SPATIAL TEXTURE ANALYSIS OF SAR DATA BY ANISOTROPIC GAUSSIAN KERNELS

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1 INTRODUCTION

Data acquired by SAR systems are directly related to the physical properties of the natural media. However, the presence of speckle, which is usually modelled as a multiplicative noise, is an obstacle for the image interpretation and understanding. Generally, on intensity images, an area is characterized by its mean reflectivity, which is computed by spatially averaging pixels. This statistic represents well homogeneous areas, but is now well established that a large amount of information can also be retrieved by studying the spatial fluctuation of the mean reflectivity within an area. This type of fluctuation is usually referred as the *texture* of the scene and is generally characterized by the *two-point* statistics, *i.e.* the auto-covariance, the auto-correlation function (ACF) and power spectrum density (PSD). Extensive studies based on two-point statistics of SAR images have been pursued in the past and generally, stationary isotropic models are considered. It has been previously shown[1,2] that a nonstationary anisotropic model called *Anisotropic Gaussian Kernel* (AGK), designed for the local auto-covariance analysis of texture, allowed a more accurate description of the image.

In this paper, we present two methods for the estimation of the AGK parameters. The first algorithm is based on the direct computation of the ACF, and is an improvement of the method developped in[1]. Such a manipulation is computationally expensive since the entire ACF has to be computed for each location of the image. In a second step, we present an alternative description of the image in terms of local orientation, based on an operator called *Gradient Structure Tensor* (GST)[3] which is widely used in computer vision. We also establish the mathematical relation between the parameters of our model and the GST, in a first step for a noise-free texture, then for a speckled texture. This relation is highly non-linear, thus we propose a scale approximation that permits our approach to be conveniently applied on real SAR data. Finally, both methods are validated and compared on SAR data.

2 THE ANISOTROPIC GAUSSIAN KERNEL MODEL

In the context of texture analysis only single-look data have been regarded, since spatial averaging tends to damage texture information. It is now well established that speckled intensity is well described by a multiplicative model, thus we adopt the following decomposition[4], maintaining that at 2-D spatial location $\mathrm{x} = [x, y]^T$, the intensity I can be written in the following form:

$$I(\mathrm{x}) = \mu_I(\mathrm{x})T(\mathrm{x})F(\mathrm{x}) \tag{1}$$

where μ_I is the local mean of terrain reflectivity, F is an uncorrelated random process modelling the speckle, with unitary mean $\mu_F = 1$ and variance $\sigma_F^2 = 1$ in the case of single-look data. The term T contains the information of interest about the texture of the scene and is modelled by a spatially correlated random field of unitary mean $\mu_T = 1$.

As multiple forms can be taken by autocorrelation and PSD, a simple parametric model is chosen for these descriptors, in order to generalize the notion of correlation length and frequency spread to the 2-D case. It has been shown in previous publications[1,2] that a well adapted model for this task is the anisotropic gaussian kernel (AGK) model, that assumes a locally stationary autocovariance with the form:

$$C_T(\mathrm{d}) = \sigma_T^2 exp\left(-\mathrm{d}^T \Sigma_T^{-1} \mathrm{d}\right) \quad (2)$$

where d is the 2-D spatial lag and Σ_T is the covariance matrix of the spatial coordinates *x* and *y*. This matrix can be decomposed in the form $\Sigma_T = \mathrm{R}_\theta^T \Lambda \mathrm{R}_\theta$ where

$$\Lambda = \begin{bmatrix} l_u^2 & 0 \\ 0 & l_v^2 \end{bmatrix} \quad (3)$$

is the covariance matrix expressed in its eigenvectors basis $[u,v]^T$. The values l_u and l_v are the principal and secondary correlation lengths in the directions given by the eigenvectors and

$$\mathrm{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (4)$$

is an unitary rotation with angle θ determining the dominant orientation of the texture. As the Fourier transform of a Gaussian function remains Gaussian and leaves rotations unchanged, the 2-D PSD has a very similar expression:

$$S_T(\mathrm{f}) = \sigma_T^2 \pi |\Sigma_T|^{1/2} exp\left(-\pi^2 \mathrm{f}^T \Sigma_T \mathrm{f}\right) + \mu_T^2 \delta(\mathrm{f}) \quad (5)$$

where $\mathrm{f} = [f_x, f_y]^T$ is the 2-D spatial frequency vector, the operator $|.|$ is the determinant and $\delta(\mathrm{f})$ is the Dirac delta distribution.

