concerns a fallacy that crept into some measurements of Q_M and Q_E. In the procedure illustrated by Fig 3, with readings taken as in Fig 4, the d.c. resistance R_E is measured then f_S the resonance frequency and R_0 the peak resistive impedance there. Next the two frequencies f_1 and f_2, at the same impedance Z_1, from which we find

$$Q_M = \frac{f_0}{f_2 - f_1}\sqrt{\frac{R_0^2 - Z_1^2}{Z_1^2 - R_E^2}} \qquad -(2)$$

Workers familiar with measurements of Q in radio frequency circuits will be familiar with the simpler expression

$$Q = \frac{f_0}{f_2 - f_1} \qquad -(3)$$

where f_1 and f_2 are taken at an impedance Z_1 that is 0.707 times the peak impedance R_O.
That method is well suited to Q measurements greater than 5, but in loudspeakers the Q's are much lower, Q_M in the range from 5 to 1 and Q_E between 2 and 0.2.
It will be seen that when Z_1^2 is equal to $R_o^2/2$, as in the r.f. measuring method, and is much greater than R_E^2, eqn (3) becomes the same as eqn (2). But the optimum value for Z_1 is $\sqrt{R_O R_E}$. In that region the slope of Z_1 vs. frequency is a maximum and the readings of f_1 and f_2 are most sensitive. At the same time the frequency difference $f_2 - f_1$ is much greater than that at 0.707 R_O. Measuring $f_2 - f_1$ to sufficient accuracy was a problem in the 50's and early 60's with a beat-frequency whose self-excited oscillators drifted, so that its zero beat needed frequent checking and frequencies integrally related to 50 Hz were corroborated by observing Lissajous figures with respect to a not very stable 50 Hz power mains.

Need it be pointed out that when individual frequencies can only be read to within 1 Hz and the difference $f_2 - f_1$ is as small as 20 Hz, the uncertainty of its reading is only ± 5%, so readings should be taken with a precision of 0.1 Hz. The dependence of these calculations on small differences between larger numbers must always be kept in mind in this work, and care taken to minimize errors in the initial readings.

With Q_M found, Q_E and Q_T are found from

$$Q_E = Q_M \frac{R_E}{R_0 - R_E} \qquad -(4)$$

and

$$Q_T = Q_M \frac{R_E}{R_0} \qquad -(5)$$

To measure the parameters, two sets of readings must be taken, with either the mass of the diaphragm or its compliance changed by a known amount between the two sets of measurements. In the method proposed in the initial publication, f_S, Q_M and Q_E are first measured with the driver un-baffled, and these "In Air" measurements called f_{SA}, Q_{MA} and Q_{EA}. Then the driver was mounted in a closed box. The measurements were repeated and called f_{SC}, Q_{MC} and Q_{EC}.

When measurements of the driver are made in the manner of Fig 4, firstly with it un-baffled and then mounted on a baffle, e.g. in a box, its resonance frequency is lower in the second measurement, because the effective mass of air loading its diaphragm is increased. The change is not great, but is often ignored, once even in standards documents.

The resulting change in the acoustical mass of the diaphragm from its value M_{ASA} measured un-baffled to its value M_{ASB} when it is mounted in the closed box, was found as

$$\frac{M_{ASA}}{M_{ASB}} = \frac{f_{SC} Q_{EA}}{f_{SA} Q_{EC}} \qquad -(6)$$

And from this, the parameters of the driver mounted in a box or on a baffle, i.e. in normal use, were found from

$$f_{SB} = f_{SA} \sqrt{\frac{M_{ASA}}{M_{ASB}}} \qquad -(7)$$

$$Q_{MB} = Q_{MA} \frac{f_{SB}}{f_{SA}} \qquad -(8)$$

$$Q_{EB} = Q_{EA} \frac{f_{SB}}{f_{SA}} \qquad -(9)$$

$$Q_{TB} = Q_{TA} \frac{f_{SB}}{f_{SA}} \qquad -(10)$$

and

$$V_{AS} = V_B \left[\frac{f_{SC} Q_{EC}}{f_{SA} Q_{EA}} - 1 \right] \qquad -(11)$$

where V_B is the volume of the testing box. The parameters described in eqns (7) to (10) as f_{SB}, Q_{MB} Q_{EB} and Q_{TB} are the ones usually known as f_S, Q_{MS}, Q_{ES} and Q_{TS} and used for design purposes.

In eqn (11), all too often, some authors have chosen to ignore the change in mass and write $(f_{SC}/f_{SA})^2$ instead of $(f_{SC}Q_{EC}/f_{SA}Q_{EA})$. But M_{ASB} is 10% to 20% greater than M_{ASA} and the effect of its ratio M_{ASA}/M_{ASB} influences the whole chain of later calculations.

Initially, the use of these two sets of measurements served well to find the parameters, but problems soon surfaced when a colleague took measurements in an supposedly "enclosed" box that had, in fact, small leaks.

Another colleague suggested later that boxes should be specified, not as “airtight” but “watertight”. Even small leaks, along the joints or where the driver mounts to the front panel, constitute a vent that can affect readings to a surprising extent. It became clear that the second set of readings could be taken much more safely in a vented box, where such inadvertent additions to the main intentional vent make a negligible contribution. The method of Fig 5 had been considered in the initial paper as of secondary importance. It now became the primary, preferred, method.

Even so, the earlier method, using two sets of measurements in the manner of Fig 4, still remains useful for measuring drivers whose back is already enclosed, such as tweeters. The testing box, with a volume, in the case of tweeters, of a fraction of a litre, is placed over the front of the driver, taking great care that the two seal well against each other.

When the second set of measurements are taken with a vented box, as in Fig 5, the frequencies are measured at the peaks of impedance f_L the lower frequency and f_H the higher frequency and at the minimum impedance between them f_B that was taken as the Helmholtz resonance frequency of the box.

Then
$$V_{AS} = V_B \frac{\left(f_H^2 - f_B^2\right)\left(f_B^2 - f_L^2\right)}{f_H^2 f_L^2} \qquad - (12)$$

and the resonance frequency of the driver when it is mounted on a baffle or in a box is

$$f_{SB} = \frac{f_H f_L}{f_B} \qquad - (13)$$

Then Q_{MB} Q_{EB} and Q_{TB}, for use in designs, are from found eqns (8), (9) and (10) as before.

But with experience of measurements with this vented box method, another problem arose. The frequency where the impedance of the driver in a vented box has a minimum or, more easily measured, a zero phase angle, is not truly the box resonance f_B so it is called f_{BO} in Fig 5.

At f_{BO}, the frequency is a little lower than f_B, where the inductive reactance of the voice coil inductance is in series resonance with a capacitive reactance of the motional impedance. Benson[14] overcame this problem by taking a further reading, when the vent is closed, of f_C the frequency of the peak impedance that is virtually unaffected by the series inductance. Then the true value of f_B is

$$f_B = \sqrt{f_H^2 + f_L^2 - f_C^2} \qquad - (14)$$

and from it V_{AS}, f_{SB}, Q_{MB} Q_{EB} and Q_{TB} are found from eqns (12), (13), (8), (9) and (10) as before.

When calculations of V_{AS} were first made using eqn (12), where squares are taken of readings and differences taken between them, the results showed random variations of ±2% that were quite unaccountable until it was realised how easily such errors can accumulate in concatenated calculations when the slide rule used for the calculations is only capable of an accuracy around 1 part in 400. This prompts the reminder that in calculations of the parameters, as in all calculations that are made in such chains from one to the next, where rounding errors easily accumulate, it is important to record all the intermediate results to 4 significant digits even though the results only merit quotation in 3. A problem that became irrelevant with the advent of computers, where intermediate results are stored in memory to a high precision..

