inter-noise 96
25th Anniversary Congress – Liverpool

INCE: 74.3.1

FAST DECONVOLUTION OF MULTI-CHANNEL SYSTEMS USING REGULARISATION

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1. INTRODUCTION

Deconvolution is useful for many practical applications, and there is a vast amount of literature covering the different aspects of the problem (see for example [1] Chapter 10, or [2]). We are interested in deconvolution techniques for the purpose of designing digital filters for multi-channel sound reproduction. More specifically, given a set of S loudspeakers, the objective is to reproduce a desired sound field at R points in space as accurately as possible. Using an approximate "least squares" method, it is possible to design a digital finite impulse response (FIR) filter, also referred to as an inverse filter, such that when the system's output is filtered through the inverse filter, the system's input is restored almost perfectly ([1] Chapter 10). The use of a modelling delay overcomes the problems caused by non-minimum phase components in the system response. An example of an inverse system used in practice is the so-called cross-talk cancellation matrix used for stereo reproduction over two loudspeakers ([3], [4], [5]). In this case a 2-by-2 matrix of digital filters is used to compensate for both the room response and the response of the loudspeakers, and also to cancel the cross-talk from the left loudspeaker to the right ear and vice versa ([6], [7]).
In this paper, we present a very fast method for deconvolving a multi-channel sound reproduction system comprising any number of acoustic sources and error sensors used in the sound field to monitor the system's performance. It combines the well-known principles of least squares inversion in the frequency domain ([8], [9]), and the zero'th order regularisation method ([10] Section 18.5) which is traditionally used when one is faced with an ill-conditioned inversion problem ([11] Chapter 2 Section 28).

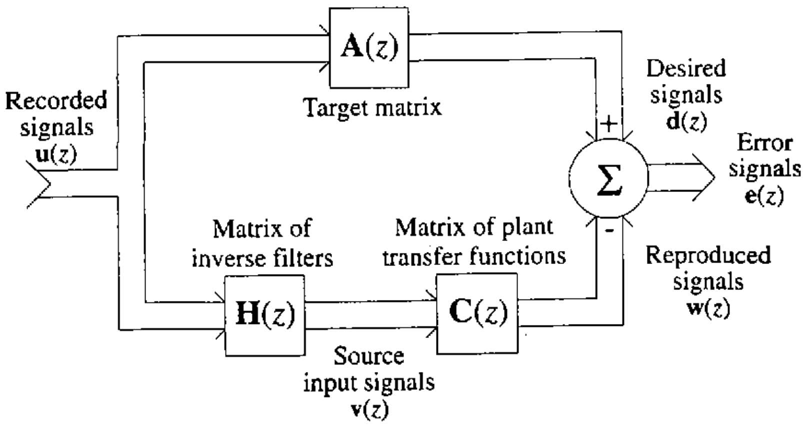

Fig.1.The discrete-time multi-channel deconvolution problem in block diagram form

2. SYSTEM DESCRIPTION

The inversion problem is shown in block diagram form in Fig. 1. The variables are defined as follows. $\mathbf{u}(z)$ is a vector of T observed signals, $\mathbf{v}(z)$ is a vector of S source input signals, $\mathbf{w}(z)$ is a vector of R reproduced signals, $\mathbf{d}(z)$ is a vector of R desired signals, and $\mathbf{e}(z)$ is a vector of R performance error signals. All vectors are column vectors. The matrices $\mathbf{A}(z)$, $\mathbf{C}(z)$, and $\mathbf{H}(z)$ represent multi-channel filters. $\mathbf{A}(z)$ is an $R \times T$ target matrix, $\mathbf{C}(z)$ is an $R \times S$ plant matrix, and $\mathbf{H}(z)$ is an $S \times T$ matrix of inverse filters. The objective of this paper is to give a method for the determination of $\mathbf{H}(z)$ given $\mathbf{A}(z)$ and $\mathbf{C}(z)$. The elements $C_{rs}(z)$ of $\mathbf{C}(z)$ are the z-transforms of the impulse responses $c_{rs}(n)$ of the multi-channel plant. Each impulse response $c_{rs}(n)$ is assumed to be a causal sequence of finite length N_c whose z-transform is of the form

$$C(z) = \sum_{n=0}^{N_c-1} c(n) z^{-n} = c(0) + c(1) z^{-1} + \cdots + c(N_c - 1) z^{-(N_c-1)} \tag{1}$$

It is not necessary to make any assumptions about the elements of $\mathbf{A}(z)$. From the block diagram shown in Fig. 1, it is straightforward to derive the following relationships:

$$\mathbf{v}(z) = \mathbf{H}(z)\mathbf{u}(z), \tag{2a}$$

$$\mathbf{d}(z) = \mathbf{A}(z)\mathbf{u}(z), \tag{2b}$$

$$\mathbf{w}(z) = \mathbf{C}(z)\mathbf{v}(z), \tag{2c}$$

and

$$\mathbf{e}(z) = \mathbf{d}(z) - \mathbf{w}(z). \tag{2d}$$

3. FAST DECONVOLUTION USING REGULARISATION

It is not difficult to derive an expression for the optimal z-transforms of the inverse filters under the constraint that they be stable [12]. However, in practice the filters also have to be causal, and, in addition, our method also requires them to have finite duration. In this section we show how to calculate a matrix of inverse causal FIR (finite impulse response) filters each containing N_h coefficients. Since this method uses Fast Fourier Transforms (FFTs), N_h must be equal to a power of two.

