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SOUND PROPAGATION IN REAL ATMOSPHERE

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Abstract

A model for sound propagation in real atmosphere is proposed. This model is based on a W.K.B. expansion of the governing equations, pointing out the role of the thermophysical properties of the fluid on the dynamics of sound propagation. A new invariant is then derived that takes into account real atmospheric lapse rates. It is shown that the well known Blokhintzev invariant should be considered only for dry adiabatic lapse rates. Comparisons between sound levels derived from wave amplitudes computed using both invariants are shown. Considering the air as inviscid fluid, measurable sound levels differences are found for sources placed in the troposphere.

Introduction

Outdoor sound propagation is strongly influenced by the thermophysical properties of the atmosphere. Existing models consider a non-uniform fluid only through a non-uniform sound speed or a horizontally stratified atmosphere. A description of the different models has been recently presented in a benchmark comparison [1], whilst [2] contains an accurate analysis of the physical phenomena involved. An approximate wave equation containing ambient properties and flows was proposed by A. D. Pierce [3]; here, an alternative wave equation is derived, using fluid- and thermodynamics and making the only assumption that acoustic length scales are smaller than ambient length scales. Solving this wave equation with a W.K.B. [4] [5] expansion, the eikonal equation and a Blokhintzev-like invariant are obtained. The difference from the classical Blokhintzev invariant [6] derives from less restrictive assumptions on the basic state of the atmosphere.

1 Mathematical model

Assuming an inviscid fluid, the governing equations are the conservations of momentum and mass, and the second law of thermodynamics:

$$\frac{D\vec{v}}{Dt}+\frac{1}{\rho}\vec{\nabla}p=0 \tag{1}$$

$$\frac{D\rho}{Dt}+\rho\vec{\nabla}\cdot\vec{v}=0 \tag{2}$$

$$\dot{q}=T\frac{Ds}{Dt} \tag{3}$$

where D/Dt is the total derivative $\partial/\partial t+\vec{v}\cdot\vec{\nabla}$, $\vec{v}$, p and ρ are the velocity, the pressure and the density of the fluid, and s, T and $\dot{q}$ are the entropy, temperature and specific heating, respectively.
Assuming that the physical system is characterized by two length and time scales, the first characteristic of the fluid and the other of the sound wave, each variable a can be separated in the two components: $a=A+a'\,,\ \langle a'\rangle=0\,,\ A=\langle a\rangle$, where $\langle\rangle$ denote time averaging.
Subtracting the time averaged equations from Eqs. (1-3), and considering only the first order terms in the sound variables, Eqs. (1-2) become

$$\frac{D\vec{u}}{Dt}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{U}+\frac{1}{\rho_0}\vec{\nabla}p'-\frac{\rho'}{\rho_0^2}\vec{\nabla}P_0=0 \tag{4}$$

$$\frac{D\rho'}{Dt}+\rho_0\vec{\nabla}\cdot\vec{u}+\rho'\vec{\nabla}\cdot\vec{U}+\left(\vec{u}\cdot\vec{\nabla}\right)\rho_0=0 \tag{5}$$

where from now on $D/Dt=\partial/\partial t+\vec{U}\cdot\vec{\nabla}$, because $\vec{U}=\langle\vec{v}\rangle$.
From Eq. (3) the well known relation between acoustic pressure p' and acoustic density ρ' can be obtained: $p'=\gamma\frac{P_0}{\rho_0}\rho'=c^2\rho'\,,$ where γ is the specific heat ratio, c is the sound speed, and $\rho_0=\langle\rho\rangle$, $P_0=\langle p\rangle$.
After some calculations Eqs. (4-5) can be reformulated for the acoustic variables $\vec{u}$ and p'/ρ_0

$$\frac{D\vec{u}}{Dt}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{U}+\vec{\nabla}\left(\frac{p'}{\rho_0}\right)-\frac{p'}{\rho_0}\vec{\alpha}=0\,,\qquad \vec{\alpha}=\frac{\vec{\nabla}\rho_0}{\rho_0}-\frac{1}{\gamma}\frac{\vec{\nabla}P_0}{P_0} \tag{6}$$

$$\frac{D}{Dt}\left(\frac{p'}{\rho_0}\right)+c^2\vec{u}\cdot\frac{\vec{\nabla}\rho_0}{\rho_0}+c^2\vec{\nabla}\cdot\vec{u}+\frac{p'}{\rho_0}\beta=0\,,\qquad \beta=\frac{c^2}{\rho_0}\vec{\nabla}\cdot\left(\frac{\vec{U}\rho_0}{c^2}\right)\,. \tag{7}$$