The parameter Q_L is derived from the resistance R_B measured at f_{BO} in Fig. 5, which is a little greater than R_E.

$$Q_L = \frac{1}{Q_{MB}} \cdot \frac{R_0 - R_B}{R_B - R_E} \qquad -(15)$$

This parameter refers the box losses to the driver parameters. The parameter Q_B refers them to the box

$$Q_B = Q_L \cdot \frac{f_B}{f_{SB}} \cdot \frac{V_B}{V_{AS}} \qquad -(16)$$

Note that in eqn (16) the ratio of air volumes V_{AS}/V_B is inverted from its usual usage. Also that Q_L and hence also Q_B depend on the small difference quantity $R_B - R_E$. Small has recommended that, when a figure for Q_L is not known, it can be taken as a mean figure of 7 with little error.

6 APPLICATION OF THE PARAMETERS

Once the parameters of the driver are known, the dimensions of the enclosure, or box, either totally enclosed or vented can be chosen, and the transfer function found, for the closed box,

$$E(s) = \frac{s^2 T_S^2}{s^2 T_S^2 + \frac{sT_S}{Q_T} + \left[1 + \frac{V_{AS}}{V_B}\right]} \qquad -(17)$$

where $$T_S = \frac{1}{\omega_S} = \frac{1}{2\pi f_S} \qquad -(18)$$

and, for the vented box,

$$E(s) = \frac{s^4 T_B^2 T_S^2}{s^4 T_B^2 T_S^2 + s^3 \left\{\frac{T_S^2 T_B}{Q_L} + \frac{T_B^2 T_S}{Q_T}\right\} + s^2 \left\{T_S^2 + T_B^2 \left[1 + \frac{C_{AS}}{C_{AB}}\right] + \frac{T_B T_S}{Q_L Q_T}\right\} + s\left\{\frac{T_S}{Q_T} + \frac{T_B}{Q_L}\right\} + 1} \qquad -(19)$$

where $$T_B = \frac{1}{\omega_B} = \frac{1}{2\pi f_B} \qquad -(20)$$

The terms of these polynomials can then be matched against those of desired polynomials, Butterworth, Chebyshev, Bessel, another that the author chose to call quasi-Butterworth 3rd order or whatever might be useful for the application.

More usually though, those ideal values serve only as a guide. Most often, the polynomials are calculated to see what can be achieved with available components under the inevitable constraints of cost, appearance and fashion, even superstition, that beset commercial designers.

Experience using eqns (17) and (19) soon showed why loudspeaker design had previously been such a chancy undertaking. It was now clear how a variety of flat, near-flat or deliberately emphasised responses could be designed as required, but each needed its own group, constellation, or what came to be called an alignment of parameters. If a suitable alignment was used, a design could be spectacularly successful. If an unsuitable alignment was chosen the result could be, and often was, a spectacular failure.

Fig 6 demonstrates a variety of closed box alignments that all produce Butterworth responses at different cut-off frequencies. They also demonstrate that the cut-off frequency with a closed box cannot be lower than the driver resonance. Table 1 and Fig 8 show that a suitable vented box alignment can extend the response at least half an octave below the driver resonance. With an even larger box, a Chebyshev response could still be maintained down to even lower frequencies, but then the response ripple increases to an extent that is unacceptable for audio applications.

Fig 10 demonstrates how, even though the parameters are selected with the best of intentions, a driver selected with a large expensive magnet that gives it a low Q_E, in a box so large that its ratio V_{AS}/V_B is small, and tuned much lower than the driver resonance, all in an effort to produce a strong bass, it may well produce a response that does extend to a low frequency but in a quite unusable manner.

On the other hand, when a driver has had a lot of use, its spider and surround often loosen up, become more compliant. More rarely they may stiffen, become less compliant. Surely either of those possibilities must affect the response of the system ? In fact the response does change, but if it is *only* the compliance that changes, then f_S, Q_T and V_{AS} all change together in a way that changes the frequency response very little. Fig 7 shows how little the response in a closed box, that was initially 2^{nd} order Butterworth, changes with a great change of compliance, when it first doubles and then halves. Fig 9 shows how little the response in a vented box, that was initially 4^{th} order Butterworth, changes with similar changes of compliance.

7 IMPEDANCE PEAKS OF EQUAL HEIGHT

In the 50's, there was a widely held belief, shared initially and embarrassingly in retrospect by the author, that in a properly designed vented box, the peaks of impedance exhibited by the driver should be of equal height.

That was fifty or more years ago, but it was rather dismaying to find recently that a number of designers still believe so. One even averred that a customer had rejected a design that failed to comply with that requirement.

The truth is, of course, that peaks of equal height simply indicate that f_B the resonant frequency of the box is identical with f_S the resonant frequency of the driver. That could indicate that the transfer function of the whole system was a 4^{th} order Butterworth but, even then, only if Q_T the total Q and the compliance ratio V_{AS}/V_B were suitable, as in the middle diagram of Fig 11.

When the higher frequency peak has the lower amplitude, the box is tuned higher than the driver, a perfectly satisfactory condition for a smallish box, e.g. with a "quasi-Butterworth 3^{rd} order" response, but again, only so long as Q_T and f_B are suitably matched to it. When the higher frequency peak has the higher amplitude, the box is tuned lower than the driver, it may indicate a perfectly satisfactory Chebyshev response, so long as the box is large, i.e. V_{AS}/V_B is small, and Q_T and f_B are suitably matched.

If the box f_B is tuned lower than $0.7f_S$, the ripple could be excessive, and all Chebyshev responses exhibit a steep rise in cone excursion below f_B but, so long as the designer had taken these factors into account, a design with a higher amplitude high frequency peak impedance could well be satisfactory.

When f_B is very low, e.g. with the leaks in the box that were described earlier, the amplitude of lower peak may become so small as to be invisible, signalling to the unwary that it is a closed box, its presence detectable only by an apparently anomalous zero crossing in the plot of its phase angle.

8 THE VOICE COIL "INDUCTANCE"

The exact nature of the voice coil inductance still remains a challenge, to this author at least. It was clear from the earliest days that it is not a pure inductance, but it seemed then that it could be approximated sufficiently well by a resistance shunting an inductance. But when he proposed a complementary network to shunt the driver, incorporating a dual of the driver impedance so as to present a pure resistance that would correctly terminate a passive crossover filter[20] , it became clear, when Small checked it, that the simple model was inadequate.

Small's interest was taken up by his student Dash[15] whose model, taking into account skin effect in the centre pole, proposed an inductive reactance that varied with the square root of frequency, together with a resistance of equal magnitude. That model was taken further by Vanderkooy[16], who dubbed this special component a "semi-inductance". However, it must be remembered that this semi-inductance is just one component of a more complicated impedance. Further work by Wright[17] and Thorburg and Unruh[18] has offered refinements, but their models include components varying with a power of frequency that is peculiar to each driver. The author is still searching for a model that more satisfactorily satisfies the physics. Quite possibly the mechanism is too complex to fathom, or be worth fathoming. For the moment, he prefers to quote the value of each inductance L_E along with the frequency at which it is measured.