It is a well-known fact that deconvolution based on matching the frequency response only at a number of discrete frequencies usually leads to an undesirable circular convolution effect, sometimes referred to as wrap-around effect, in the time domain ([13] Sections 3.2.4 and 3.6.4). The basic idea behind our method is to reduce the effects of circular convolution by using regularisation to reduce the effective duration of each of the inverse filters to approximately $N_h/2$.

The implementation of the method is straightforward. FFTs are used to get in and out of the frequency domain, and the system is inverted for each frequency in turn. A "cyclic shift" of the inverse FFTs of the optimal frequency responses is used to implement a modelling delay ([14]).

The filter matrix $\mathbf{H}_0$ that minimises the quadratic cost function

$$J(e^{j\omega\Delta}) = \mathbf{e}^{\mathrm{H}}(e^{j\omega\Delta})\mathbf{e}(e^{j\omega\Delta}) + \beta\, \mathbf{v}^{\mathrm{H}}(e^{j\omega\Delta})\mathbf{v}(e^{j\omega\Delta}) \quad (3)$$

can be shown to be given by ([12])

$$\mathbf{H}_0(e^{j\omega\Delta}) = \left[\mathbf{C}^{\mathrm{H}}(e^{j\omega\Delta})\mathbf{C}(e^{j\omega\Delta}) + \beta\,\mathbf{I}\right]^{-1} \mathbf{C}^{\mathrm{H}}(e^{j\omega\Delta})\mathbf{A}(e^{j\omega\Delta}). \quad (4)$$

Here, Δ is the sampling interval, ω is the frequency, and β is the regularisation parameter which is a small positive number. Note that $\mathbf{H}_0$ is a continuous function of frequency. If an FFT is used to sample the frequency response of $\mathbf{H}_0$ at N_h points around the unit circle, then the value of $\mathbf{H}_0(k)$ at those frequencies is given by

$$\mathbf{H}_0(k) = \left[\mathbf{C}^{\mathrm{H}}(k)\mathbf{C}(k) + \beta\,\mathbf{I}\right]^{-1} \mathbf{C}^{\mathrm{H}}(k)\mathbf{A}(k), \quad (5)$$

where k denotes the k'th frequency line, that is, the frequency corresponding to the complex number $\exp(j2\pi k/N_h)$.

Thus, in order to calculate the impulse responses of the inverse filters $\mathbf{h}(n)$ for a given value of β, the following steps are necessary.

1. Calculate $\mathbf{C}(k)$ by taking $R \times S$ N_h-point FFTs of the plant impulse responses $c_{rs}(n)$.
2. For each of the N_h values of k, calculate the $S \times R$ matrix $\mathbf{H}(k)$ from Eq. 5.
3. Calculate $\mathbf{h}(n)$ by taking $S \times R$ N_h-point inverse FFTs of the elements of $\mathbf{H}(k)$ followed by a cyclic shift of $N_h/2$. For example, if the inverse FFT of $H_{sr}(k)$ is {3,2,1,0,0,0,1,2}, then $h_{sr}(n)$ is {0,0,1,2,3,2,1,0}.

It is important to note that in order to achieve an accurate inversion, the regularisation parameter β must be set to an appropriate value. Fortunately, though, the exact value of β is usually not critical. For example, if the optimal value of β for a given plant is 1, then values in the range between 0.8 and 1.2 will usually work just as well, and values in the range between 0.2 and 5 are likely to produce acceptable results. In practice, β is most conveniently determined by trial-and-error experiments.

4. CONCLUSIONS

An FFT-based deconvolution method can be used to deconvolve both single- and multi-channel systems with a matrix of causal FIR (finite impulse response) filters. The method is typically several hundred times faster than a conventional steepest descent algorithm implemented in the time domain. However, the method works well only when it is possible to use relatively long inverse filters. The circular convolution effect in the time domain is controlled by using regularisation. In practice, the regularisation works by making the inverse filters decay away quickly enough to ensure that the circular convolution effect is insignificant. In order to achieve an accurate inversion, the regularisation parameter β must be set to an appropriate value, but the exact value of β is usually not critical.

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