Modeling of dynamic noise scattering in a layered fluctuating ocean

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Abstract

This paper is devoted to studying the statistical characteristics of an ocean acoustic noise field. The field in question is generated by force sources, distributed on a surface, and the environment is subject to random fluctuations in sound speed. Earlier papers have investigated this problem analytically under certain approximations. Here the problem is solved exactly using statistical modeling and we compare the results of such analysis with the previous analytical ones.

1. Introduction

We consider the problem of dynamic noise scattering in the ocean by the random layered fluctuations of a sound speed profile. These noises themselves may be generated in the near surface layer of the ocean, for example by atmospheric pressure pulsations in turbulent wind. The specific mechanism of noise generation is not important for the purposes of this paper: the only significance that such generation is the result of some external force sources acting on the surface. It has been shown that this circumstance provides the key distinction between the problem of noise generation and propagation in the fluctuating ocean, and that of acoustic wave propagation and scattering when the radiation is from point mass sources.

2. General Formulation of the Problem

Acoustical waves can be excited in the oceanic water column by force sources. These might, for example, arise from atmospheric pressure fluctuations $p_a(t, \mathbf{r})$ (here t is the temporal, and $\mathbf{r} = \{x, y, z\}$ are the spatial, coordinates). Such excitation is described by the correspondent boundary value problem for the wave equation. For the boundary condition corresponding to the ocean surface $z = H$, we have continuity of the sound pressure as $p_H = p_a(t, x, y)$. At the bottom boundary we have continuity of the vertical component of the oscillatory velocity. For the spatial-time spectral components of the fields in the layered case, this boundary value problem reduces one of solving the following set of imbedding equations for a water layer (for example, see [1,2]):

$$\frac{d\phi_H}{dH} = -q^2(H) - \phi_H^2, \quad \phi_0 = i\alpha \quad , \tag{1}$$

$$\frac{\partial \psi(z,H)}{\partial H} = -\phi_H \psi(z,H), \quad \psi(z,z) = 1 \quad . \tag{2}$$

Here $q^2(z) = (k^2(z) - æ^2)$ is the square of vertical wave number, $k^2(z) = \omega^2 / c^2(z)$, $æ = |æ|$ is the horizontal wave number, $\omega = \omega_0(1 - i\gamma)$ is the sound frequency, where ω_0 is its real part and where γ is the small medium dissipation. The parameter $c^2(z)$ describes the sound speed layered stratification inside the ocean medium, having a certain mean value c_0. An initial condition to the Riccati equation (1) is determined via the bottom impedance at $z = 0$: $\alpha = k_0 \alpha_1 \sqrt{\alpha_2^2 (1 - i\gamma_1)^2 - \cos^2\theta}$, where $k_0 = \omega_0 / c_0$, while the parameters α_1 and α_2 represent correspondingly the ratios of the water density and sound speed to the same ones of the liquid bottom $\alpha_1 = \rho_0 / \rho_1$, $\alpha_2 = c_0 / c_1$, and where γ_1 is the bottom dissipation, and θ is the angle from the surface of propagating plane waves. In (1) the function ϕ_H delineates a perturbation field at the surface produced by point force sources induced there. As it was shown in [1,2] ϕ_H is a transfer function and characterizes the process of excitation transmission from the atmosphere to ocean surface, taking into account layered stratification and all other properties of the ocean. The function $\psi(z,H)$ in (2) characterizes the wave field inside the medium resulting from a point surface sources in terms of the spectral components. It is seen from (2) that $\psi(z,H)$ is defined as a quadrature of the transfer function ϕ_H.

Let further assume that field of random surface sources of noise is statistically stationary, uniform and it is described by the spatial-time spectral density $S_a(\omega, æ)$. The spectral amplitudes of the pressure and oscillatory velocity are determined via the solutions of (1)-(2) by the expressions $p(\omega, æ; z) = p_a(\omega, æ)\psi(z,H)$, $w(\omega, æ; z) = -i\phi_z p_a(\omega, æ)\psi(z,H)/k_0$, where the oscillatory velocity is normalized by the medium wave resistance $\rho_0 c_0$. The spatial-time spectral densities of the pressure and average vertical power flux (the real part of mutual p-w spectrum) then take the following form:

$$S_{pp}(\omega, æ; z) = S_a(\omega, æ)<|\psi(z,H)|^2> \ , \quad S_{pw}(\omega, æ; z) = k_0^{-1} S_a(\omega, æ)<|\psi(z,H)|^2 \, Re(-i\phi_z)> \ . \quad (3)$$

Our purpose is to investigate the behaviour of the spectra given by (3) in the deep fluctuating ocean, both under the diffusion approximation and with statistical modelling of equations (1)-(2).