One useful figure is the value at the frequency f_Z where the driver impedance goes through a zero phase angle due to series resonance between the inductance and the capacitive component of the motional impedance

$$L_E(\mu H) = \frac{10^6 R_E}{2\pi f_{SA} Q_{EA}\left[\left(\frac{f_Z}{f_{SA}}\right)^2 - 1\right]} \qquad -(21)$$

Thus the figure above should be described as "L_E at f_Z".

The minimum impedance R_{EZ} of a driver near f_Z, usually taken as a guide to its rated impedance, is greater than its d.c. resistance R_E by an increment of 10% to 20% that is largely the resistive component of the voice coil "inductance". The motional impedance at f_Z also contributes another but smaller component.

The time constant $\mathbf{T_E = L_E/R_E}$ **(μs)** at f_Z allows useful comparisons to be made between drivers with different values of R_E.

9 SENSITIVITY OF THE DRIVER

During the initial development of the parameters, it was found that the efficiency η of a driver expressed as Acoustic Watts out / Electrical Watts in could be calculated from the other parameters as

$$\eta = \frac{487 f_{SB}^3 V_{AS} R_E}{10^{12} Q_{EB} R_Z} \qquad -(22)$$

where V_{AS} is in litres (dm^3), initially with the R_E and R_Z terms omitted. At first sight, such a derivation seemed incredible, as it arose out of a series of only electrical measurements. Eventually though, it was realized that it *did* include one acoustical measurement, of V_{AS} that depended on the acoustical compliance V_B of the measuring box.

Later on, dissatisfaction with the use of the Bl product, with the dimensions of Newtons/Ampere, as a force factor that takes no account of the different resistances of different voice coils, led to the derivation of two separate force factors[19], firstly F_{ME}, a "mechano-electrical" force factor

$$F_{ME} = \frac{Bl}{\sqrt{R_E}} = \frac{149 S_D}{\sqrt{f_{SB} V_{AS} Q_{EB}}} \sqrt{\frac{R_E}{R_Z}} \qquad - (23)$$

where S_D is the area of the diaphragm. This factor, with the dimensions of Newtons/Watts^{½} (electrical input), expresses the effectiveness of the motor, initially with the R_E and R_Z terms omitted. Secondly an "acousto-mechanical force factor

$$F_{AM} = \frac{0.00595 f_{SB}^2 V_{AS}}{d_C^2} \qquad - (24)$$

with the dimensions of Watts^{½} (acoustic output) / Newton, expresses the effectiveness of the acoustic radiating system.

Their product expresses the overall efficiency

$$\eta = (F_{ME} F_{AM})^2 \qquad - (25)$$

Some time after the publication of these factors, it was learned that some British engineers already used the square of F_{ME} for an expression that they called "Thrust".

$$F_{ME}^2 = \frac{(Bl)^2}{R_E} = Thrust \qquad - (26\ )$$

The term "electrical watts input" needs a little clarification. The electrical input impedance varies so greatly with frequency that any estimate of the power that it absorbs from an electrical input must be difficult, not to say pointless. For this reason, Beranek[3] proposed that the input power be taken as the input power *that would have been absorbed* by a pure resistance that represented the driver. For eqns (22),(23) and (26) this was taken initially as the d.c. resistance $R_{E.}$.

Now a power of 1 acoustic watt, radiated isotropically into half-space (2π steradians), as from a driver mounted in an infinite baffle, produces a sound pressure level SPL at 1 metre distance of 112.1 dB above reference level, so it becomes more convenient to express the sensitivity of a driver as

$$dB\ SPL, 1W, 1m = 112.1\ d\mathbf{B} + \eta\,(dB) \qquad - (27)$$

Thus a driver with an η of 1% is rated as having either an efficiency η of -20.0 dB, or alternatively, an output SPL at 1 metre distance, with 1 Watt input, of +92.1 dB.

Even more conveniently, the sensitivity may be specified as the **SPL** at **1 metre** resulting from an input voltage of **2.83 Volts**, the voltage that produces 1 Watt power in a resistance of 8 ohms. Such a specification obviates the need, in design calculations, to consider the value of the series voice coil resistance R_Z.

It also rates a driver with a low voice coil resistance as more sensitive because it draws more power from a transistor amplifier, which is a very low output impedance source of voltage. By this criterion then, which accords with practice, a driver with a series resistance of 4 ohms is rated 3.0 dB more sensitive than if its resistance had been 8 ohms with nothing else changed, and we derive the expression

$$dB\ SPL,\ 2.83\ V,\ 1m = 112.1\ d\mathbf{B} + \eta\ (dB) + 10\log_{10}\frac{8}{R_Z} \qquad -(28)$$

Recently though, Peter Larsen informed the author, over a most hospitable dinner, that the sensitivities that he had calculated for drivers using the original eqn (28) were always "1 or 2 dB more optimistic" than the figures that he had measured. It soon became apparent that the appropriate reference impedance was nor the d.c. resistance R_E but R_Z the resistance at f_Z where sensitivity figures are universally measured. Hence the use of R_Z in eqn (28) and the paragraphs above and the addition of R_E and R_Z to eqns (22) and (23).

10 RELATIONSHIP OF A DRIVER WITH A PASSIVE CROSSOVER

A two-way loudspeaker system requires a woofer, a tweeter and a pair of filters, high-pass and low-pass, in the hope that when their individual acoustic outputs sum together, the amplitude response will be ideally flat.

In spite of the advantages of active systems, the great majority of loudspeakers are designed as stand-alone devices, each an entity in itself with a flat response, or one that sounds even better than flat to the designer, when it is fed by any amplifier, so long as that amplifier has a low output impedance, and so long as the connecting cables are of sufficiently low resistance, though not necessarily monstrous.

The crossover filters then are passive, requiring fairly large inductors and capacitors, especially now that voice coil resistances are lower so as to extract more power from transistor amplifiers. The design procedures for passive crossovers that realize Butterworth, Linkwitz-Riley or notched NTM transfer functions are well-known. All are designed to feed a purely resistive impedance at their outputs. But a loudspeaker presents anything but a resistive impedance.

To handle this problem, its impedance can be characterized as a pure resistance R_E, the d.c. resistance of its voice coil, in series with an impedance that we can generalise as Z_1. If such an impedance $R_E + Z_1$ is shunted with another impedance $R_E + R_E^2/Z_1$, the combination presents a resistance R_E. The impedance R_E^2/Z_1 is the dual of Z_1, in which resistances R_X in Z_1 become R_E^2/R_X, inductances L_X become capacitances L_X/R_E^2, capacitances C_X become inductances $C_XR_E^2$, circuit elements in series become their duals in parallel and circuit elements in parallel become their duals in series, as in Fig 12(a). In general, if Z_B is the dual of Z_A with respect to R_E , i.e if $Z_B = R_E^2/Z_A$, then also Z_A is the dual of Z_B with respect to R_E .

Such a shunt compensation network[20] is easily devised for a driver, see Figs 12(b), 12(c) and 13, so that it terminates a passive crossover filter resistively. All its elements are calculable from the driver parameters, except for those that compensate for the voice coil inductance, a resistor and a capacitor in series, or two such elements at most. The most accurate compensation is needed around the crossover frequency, which is where the filter networks are most sensitive to mis-termination and where also, with crossovers so often in the region of 2 kHz to 4 kHz, the ear is most sensitive to errors in frequency response.

