BACKWARD SCATTERED PULSES FIELD STATISTIC MODELING IN THE TIME DOMAIN IN STOCHASTIC PROBLEM OF VOLUME SCATTERING

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1. INTRODUCTION

In the series of papers [1-3] the non-stationary problem of pulses propagation and scattering in layered-inhomogeneous media was studied. It was described by one-dimensional on the space wave equation with boundary and initial conditions. For some classes of sound velocities profiles analytical solutions were obtained based on which analytical-numerical method of treating such problems in exact formulation has been elaborated [3]. Formulation of the problem about pulses interaction with plane-layered periodic medium, obtaining in the time domain of complete wave image of the formation of parametric resonance and appearance of Bragg resonance became one of the important applications of this method. So, continuing these investigations [1-3] we shall study in this paper the behaviour of statistic characteristics of the wave field formed as the result of scattering on randomly fluctuating medium of various type incident pulses. Our consideration involves statistic simulation method too, that has been developed in paper [4] for the solution of steady-state problem of monochromatic plane waves scattering on a randomly layered medium. Some not numerous papers have been devoted (for instance, [5,6]) to our problem of pulse scattering on a random volume irregularities in similar wave setting. However, results, obtained there, are the approximate ones and they not allow to imagine the back -scattered field behaviour in the most interesting transitional domain.

2. THE MATHEMATICAL PROBLEM FORMULATION AND THE MAIN METHODS OF INVESTIGATION

Original boundary value problem, which is the mathematical description of considering problem of the medium pulse probing is formulated below (see Figure 1, too):

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2(z)}\frac{\partial^2}{\partial t^2}\right)U(z,t)=0,$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{c_0}\frac{\partial}{\partial t}\right)U(z,t)\bigg|_{z=L}=\frac{2}{c_0}\frac{\partial}{\partial t}\varphi(L+c_0t),$$

$$\left(\frac{\partial}{\partial z}-\frac{1}{c_0}\frac{\partial}{\partial t}\right)U(z,t)\bigg|_{z=L_0}=0.$$

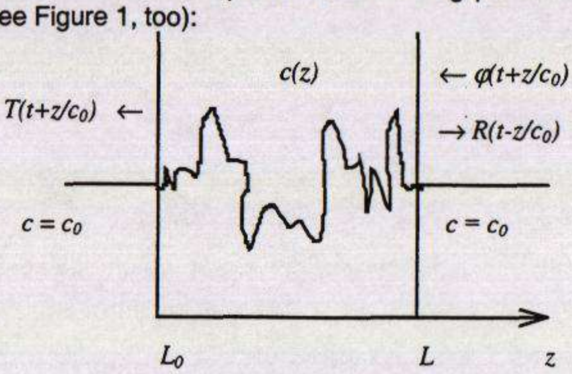

Fig.1. Illustration to problem setting.

Here a pulse $\varphi(t)$ is incident at a time moment $t=+0$ on the medium slab with the random profile of $c(z)$. As an example, one of these random function realizations is represented in Figure 2. Our special interest is connected with the studying of back-scattered field $R(z-c_0t)$ fluctuations. Using the imbedding method [2] it is easy to write down the closed-form first order equation for Green's function