3 PARAMETER ESTIMATION

3.1 Direct estimation

Previously, a parameter estimation procedure based on the direct computation of the local sample autocovariance was proposed[1]. Here, we present an enhanced algorithm based on the original one, where a new correction step for the estimated correlations lengths is introduced.

3.1.1 Local autocovariance estimation

The most straightforward method to estimate the parameters of our model is based on the explicit computation of the local autocovariance on a $N \times N$ sliding window. As the autocovariance is assumed locally stationary, it is estimated by the well-known ergodic estimator of sample autocovariance for the discrete 2-D lag $[p,q]^T$:

$$\hat{C}_I(p,q) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} (I(i,j) - \hat{\mu}_I)(I(i+p, j+q) - \hat{\mu}_I) \quad (6)$$

where $\hat{\mu}_I$ is the sample mean.

The autocovariance C_T of texture is not directly available due to the model (1). As *T* and *F* are assumed to be statistically independent, the autocorrelations of *T* and *I* are linked by the relation:

$$R_I(p,q) = \mu_I^2 R_T(p,q) R_F(p,q). \quad (7)$$

The autocorrelation R_F is determined under the hypothesis of spatially uncorrelated speckle:

$$R_F(p,q) = \sigma_F^2 \delta(p,q) + \mu_F^2, \quad (8)$$

where $\delta(\mathrm{p,q})$ is the discrete Kronecker delta. Then, it can be easily shown that the relation between the autocovariances C_I and C_T is:

$$C_I(p,q) = \mu_I^2 \left[\delta(p,q)(\sigma_T^2 + 1) + C_T(p,q)\right]. \quad (9)$$

It can be observed that the speckle contribution affects only the (0,0) coefficient of C_I. Moreover, as $\sigma_T^2 = C_T(0,0)$, the simple following speckle correction on has to be applied to retrieve C_T from C_I:

$$C_T(0,0) = \frac{\left(C_I(0,0)/\mu_I^2\right) - 1}{2}. \tag{10}$$

3.1.2 Parameter estimation by geometrical moments

The previously introduced correction permits to retrieve texture statistics which are here described by the AGK model (2). To estimate the spatial parameters l_u, l_v and θ, the shape of the measured autocovariance C_T is analysed by means of the geometrical moments[5], where the expression of the raw moments is :

$$m_{pq} = \frac{\int_{(x,y)\in R^2} x^p y^q C_T(x,y)dxdy}{\int_{(x,y)\in R^2} C_T(x,y)dxdy} \tag{11}$$

and the central moments are expressed by :

$$\mu_{pq} = \frac{\int_{(x,y)\in R^2} (x - m_{10})^p (y - m_{01})^q C_T(x,y)dxdy}{\int_{(x,y)\in R^2} C_T(x,y)} \tag{12}$$

where integrals can be replaced by discrete sums for computation on the image. Based on these moments, the following dispersion matrix can be defined:

$$\mathrm{V} = \begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} \tag{13}$$

The eigenvalues λ_1 and λ_2 of V correspond to the spatial variances of the AGK autocovariance, *i.e.* $\lambda_1 = l_u^2/2$ and $\lambda_2 = l_v^2/2$ and the orientation angle θ is given by the eigenvector $\mathrm{k}_1 = [k_{1,x}, k_{1,y}]^T$ corresponding to the largest eigenvalue λ_1 since:

$$\theta = arctan\left(\frac{k_{1,y}}{k_{1,x}}\right). \tag{14}$$

3.1.3 Thresholding of the autocovariance

From a theoretical point of view, the autocovariance C_T should only be affected at coefficient $(0,0)$. In practice, the estimate is performed locally on a restricted number of samples, which provokes uncertainties on each coefficient of $\hat{C}_T$. These perturbations may have strong effects on the estimation of the geometrical moments. Therefore, a thresholding at e^{-1} of the sample autocovariance $\hat{C}_T$ is performed in order to stabilize the estimation and only the central lobe is used for the estimation of the geometrical moments. The estimated parameters are then a fraction of the real parameters, which can be evaluated by resolving the integrals:

$$\begin{aligned} I_{20}(l_u,l_v) &= \int_{\frac{x^2}{l_u^2}+\frac{y^2}{l_v^2}\le 1} x^2 exp\left[-\left(\frac{x^2}{l_u^2}+\frac{y^2}{l_v^2}\right)\right]dxdy, \\ I_{00}(l_u,l_v) &= \int_{\frac{x^2}{l_u^2}+\frac{y^2}{l_v^2}\le 1} exp\left[-\left(\frac{x^2}{l_u^2}+\frac{y^2}{l_v^2}\right)\right]dxdy \end{aligned} \tag{15}$$

and the symetrical I_{02} for the *y* direction moment. Based on these integrals, the relation between the estimated fraction $\tilde{l}_u$ and the real values l_u are:

$$\tilde{l}_u = \sqrt{\frac{I_{20}(l_u,l_v)}{I_{00}(l_u,l_v)}} = l_u\sqrt{\frac{(1-2e^{-1})}{2(1-e^{-1})}}. \tag{16}$$

and the same relation stands for $\tilde{l}_v$ and l_v.

3.2 Gradient Structure Tensor based estimation

As previously explained, the above algorithm is computationally expensive. We expose here a second approach of texture estimation based on the GST operator.

3.2.1 The Gradient Structure Tensor operator

The GST is a well-know operator in the field of computer vision and is defined in[3]. Here we adopt a different formulation of this operator, that allows the analysis of random fields :

$$\mathrm{J} = E\left[\nabla I \nabla I^T\right] \tag{17}$$

where $\nabla I = [\partial I / \partial x, \partial I / \partial y]^T$ is the 2-D gradient of the intensity. By construction, **J** is a 2×2 real symmetric matrix with diagonal terms J_{11}, J_{22} and off-diagonal terms $J_{12} = J_{21}$. By applying the power conservation theorem[6] the GST may be rewritten in the spectral domain as :

$$\mathrm{J} = 4\pi^2 \int_{R^2} \mathrm{f}\mathrm{f}^T S_I(\mathrm{f}) d\mathrm{f} \tag{18}$$

where S_I is the PSD of the intensity. Thus, we have shown that the structure tensor is proportional to the second order moment matrix of the PSD and is directly related to second-order statistics of the data.

3.2.2 Noise-free texture

In a first step, we consider the case of a noise-free texture, which corresponds to *I=T*. If the local autocovariances of the texture follows the AGK model given by (2). Inserting the PSD (5) into the equation (18) leads to the relation :

$$\mathrm{J} = 2\sigma_T^2 \Sigma_T^{-1}. \tag{19}$$

Therefore, eigenvalues λ_1 and λ_2 of the tensor are related to the correlation lengths l_u and l_v by :

$$\lambda_1 = \frac{2\sigma_T^2}{l_v^2}, \lambda_2 = \frac{2\sigma_T^2}{l_u^2} \tag{20}$$

where σ_T^2 can be estimated by sample variance. Besides, it can be shown[7] that the angle θ is given by the relation :

$$\theta = \frac{1}{2} arctan \frac{2J_{12}}{J_{11} - J_{22}}. \tag{21}$$

Thus, for such a texture, parameters can be estimated without the explicit computation of the autocovariance or PSD.

3.2.3 Multiplicative model

In the case of SAR intensity, texture is affected by speckle and described by the multiplicative model (1). Such a process is not differentiable and has to be smoothed before the estimation of gradient. The GST is thus computed on the image $I_\sigma = K_\sigma * I$ where K_σ is an isotropic gaussian kernel. The GST of the pre-smoothed image is then:

$$\mathrm{J} = E\left[\nabla I_\sigma \nabla I_\sigma^T\right]. \tag{22}$$

The power density spectrum of the pre-smoothed intensity is then given by the relation:

$$S_{I_\sigma} = S_I(\mathrm{f}) \mid K_\sigma(\mathrm{f}) \mid^2 \tag{23}$$

where $K_\sigma(\mathrm{f}) = exp(-2\pi^2\sigma^2\mathrm{f}^T\mathrm{f})$ is the 2-D frequency response of the isotropic gaussian pre-smoothing filter.