Lack of care in this aspect of design, or for the time alignment of drivers with each other or consideration of the tweeter response as part of the crossover's high-pass transfer function, or the easily calculable phase rotation of the acoustic output of tweeters, even when their amplitude response is close to flat, led an earlier generation of designers to depend heavily on cut and try selection of component values, sometimes even reversing the sense of tweeter connections, in efforts to make a poorly conceived crossover a little less imperfect. With the great number of variables in realizing a crossover, some tweaking is still sometimes necessary, but when all the aspects discussed above are covered, a satisfactory result is more easily possible.

11 INCORPORATING A DRIVER INTO A PASSIVE CROSSOVER

We discussed earlier how the 2nd order high-pass response of a closed back loudspeaker, e.g. a tweeter, could be constrained into a desired 3rd order function. It can likewise be constrained into a 5th order high-pass function that can incorporate the native amplitude response and impedance characteristics of a loudspeaker without further need of impedance compensation.

Suitable choice of the 5th order function allows it to be teamed up with a 3rd order low-pass function in an asymmetrical crossover[21,22,23] that adds nearly in-phase, with only a tiny quadrature component, so that its insensitivity to errors in time alignment is similar to the Linkwitz-Riley crossover[24] . That particular asymmetrical crossover leads in fact to a near-paradox that the one low-pass Butterworth function can be used in crossovers to produce true all-pass summed outputs with two different high-pass functions.

One, a 3rd order Butterworth adds, preferably out-of-phase, in quadrature. The other takes the same 3rd order function and convolves it with a 2nd order Butterworth whose cut-off frequency is half an octave higher and adds nearly in-phase at the crossover frequency.

LOW-PASS HIGH-PASS ALL-PASS SUM

$$F(s) = \frac{1}{1+2sT+2s^2T^2+s^3T^3} - \frac{s^3T^3}{1+2sT+2s^2T^2+s^3T^3} = \frac{1-sT}{1+sT} \qquad - (29)$$

OR

$$F(s) = \frac{1}{1+2sT+2s^2T^2+s^3T^3} + \frac{s^5T^5}{(1+2sT+2s^2T^2+s^3T^3)(2+2sT+s^2T^2)}$$
$$= \frac{2-2sT+s^2T^2}{2+2sT+s^2T^2} \qquad - (30)$$

The crossover frequency is then constrained by the resonance frequency of the tweeter, and when this is the typical figure of 1500 Hz, the nominal crossover frequency (1/2πT) of the crossover of eqn (30) will be either 1061 Hz or 1500 Hz, depending on which of the two 2nd order factors of the high-pass denominator is assigned to the tweeter, and the crossover frequency where the low- and high-pass outputs are equal is either 1270 Hz or 1796 Hz. With the tweeter embedded in the crossover in the manner of ref. 22, the actual crossover frequency would be around 1920 Hz depending on the ratio of Q_M to Q_E. Even that last frequency is rather lower than many designers would prefer, from considerations of power handling and excursion, but on the other hand the lower the crossover frequency the less are the effects of time alignment[24] or the audible artefacts of response errors.

Above all, though, this procedure enables the native response and impedance of a tweeter, so long as its resonance lies within a suitable frequency range, to become not a problem but part of an economical solution.

12 CONCLUSION

Since the first appearance in 1961 of a complete set of loudspeaker parameters with guidance for their use, many workers have made valuable further contributions to their measurement and application. This brief and highly selective account has tried to describe some of the most important.

Use of these "small signal' parameters assumes that a loudspeaker is a linear device, an approximation that is completely valid only at the lower signal levels, though how low that level extends varies greatly with the quality, and sometimes the expense, of each design.

The study of driver non-linearities is another field altogether that Klippel[25], in particular, has pursued with great skill and energy.

It must be remembered that for much of the time the instantaneous level of an audio signal is comparatively low – broadcasters and recording engineers allow for peaks up to 20 dB above the mean, Alignment, level - so the "small signal" parameters remain an essential feature of loudspeaker design.

13 ACKNOWLEDGMENT

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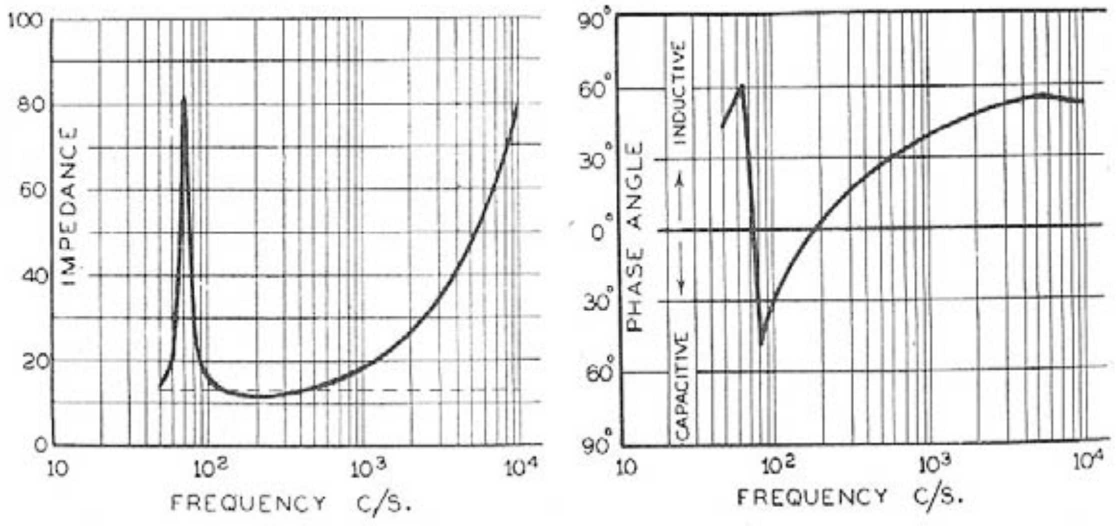

Fig. 1 from Chapter 3 – The Relationship between the Power Output Stage and the Loudspeaker
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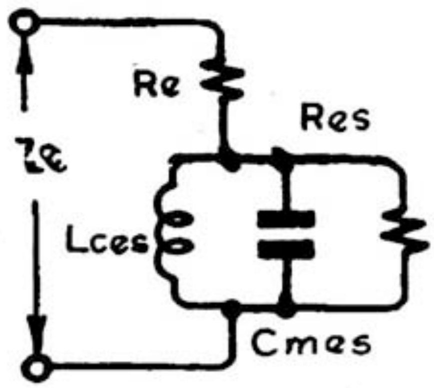


Fig 2. Simplified Model of Driver Impedance for Derivation of Parameters
L_{CES}, R_{ES} and C_{ES} are the electrical components of the motional impedance due respectively to the acoustical compliance, resistance and mass of the driver.
R_E is the d.c. resistance of he voice coil

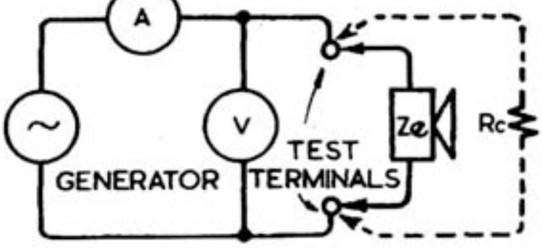


Fig 3. Test Equipment for Parameter Measurement

Voltmeter V, whose Impedance is much higher than the driver, e.g. 20Kohms, serves only for observation that the testing voltage remains constant.
If a reading is taken with calibrated resistance R_C substituted, then the absolute calibration of ammeter A is not necessary.
Generator includes a frequency counter, preferably with 0.1 Hz precision