2 W.K.B. approximated wave equation

For p'/ρ_0 and $\vec{u}$ a time dependence $e^{-i\omega t}$ is assumed. Scaling variables and operators with the ambient variations scale L and denoting by c_0 and $k_0 = \omega/c_0$ two reference values for the sound speed and the wave number, the non-dimensional parameter $\delta = (k_0 L)^{-1}$ can be introduced.
Neglecting the rotational part of the sound velocity field, the potential ϕ such that $\vec{u} = \vec{\nabla}\phi$ can be introduced. To solve the equations with the W.K.B. expansion the assumptions are made

$$\phi = -i\delta\Phi e^{\frac{i}{\delta}S+\phi_1-i\omega t}, \quad \frac{p'}{\rho_0} = \Pi e^{\frac{i}{\delta}S+p_1-i\omega t}. \tag{8}$$

Introducing $D_t = -ic_0/\delta + \vec{U}\cdot\vec{\nabla}$, neglecting the second order terms, and making some manipulations a wave equation is obtained:

$$D_t^2\phi - c^2\nabla^2\phi - \frac{i}{\delta}\phi\left[c^2\vec{\nabla}S\cdot\frac{\vec{\nabla}\rho_0}{\rho_0} + c|\vec{\nabla}S|\beta + c|\vec{\nabla}S|\,c\hat{n}\cdot\vec{\alpha}\right] = 0 \tag{9}$$

where $\hat{n} = \vec{\nabla}S/|\vec{\nabla}S|$ is the outward normal to the wave front. δ^{-1} terms give the eikonal equation $\left(c_0 - \vec{U}\cdot\vec{\nabla}S\right)^2 = c^2|\vec{\nabla}S|^2$, and at the next order, after lengthly calculations

$$\vec{\nabla}\left(\vec{V}B^2A\right) = -B^2A\,c\hat{n}\cdot\vec{\alpha}, \qquad A = \frac{\rho_0|\vec{\nabla}S|}{c} \tag{10}$$

where $\vec{V} = \vec{U} + c\hat{n}$ is the wave front velocity, and B is such that $e^{2\phi_1} = B^2$.
If the ambient state is adiabatic, the *Blokhintzev invariant* [2], [6], [7] would result:

$$\vec{\nabla}\left(\vec{V}B^2A\right) = 0 \tag{11}$$

Eq. (10) can be written as $\vec{\nabla}\left(\vec{V}B^2A\right) + B^2A\vec{V}\cdot\vec{\alpha} = B^2A\,\vec{U}\cdot\vec{\alpha}$. Observing that $\vec{V}\cdot\vec{\alpha} \simeq c\hat{n}\cdot\vec{\alpha} \gg \vec{U}\cdot\vec{\alpha}$ and $c \gg |\vec{U}|$, a better approximation can be obtained to propose a new invariant:

$$\vec{\nabla}\left(\vec{V}B^2A\frac{\rho_0}{P_0^{1/\gamma}}\right) = 0 \tag{12}$$

This is a generalisation of the Blokhintzev invariant and reduces to it when the atmospheric lapse rate is adiabatic.

From Eq. (12) a numerical model has been derived, and applied to the analysis of sound propagation from a monochromatic point source, producing the sound pressure levels shown in Fig. 1. Realistic atmospheric data has been considered [8]. Fig. 2 shows the amount of correction on sound pressure levels introduced using the new model, including the factor $\rho_0 P_0^{-1/\gamma}$ in the invariant. This factor is negligible only if the atmosphere is dry, i.e. in particular ideal meteorological conditions.

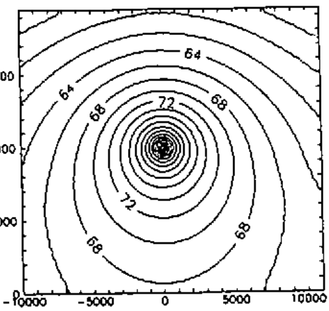

Fig. 1: sound pressure levels computed with the new invariant.

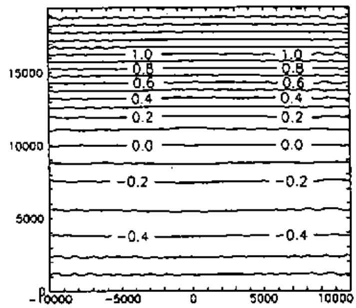

Fig. 2: difference between sound pressure levels computed with new and classical invariant

References

[1] K. Attenborough et al., J. Acoust. Soc. Am., 97, 173 (1995).

[2] A. D. Pierce, *Acoustics: An Introduction to Its Physical Principles and Applications* (Acoustical Society of America, Woodbury, N.Y., 2nd ed., 1989).

[3] A. D. Pierce, J. Acoust. Soc. Am., 87, 2292 (1990).

[4] C. M. Bender and S. A. Orszag, *Advanced mathematical methods for scientists and engineers* (Mc Graw - Hill, Inc., NY, 1978).

[5] S. Weinberg, Phys. Rev., 126, 1899 (1962).

[6] D. Blokhintzev, J. Acoust. Soc. Am., 18, 322 (1946).

[7] W. D. Hayes, Phys. Fluids, 11, 1654 (1968).

[8] A. H. Oort, E. M. Rassmussen, NOAA Professional Paper, 5 (1971).