By computing the integral (18) for the PSD (23) and considering that, for the product model, the expression (7) leads to

$$S_I(\mathrm{f}) = \mu_I^2(S_T * S_F)(\mathrm{f}), \tag{24}$$

where the symbol * denotes the spatial convolution operator, it is thus possible to obtain the expression of **J** as a function of the anisotropic gaussian kernel model parameters l_u, l_v, θ and the scale parameter σ:

$$\mathrm{J} = \mu_I^2\left[2\sigma_T^2\sqrt{\frac{l_u^2 l_v^2}{(l_u^2+4\sigma^2)(l_v^2+4\sigma^2)}}\Sigma_S^{-1} + \frac{1+\sigma_T^2}{8\pi\sigma^4}\mathrm{I}_2\right] \tag{25}$$

with $\Sigma_S = \Sigma_T + 4\sigma^2\mathrm{I}_2$, where I_2 is the 2×2 identity matrix.

At this point, it may be observed that, due to the presence of speckle and the effect of pre-smoothing, the relation between the structure tensor and the model parameters is highly nonlinear. Thus, since the solution of such an equation has no close analytic form, numerical optimization is necessary to obtain the correlation lengths l_u and l_v from the structure tensor. Moreover, the observation of the GST eigenvalues as a function of these parameters as displayed on figure 1 permits to notice that the solution is not unique.

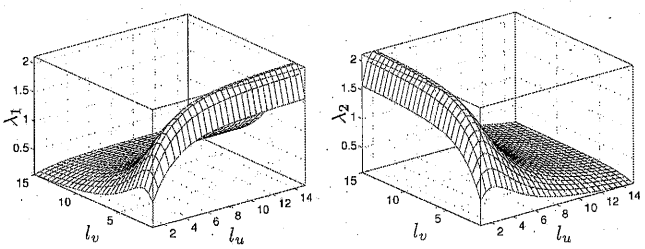

Figure 1: Eigenvalues of the structure tensor as a function of the parameters l_u and l_v for a speckled texture with pre-smoothing σ=1.2.

3.2.4 Scale approximation

To apply the GST estimation method on real data, we make the approximation that the spatial information of texture at scale σ is included in the smoothed signal I_σ, which is considered as a noise-free signal for a sufficient amount of smoothing (i.e. $T = I_\sigma$). The GST and the local variance $\sigma_{I_\sigma}^2 = \sigma_T^2$ are then computed on the filtered image on a *N*×*N* square window and the parameters l_u, l_v, θ at scale σ of the AGK model may then be estimated using the previously established relations (20) and (21).

4 VALIDATION ON SAR DATA

The two presented approaches have been validated and compared on the SAR intensity channel $I_{HH} =| S_{HH} |^2$, for a subarea extracted from the L-band *Trauntstein* dataset that was acquired by the ESAR sensor from the DLR (German Aerospace Center). The results are displayed on figures 2 and 3. The two methods have been applied with a 31×31 sliding window. The smoothing parameter for the GST method is σ=8. For a better visulisation of the results, the kernels are displayed for sampled positions with a step of 15 pixels. Moreover, for display purposes, four ranges of pixels have been determined by thresholding the coefficient of variation $CV_I = \sigma_I / \mu_I$. First, the median value of the coefficient of variation has been calculated, giving two ranges, containing each 50% of the points of the image. Then, another computation of the median on these two subsets permits to distinguish four ranges of coefficients of variation in the image, which are related to the power of T, the texture process. The ranges 1 and 2 (top row of figures 2 and 3) stand for the less heterogenous zones of the image, as the ranges 3 and 4 (bottom row of figures 2 and 3) stand for strongly textured areas. This representation permits us to observe the fact that spatial correlation can occur even for zones that are not stronlgy textured, although the most anisotropic behaviours are measured for ranges 3 and 4, *i.e.* zones whith strong variability. Since random heterogeneities may be responsible for the measurement of strong anisotropy (as it can be observed for the forested area), our method is also detecting deterministic structures like edges as they provoke steep changes in the structure of the signal. From a quantitative point of view, the first method gives an estimate of the underlying texture correlation lengths, whereas the second one provides only the values at scale σ for the filtered speckle/texture ensemble. Moreover, the filtering process used in the second method introduces correlation in the signal. This explains the differences of estimates between the two approaches. In particular, for ranges 1 and 2, one can observe more dissimilarities between the two methods due to the influence of this pre-smoothing in the GST-based method.