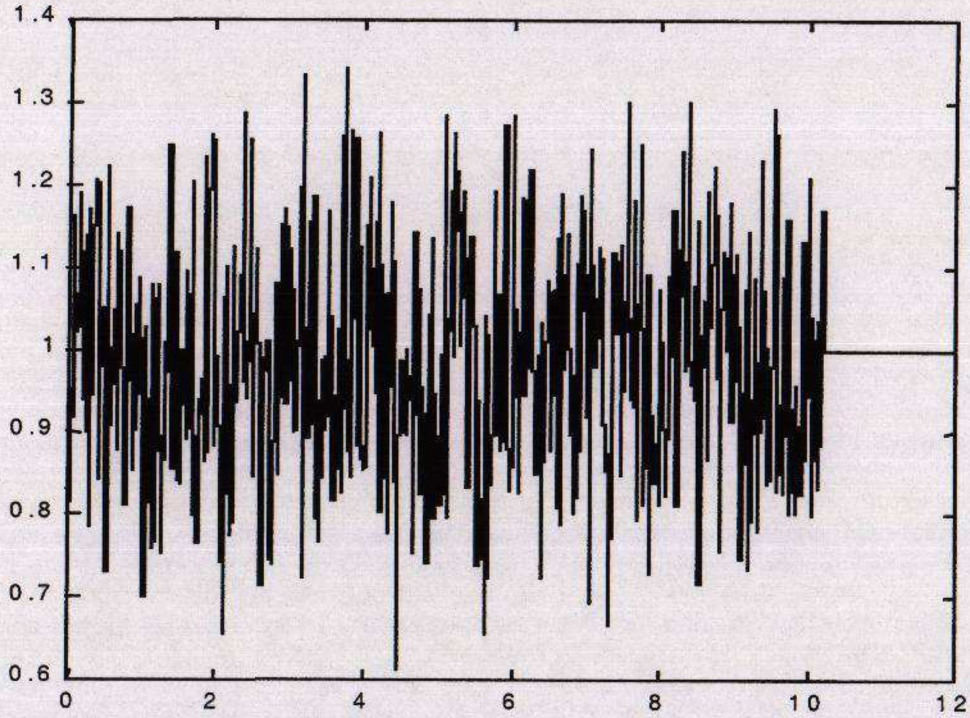

Fig.2. One of random function realizations. Along horizontal axis the dimensionless distance $(L\text{-}L_0)D(\Omega)$ (see below) is designated.

$G_L(t) = U(L,t) = \varphi(t) + R(t)$, describing the field on the right slab boundary L in the case of incidence on a random medium of the pulse $\varphi(t) = \theta(t)$ i.e. in the form of Heaviside unit function:

$$\left(\frac{\partial}{\partial L} + \frac{2}{c_0}\frac{\partial}{\partial t}\right)G_L(t) = \frac{2}{c_0}\delta(t) + \frac{1}{2c_0}\left(1 - \frac{c_0^2}{c^2(L)}\right)\int dt_1 \frac{\partial}{\partial t}G_L(t-t_1)\frac{\partial}{\partial t_1}G_L(t_1),$$
$$G_L(t)\big|_{L=L_0} = \theta(t) \qquad (1).$$

Knowing the latter equation solution, it is easy to obtain the field for arbitrary form pulses by the convolution $U(L,t) = \int d\xi G_L(t-\xi)(\partial\varphi / \partial\xi)$. As it has been showed in papers [2,3], owing to analytical-numerical method for determinated inhomogeneities $c(z)$ the problem solution $G_L(t) = \Phi_L(t)\theta(t)$ on the nonuniform space grid $\{z_i\}_{i=0}^N$, $z_0 = L_0$, $z_N = L$ can be obtained in the following form:

$$\Phi_{z_i}(t) = \sum_{j=0}^{i} a_{i,j}\theta(t - 2j\Delta_i) + \sum_{j=1}^{i} b_{i,j}\Phi_{z_i}(t - 2j\Delta_i)\theta(t - 2j\Delta_i) \qquad (2).$$

Here $\Delta_i = (z_i - z_{i-1})/c$ and it is supposed a piecewise approximation on the grid of inhomogeneous profile $c(z)$ by determined type of functions, allowing an analytical step-by-step solution of equation (1) [1]. Coefficients $a_{i,j}, b_{i,j}$ are expressed via values $c(z_i)$ in the framework of that or other assigned approximation. Transiting further elsewhere from space variables to temporary ones

$x(z) = \int_{L_0}^{z} d\xi c^{-1}(\xi),\ x(L_0) = 0,\ x(L) = H,\ \Psi_x(t) = \Phi_z(t),$ and supposing without restriction of generality but for a conveniency that $\Delta x_i = \Delta x = const$, and $c(x_i) = const$ in the limits of every Δx (discontinuous piecewise-constant function), the problem (1) can be investigated on the basis of the method of stochastic simulation [4] in the case, when the function $c(z)$ randomly fluctuates, and solution (2) is the random function.