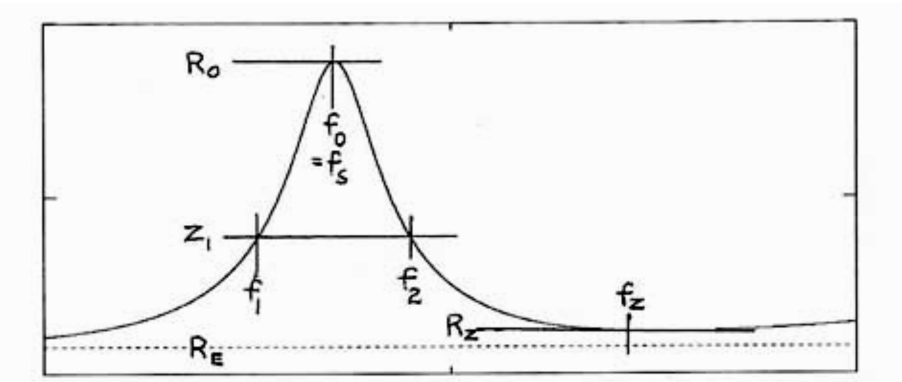


Fig 4. Readings of driver taken unbaffled (In Air) and in a closed box

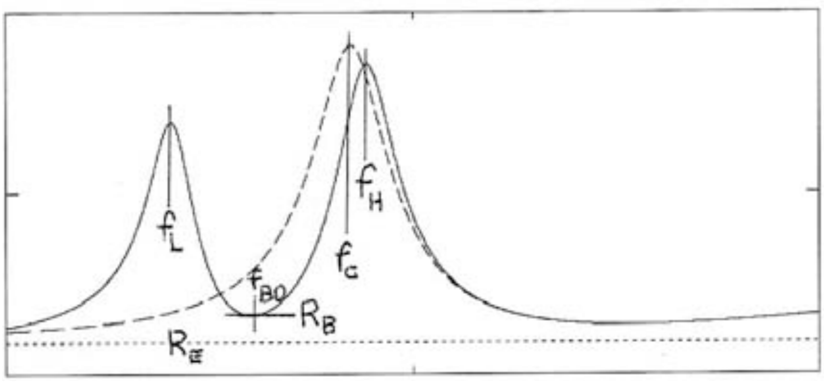


Fig 5. Readings taken in vented box – solid curve
When vent is closed up, f_C is read on the dashed curve
f_{BO} is the frequency of minimum impedance a little higher than the true box resonance f_B

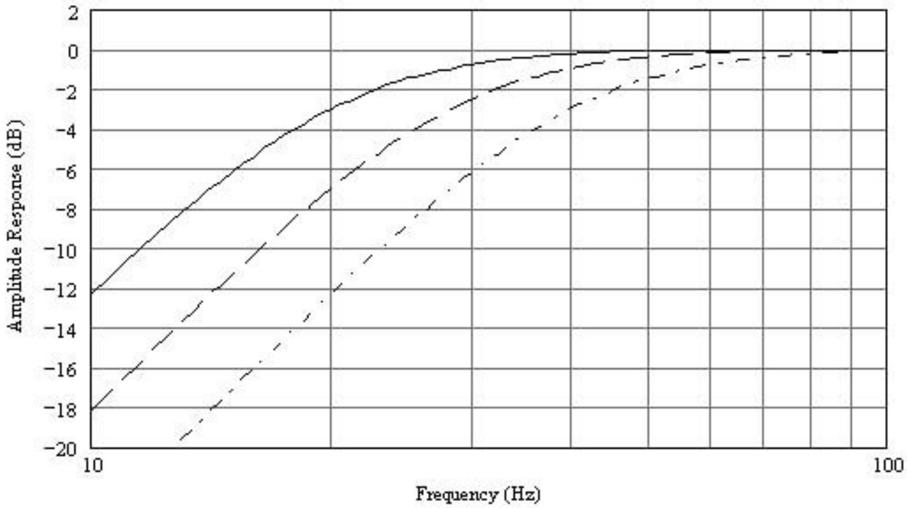


Fig 6. Amplitude Responses of a Closed Box Loudspeaker - 2nd Order Butterworth Alignments
solid curve : f_S = 20 Hz : Q_T = 0.707 : $V_{AS}/V_B \rightarrow \infty$, i.e. box is very large.
dashed curve: f_S = 20 Hz : Q_T = 0.500 : V_{AS}/V_B = 1 , medium sized box
dash-dot curve: f_S = 20 Hz : Q_T = 0.354 : V_{AS}/V_B = 3 , small box

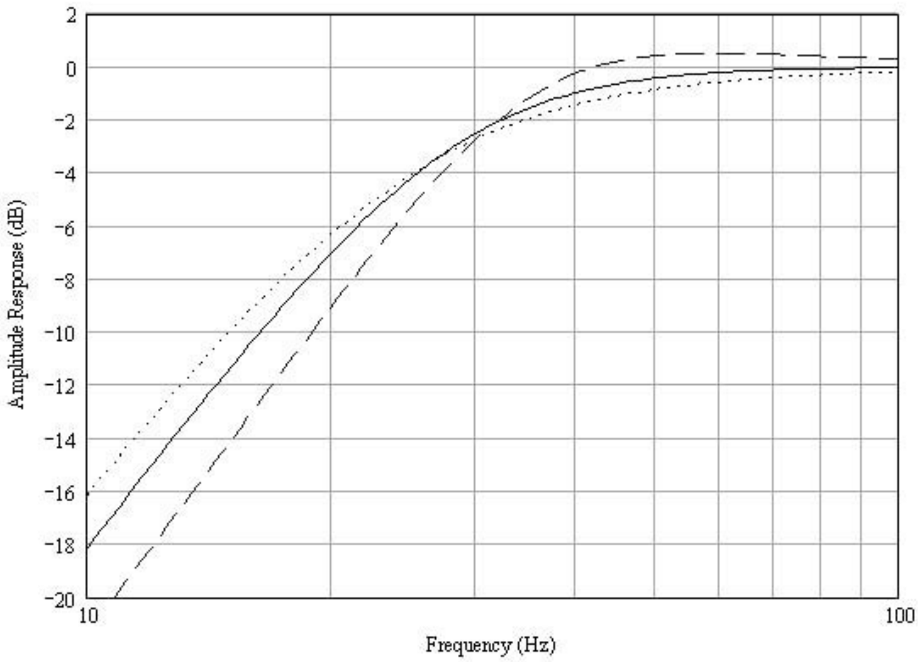


Fig 7. Response Change in a Closed Box when only the Driver Compliance Changes
solid curve - initial 2nd Order Butterworth response
dotted curve - when the compliance is doubled
dashed curve - when the compliance is halved

Table 1. SUMMARY OF LOUDSPEAKER ALIGNMENTS
adapted from Table1, Loudspeakers in Vented Boxes