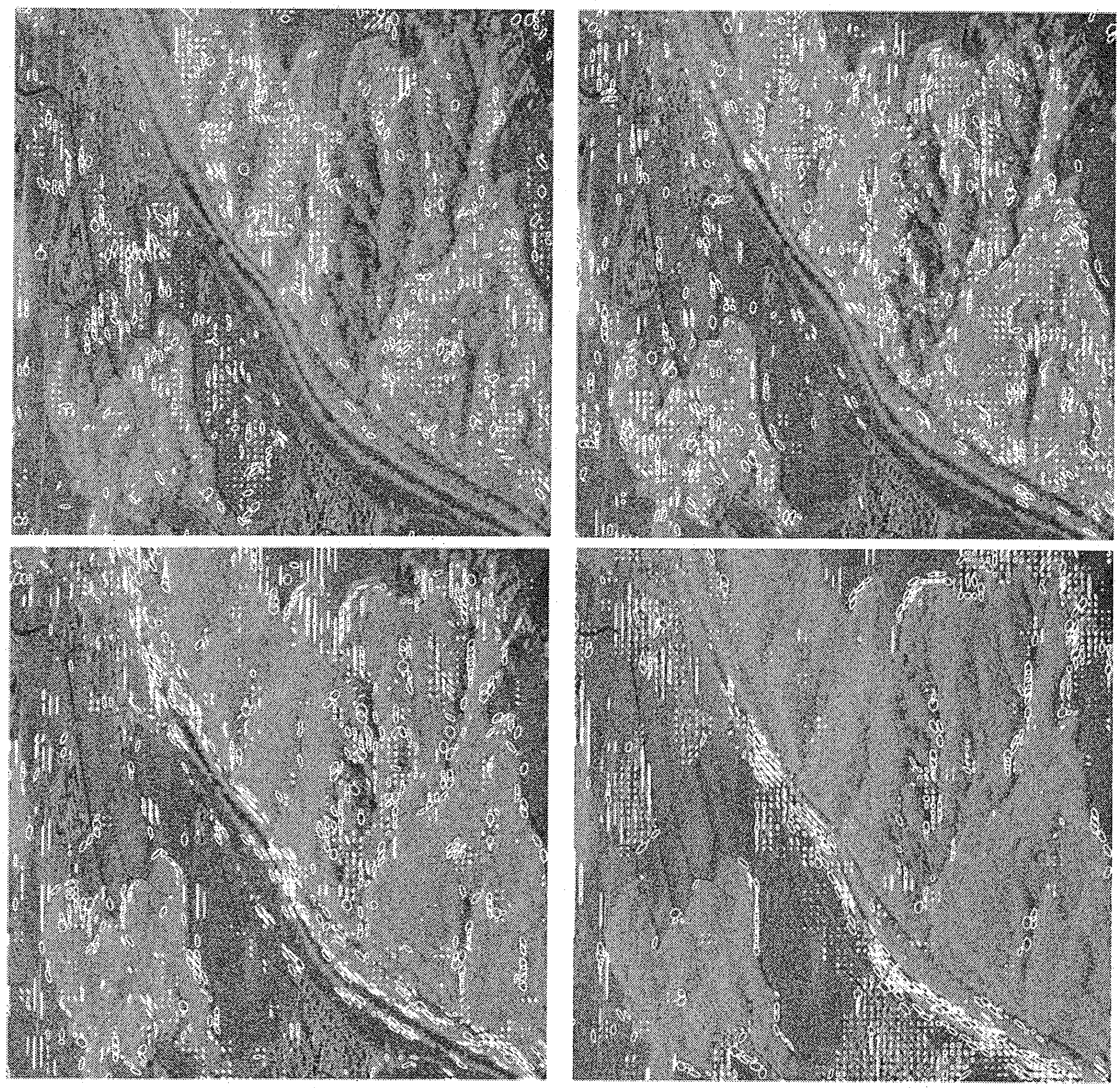

Figure 2: Estimated kernels with the direct estimation method. The kernels are displayed with a scale of 1. *Top left:* range 1. *Top right:* range 2. *Bottom left:* range 3. *Bottom right:* range 4.