3. ASSIGNMENT OF PARAMETERS AND SIMULATION RESULTS

While studying stationary problem of monochromatic wave of frequency ω ($\exp(-i\omega[z-L]/c_0)$) incidence on a half-space of randomly inhomogeneous medium fluctuations of which are defined by Gaussian «white» noise, the following main spatial scales appear [7]. These are: inhomogeneities correlation radius l, as the least (in comparison with all the rest ones) scale; wave parameter $\mu = \omega l/c_0$ characterizing the relation between the wave length λ and correlation radius l, $\lambda >> l$, diffusion coefficient $D(\omega) = 2\sigma_\varepsilon^2 \omega^2 l/(c_0^2 + 4\omega^2)^2)$ $(D^{-1} >> \lambda)$, medium layer thickness $L-L_0$. $D^{-1}(\omega)$ is the characteristic spatial scale of random inhomogeneities influence to the wave with frequency ω is propagated in the medium. While constructing the scheme of a stationary problem numerical modeling the discretization interval is chosen based on the condition $\Delta z < l$. Analyzing our non-stationary problem and implying the possibility of the further comparison of the results, it is sensible to make the similar referencing to the scales characterizing the scattering process of a narrow-band signal with a certain high carrier frequency Ω. So, let us be the reference frequency Ω, then $L_\Omega = D^{-1}(\Omega)$ is the spatial scale of influence of random medium inhomogeneities characterizing effects of accumulated influences of low intensity fluctuations σ_ε^2 on the propagating wave. Average value of the sound velocity in the medium will be $<c(z)> = c_0$, so the time scale corresponding to L_Ω will be $T_\Omega = L_\Omega / c_0$. Let us perform normalization of all the spatial scales of the problem as L_Ω and those of time scale as T_Ω introducing the following evident designations: $\Omega = \Omega T_\Omega$, $\tau = t/T_\Omega$, $\Delta = \Delta x / T_\Omega$,

$\boldsymbol{H} = H / T_\Omega$. Numerical values of the parameters for performing model calculations have been chosen as the following: $\Omega = 100$, $\mu = 1$, $\sigma_\varepsilon^2 = 0{,}025$, $\boldsymbol{H} = 20$. It means that in terms of monochromatic wave with the frequency Ω we have the situation, when the wave length is about the order higher than the radius of correlation of sound velocity random fluctuations. About 16 wave periods take place at the time scale of diffusion T_Ω. The medium layer thickness at this is 20 times larger than the diffusion scale L_Ω. Studying the stationary problem of scattering on the fluctuation medium of the incident plane monochromatic wave of type $\exp(-i\omega[z-L]/c_0)$ will yield us as one of the main results [7] total reflection of this wave, the probability being close to 1 for the medium layer thickness $L - L_0 > 5L_\Omega$, i.e. from this point of view the layer becomes equivalent to half-space. Low-frequency spectral components with $\omega < \Omega$ will take the medium as a layer of finite thickness. The accumulating influence of fluctuations on them will not be enough to be fully scattered backwards, because the layer ceases to be thick in the scales L_ω.