No.	*Type*	*Ripple (dB)*	f_3/f_S	f_B/f_S	C_{AS}/C_{AB}	Q_T	f_{AUX}/f_S
1	QB3	-	2.68	2.00	10.48	0.180	-
2	QB3	-	2.28	1.73	7.48	0.209	-
3	QB3	-	1.77	1.42	4.46	0.259	-
4	QB3	-	1.45	1.23	2.95	0.303	-
5	**BW4**	**-**	**1.000**	**1.000**	**1.414**	**0.383**	**-**
6	CH4	-	0.852	0.927	1.055	0.415	-
7	CH4	0.07	0.724	0.829	0.729	0.466	-
8	CH4	0.25	0.704	0.757	0.559	0.518	-
9	CH4	0.51	0.685	0.716	0.485	0.557	-
10	**BW5**	**-**	**1.000**	**1.000**	**1.000**	**0.447**	**1.00**
11	CH5	-	0.850	0.912	0.583	0.545	1.22
12	CH5	0.25	0.698	0.814	0.273	0.810	1.81
13	CH5	0.5	0.620	0.798	0.227	0.924	2.06
14	CH5	1.0	0.554	0.781	0.191	1.102	2.47

QB3 ≡ Quasi-Butterworth 3rd order
BW4 ≡ Butterworth 4th order, maximally-flat amplitude response
CH4 ≡ Chebyshev 4th order, equal-ripple amplitude response
BW5 ≡ Butterworth 5th order, maximally-flat amplitude response
CH5 ≡ Chebyshev 5th order, equal-ripple amplitude response

For the 5th order responses, Nos. 10 – 14, the capacitance C_X coupling to R_X, the input impedance of the driving amplifier, is chosen so that the time constant $C_XR_X = 1/2\pi f_{AUX}$

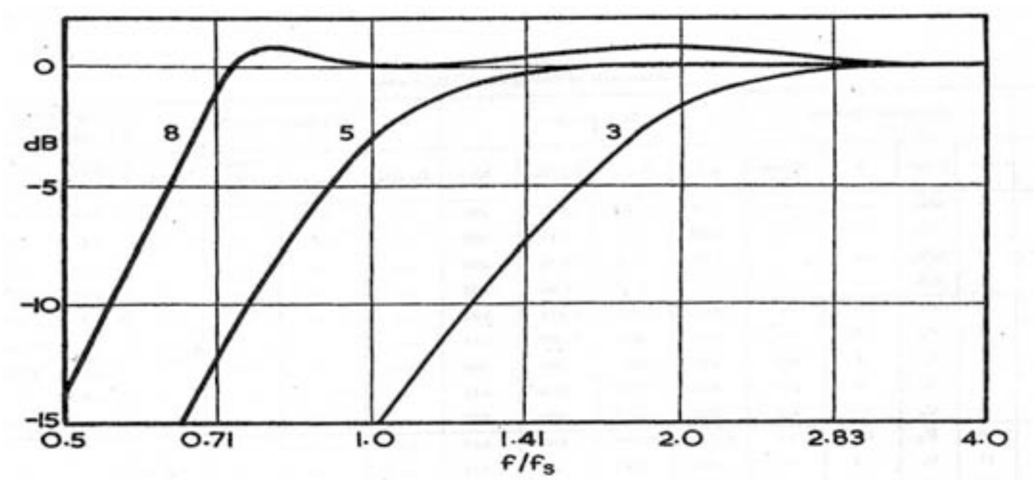


Fig 8 Amplitude Responses of a Vented Box Loudspeaker, with alignments numbered as in Table 1

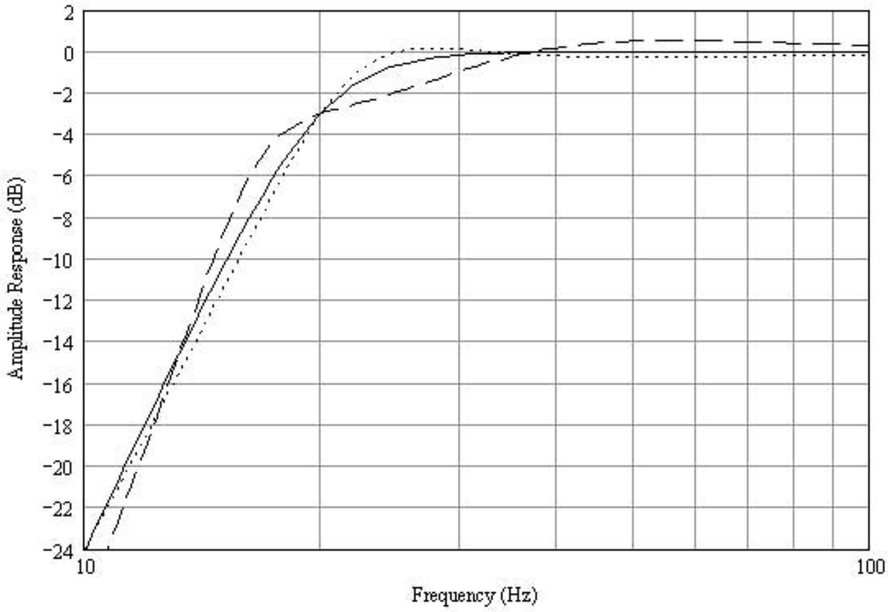


Fig 9. Response Change in a Vented Box when only the Driver Compliance Changes
solid curve - initial 4th Order Butterworth response
dotted curve - when compliance is doubled
dashed curve - when compliance is halved

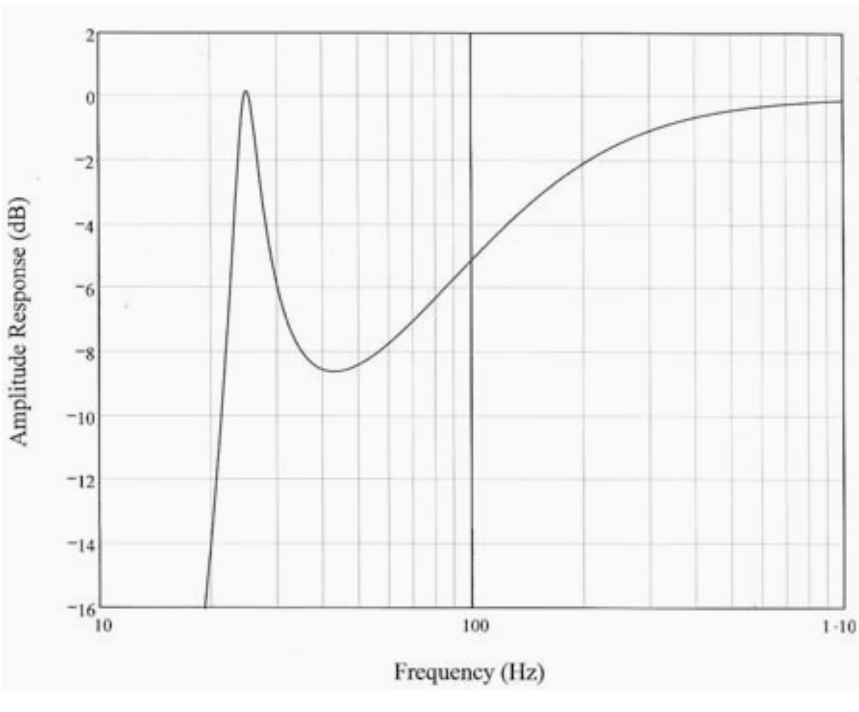


Fig 10 A Big Driver in a Big Vented Box Tuned Low **Must** Make Big Bass **?**
F_S = 35 Hz: F_B = 25 Hz: Q_T = 0.2 : V_{AS}/V_B = 0.5
What can happen when the alignment chosen is not near optimum