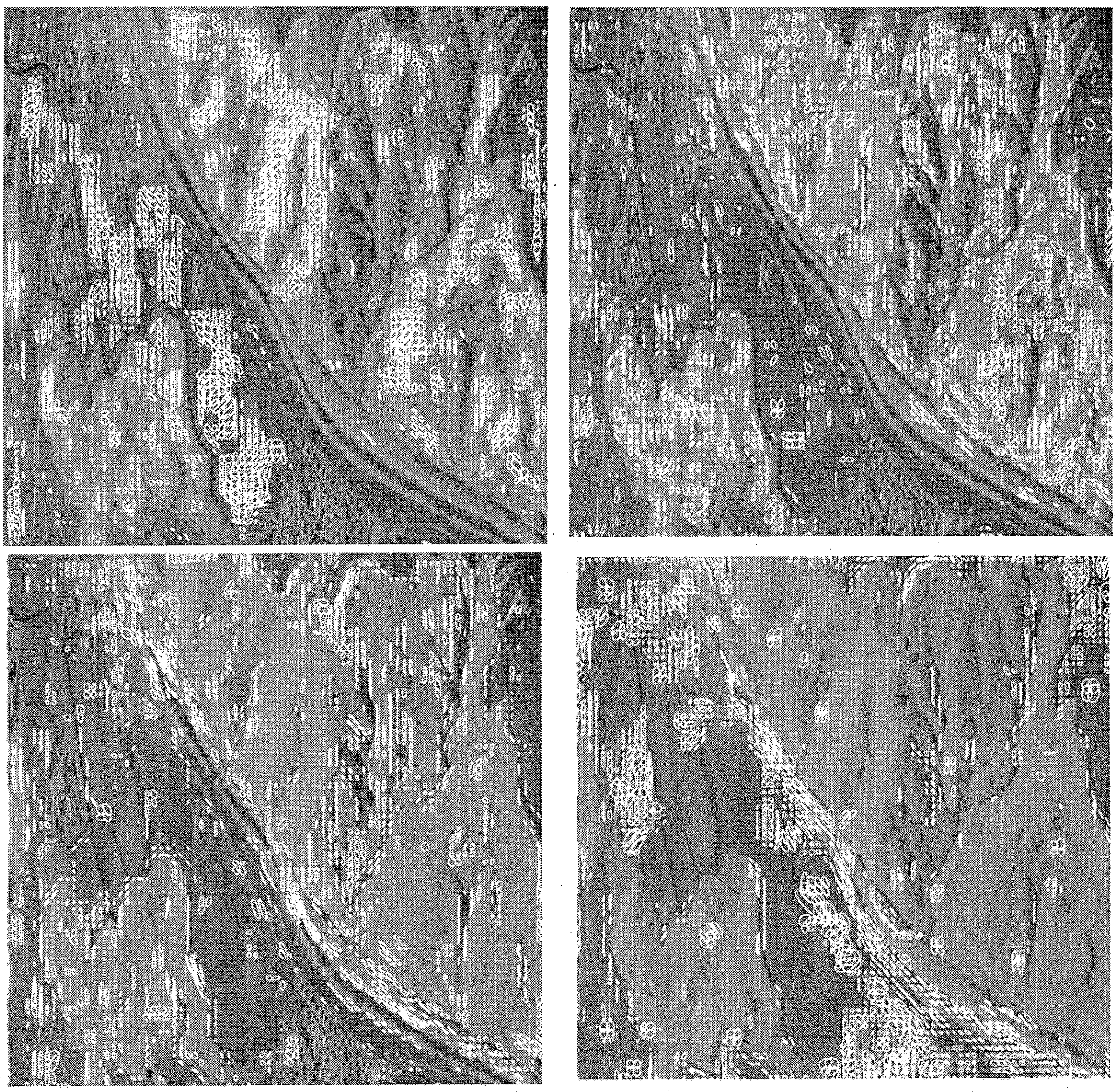

Figure 3: Estimated kernels with the GST based estimation method. The kernels are displayed with a scale of 0.05. *Top left:* range 1. *Top right:* range 2. *Bottom left:* range 3. *Bottom right:* range 4.

5 CONCLUSION

In this paper, two methods for the analysis of nonstationary spatial texture from SAR data have been introduced. Both are based on an original parametric model named AGK for the local autocovariance of texture intensity. The first method extracts parameters from direct estimation of the autocovariance whereas the second one is based on the Gradient Structure Tensor (GST). This operator has been adapted to stochastic processes and the relation with the parameters of the AGK model has been established. In the case of a speckled process, a pre-smoothing operation is necessary, leading to a nonlinear equation with no simple analytic solution. Therefore, a scale approximation is formulated in order to apply the method to experimental SAR data. These two approaches are then validated and compared on experimental data. They both provide a compact representation of the spatial information of the image which could be used for applications as for instance speckle filtering[8], detection of linear structures or texture classification. Moreover, for a more complete representation of the data, the GST method could be improved by considering multiscale analysis approaches.

6 REFERENCES

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