Now we go over to the analysis of simulation results. Let us consider a situation when the short-time pulse is incident on a random medium layer. For the rectangular pulse with duration η we have the simple relation $U_L(\tau) = (G_L(\tau) - G_L(\tau-\eta))/\eta$. With the help of it the scattering of a delta-pulse is well simulated as follows: $\lim_{\eta\to 0} (G_L(\tau) - G_L(\tau-\eta))/\eta$. Figures 3 (*a-c*) show the behavior of statistic moments of back-scattered field $R(\tau)$ for the small value of η. Since the spectral components of short-time pulses have no increasing amplitude in a low-frequency domain, the quantity of energy in them is limited. This results in strong scattering of an initial pulse on the random irregularities of medium and as a consequence, in its almost total egress. For a delta-pulse model ($\eta << T_\Omega$) the behavior of the second statistic moment of a back-scattered field testifies for considerable spreading over the medium of the initial pulse energy thus the pulse width is increasing in tens times (at the level of 0,5 it is $\sim 150\eta$). Overwhelming part of the δ - pulse energy egresses backside the medium to the moment $\tau \sim 20$. For $\tau > 20$ only energy of a deeply penetrating inside the medium low-frequency spectral components is remains and this energy is negligibly small. As we mentioned in the introduction, approximate analysis of the considered problem was carried out in papers [5,6]. For this purpose in paper [5] special mathematical techniques, based on the asymptotic method of averaging (Bogolyubov and Mitropolski method) were developed. One of the main results of these investigations is assertion that for the η larger in comparison with a time scale of one correlation length (that is our scale l/c_0) the back-scattered field is a Gaussian random process with mean value equal to zero. For verification of this assertion the series of modeling computations for different values η (both larger than l/c_0 and smaller, too) were carried out. It is obtained that for any $\eta << T$ the back-scattered field $R(\tau)$ is really Gaussian stochastic process, but with non-zero mean value. This verification of $R(\tau)$ correspondence to Gaussian process was carried out at selected time moments on the base of Kholmogorov-Smirnov criterion. As to the mean value $\langle R \rangle$, it is non equal to zero also in the case of large η. The latter assertion in particular follows from the analysis of the Green's function behaviour. In addition, we have been carried out the comparison of our simulation results with the laws of back-scattered field mean intensity $\langle R^2(\tau) \rangle$ asymptotic behaviour, obtained in papers [5,6]. For this aim, based on the method of least squares we made the approximation of mean value intensity simulative time dependencies for various time regions and number of incident pulse

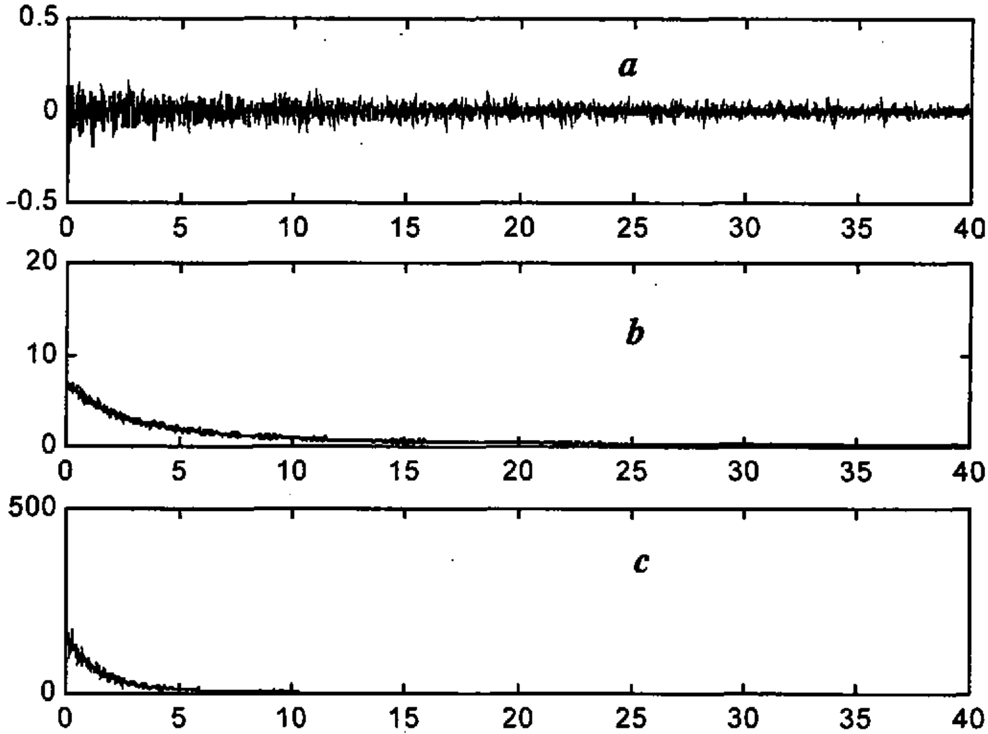

Fig. 3. $\eta = 0{,}02$; a - $<R>$, b - $<R^2>$, c - $<R^4>$.