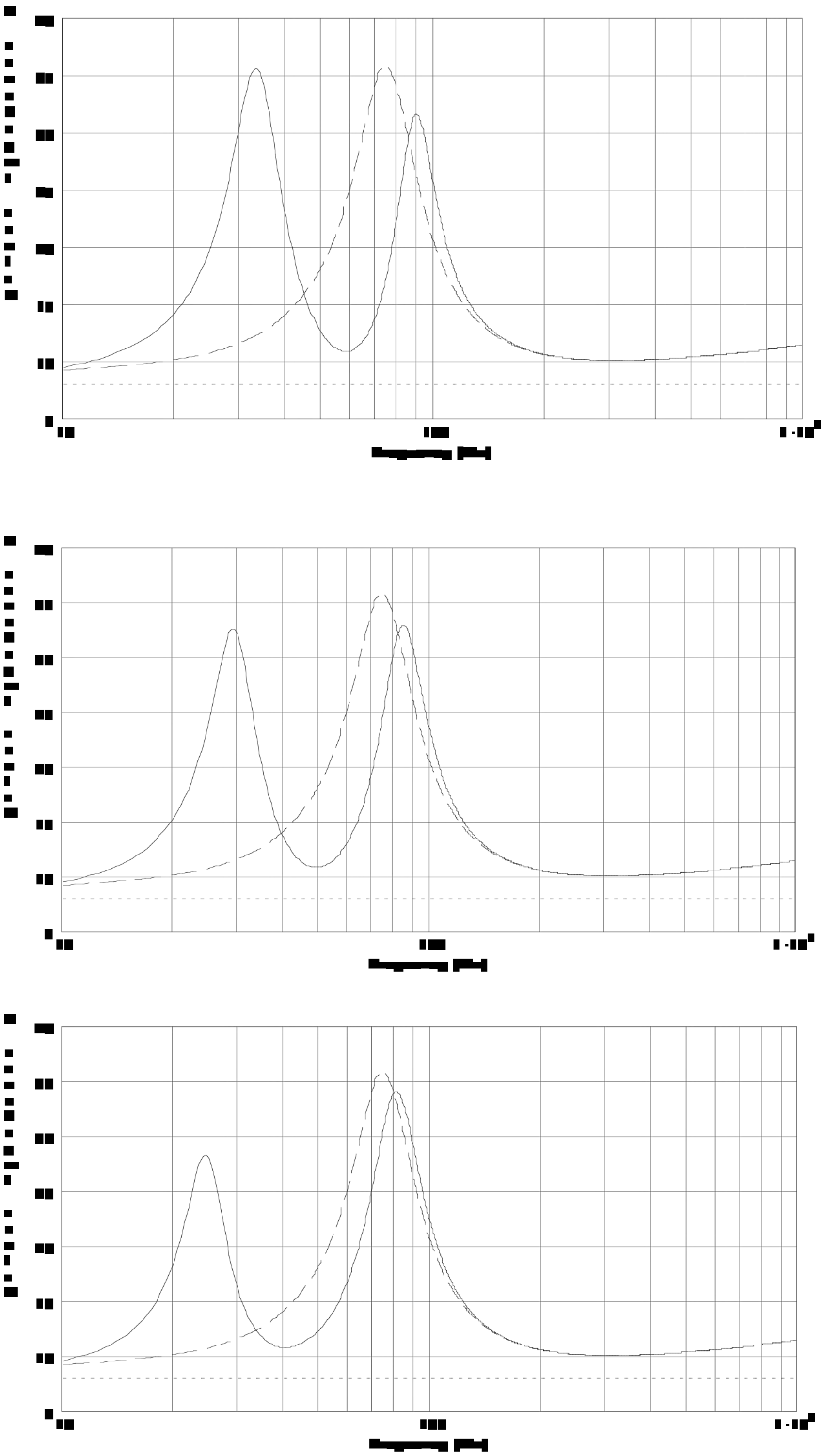
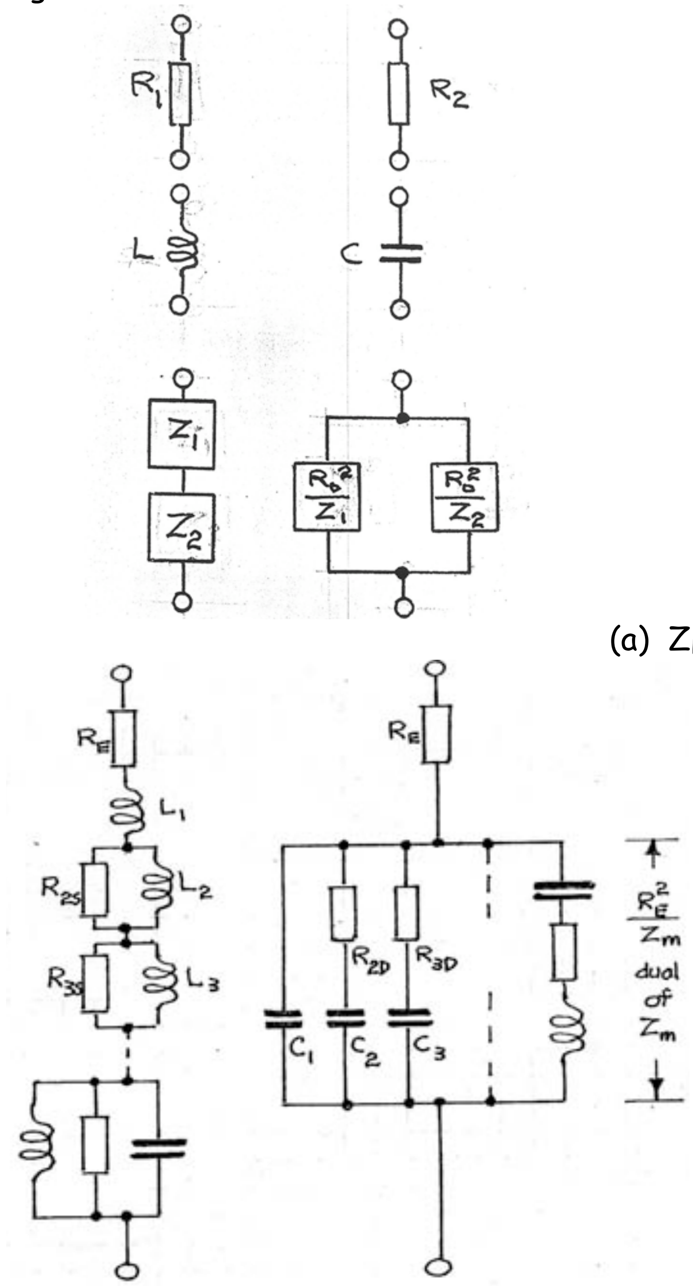


Fig. 11. Change in Symmetry of Impedance Curves as f_B Changes:
Q_T = 0.383 : f_S = 50 Hz : top - f_B = 60 Hz : mid - f_B = 50 Hz : bottom - f_B = 40 Hz

Original - Z_A Dual - Z_B



(a) $Z_B = R_O^2/Z_A$

(b) Approx. Model of Driver **(c) Impedance Equalizer**

Fig 12. Driver Impedance Equalizer

All elements of (c) are duals with respect to R_E of the corresponding elements of (b)
When the two are connected in parallel, they present a pure resistance R_E

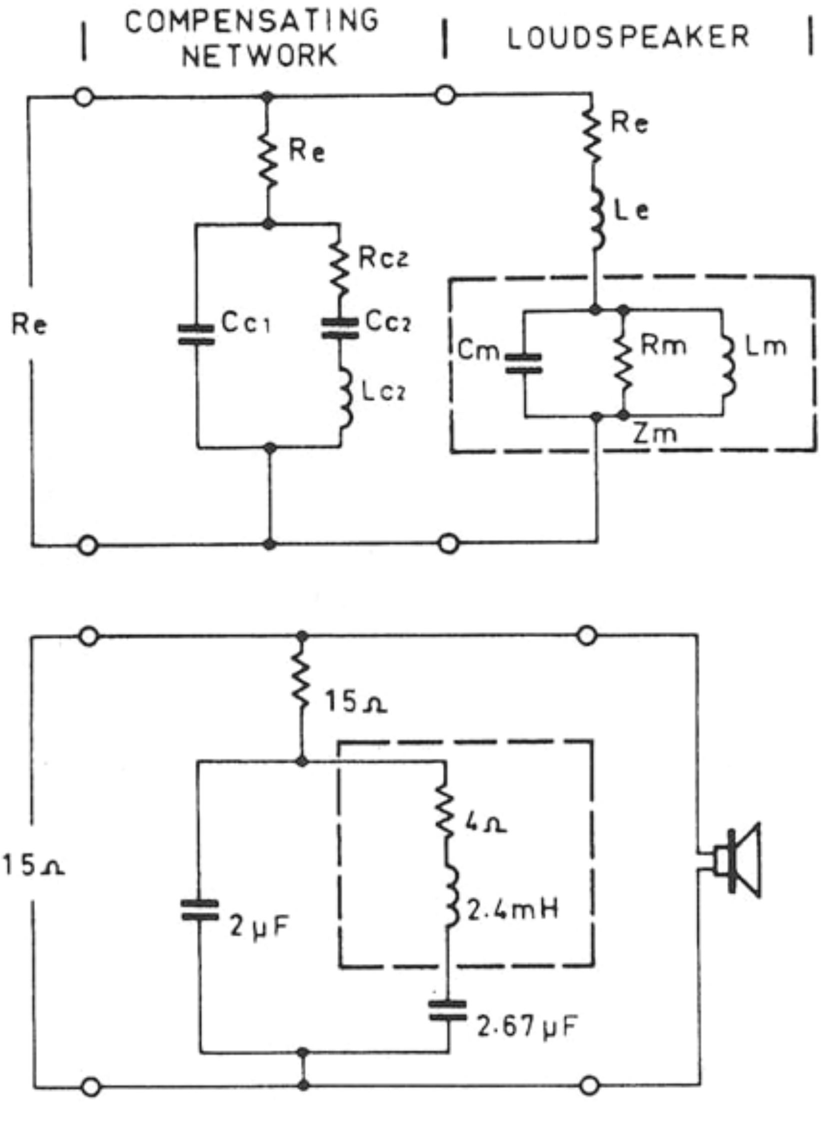


Fig 13. Loudspeaker with Impedance Equalizer – from ref.. 20
Within the dashed box, the 4 0hm component is realised in the winding resistance of the 2.4 mH inductance, thus minimising the mass of its wire

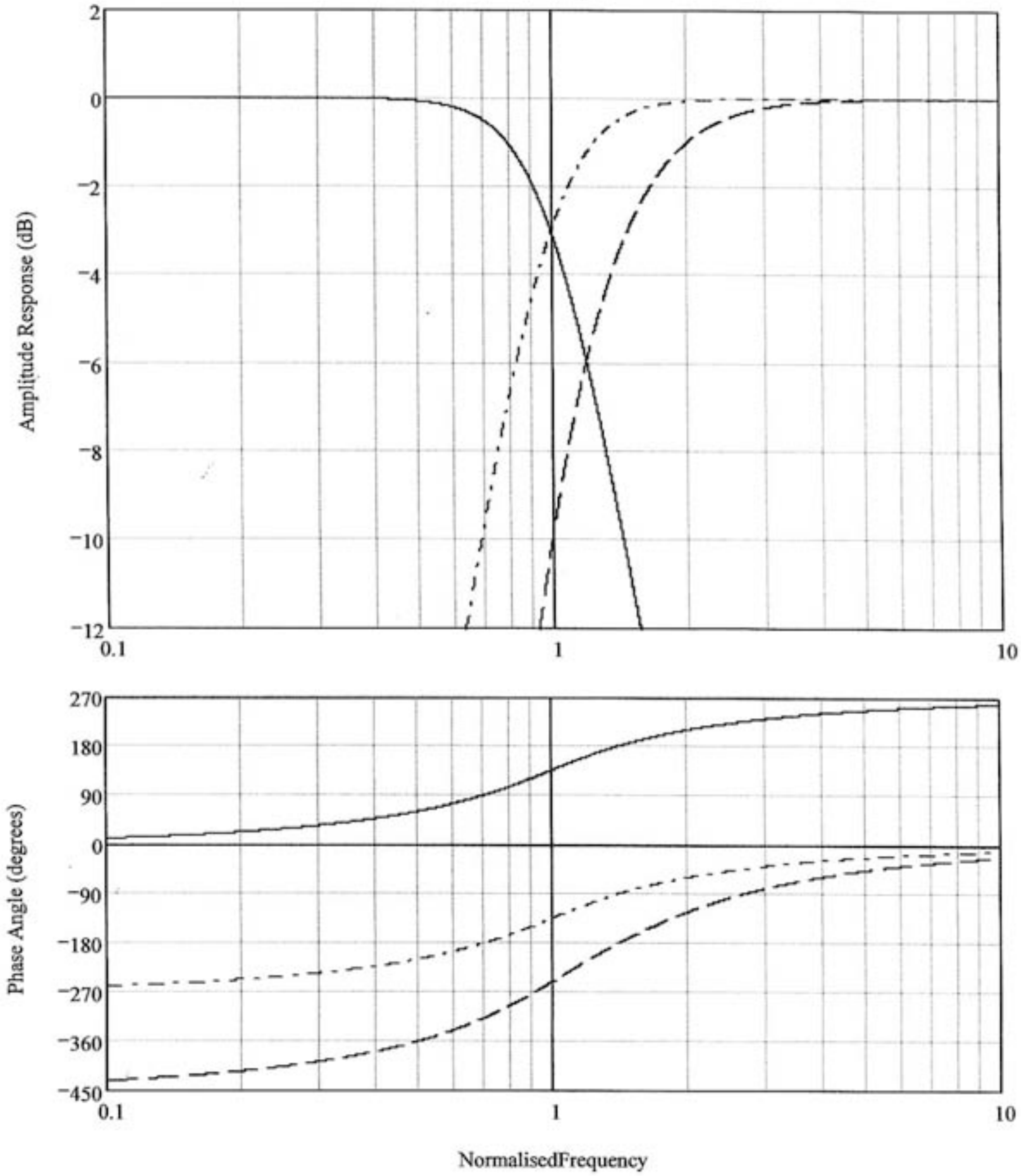


Fig 14 Amplitude and Phase Responses of Two All-Pass Crossovers
(a) solid curves – 3rd order Butterworth low-pass responses, common to both
(b) dash-dot curves - 3rd order Butterworth high-pass responses, as in eqn (29)
(c) dashed curves - 5th order high-pass response as in eqn (30)
Note that, at their crossover points of equal amplitude response,
the phases of (a) and (b) are 270º apart, so that they add in quadrature,
while (a) and (c) are 373º (≡ 13º) apart, so that they add nearly in-phase.