durations. As it must be, the initial part $0 < \tau < 1 \div 2$ corresponds to exponential law of the functions $<R^2(\tau)>$ decreasing, according to transport theory. Further, this dependence acquires the character of power-type function $\sim \tau^{-a}$. While modeling delta-pulse at the interval $20 < \tau < 40$ the law of back-scattered field averaged intensity behaviour is well described by the function $\sim \tau^{-1,3}$, whereas for the pulse with larger duration ($\eta = 0{,}2$) the power index a is lesser: $a \approx 0{,}9$. For the Green's function, as a long-duration pulse, it is obtained the power index $a \approx 0{,}5$. Thus, the asymptotic behavior of the curves is considerably different from the result of papers [5,6], where one can find the following laws: $\sim \tau^{-1,5}$ for rectangular pulses with not small duration and $\sim \tau^{-2}$ for the such ones with high frequency content. Results of statistical simulation show that much more slow decreasing character is observed, than it has been obtained in these papers. The latter means that going out to the asymptotic in the considered problem is very long and the approximate analysis done in papers [5,6] corresponds to such prolonged observation times τ, for which the energy of initial incident pulse is almost not remained. The general transitional scattering processes as showed in Figures 3 b,c , 4 b,c take place in the interval $0 < \tau < 20$.

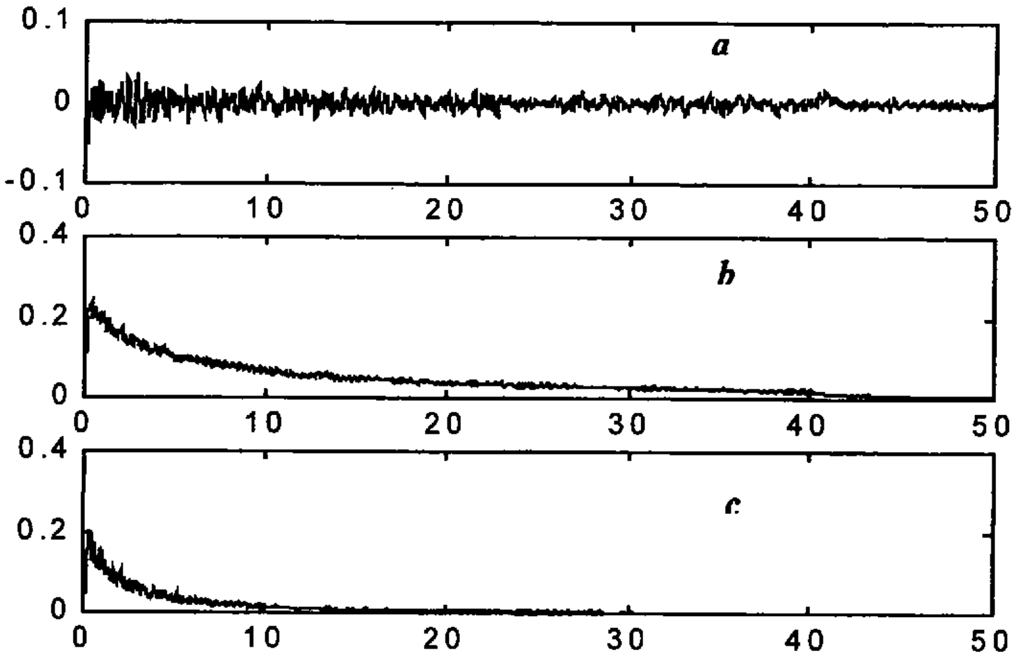

Fig.4. $\eta = 0{,}2$; a - $\langle R \rangle$, b - $\langle R^2 \rangle$, c - $\langle R^4 \rangle$.

4. REFERENCES

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