3. Assignment of the parameters and some analytical studying of the problem

Let us assume that sound speed in the water column, at depth z, is $c(z) = c_0(1 + \varepsilon(z))$, where the function $\varepsilon(z)$ describes the profile fluctuations relative to the mean value c_0. Let $c_0 = 1500$ m/s, and the depth of the ocean be $H = 6000$ m. We imply that fluctuations of inhomogeneities are described by a Gaussian random process having zero mean value, and having variance $\sigma^2 = 5\times10^{-9}$ and correlation radius $l = 0.6$ m. These parameter values correspond, for example, to the microstructure of the sound speed, in particular with respect to the ocean subsurface layer [2,3].

We seek a solution to the Riccati equation which has the form $\phi_H = iq_0(1+\chi_H)/(1-\chi_H)$. It follows from equation (1) that the function χ satisfies:

$$\frac{d\chi_H}{dH} = -2iq_0\left(1 - i\gamma k_0^2/q_0^2\right)\chi_H - i\varepsilon(H)\left(k_0^2/q_0^2\right)\left(1-\chi_H\right)^2, \qquad \chi_0 = (\alpha - q_0)/(\alpha + q_0). \qquad (4)$$

In the last equation we have neglected those terms having second order in γ and ε. We have also introduced the notation $q_0^2 = k_0^2 - æ^2$. Equation (4) describes the complex monochromatic plane wave reflection coefficient from a layered fluctuating medium $(0, H)$ [4]. The following equation for the power reflection coefficient $\eta_H = \chi_H\chi^*_H$ results from (4):

$$\frac{d\eta_H}{dH} = -4\gamma\frac{k_0^2}{q_0}\eta_H - i\varepsilon(H)\frac{k_0^2}{q_0}\left(\chi_H^* - \chi_H\right)\left(1-\eta_H\right), \qquad \eta_0 = \chi_0\chi_0^* \qquad (5)$$

The statistical characteristics of η_H are completely described by the probability density function $W_H(\eta) = <\delta(\eta_H - \eta)>$, that satisfies to the Einstain-Fokker equation:

$$\frac{\partial}{\partial H}W_H(\eta) = 4\gamma\frac{k_0^2}{q_0}\frac{\partial}{\partial\eta}\eta W_H(\eta) - 2D\frac{\partial}{\partial\eta}\eta(1-\eta)W_H(\eta) + D\frac{\partial}{\partial\eta}\eta(1-\eta)^2\frac{\partial}{\partial\eta}W_H(\eta) \ , \qquad (6)$$

$$W_0(\eta) = \delta(\eta_0 - \eta) \ .$$

Equation (6) can be obtained from equations (4) and (5) under the diffusion approximation, with averaging over the fast oscillations of the function χ_H. The necessary condition for the fast oscillation averaging method is $q_0 >> D$ and in the majority of real cases it usually is valid except enough small grazing angles θ. For the Gaussian Markovian fluctuations $\varepsilon(z)$ the coefficient of diffusion has the form $D = 2k_0^4\sigma^2 l/q_0^2$. Note that equation (6) corresponds to the model of medium random inhomogeneities in the form of "white noise":

$$<\varepsilon(z)> = 0 \ , \ <\varepsilon(z_1)\varepsilon(z_2)> = 2\sigma^2 l\delta(z_1 - z_2) \ , \qquad (7)$$

The diffusion coefficient determines the spatial scale of the problem over which the fluctuations are important. For instance, if the thickness of a random layer H is such that $H >> D^{-1}$ (in fact for $HD \geq 5$, see [4]), then equation (6) has a stationary solution, $W_\infty(\eta)$, that is independent on the impedance conditions at the ocean bottom. Besides, in such problems there is the "stochasticity" parameter to consider. It is equal to the spatial scale of wave power attenuation (owing to the medium dissipation), divided by the spatial scale of its (wave power) scattering by the random inhomogeneities. It follows from equation (6) that in our case this parameter is equal to

$$\beta = \frac{2\gamma k_0^2 q_0^{-1}}{D} = \frac{\gamma \sin\theta}{k_0\sigma^2 l}, \qquad \left(\text{here } D = \frac{2k_0^2\sigma^2 l}{\sin^2\theta}\right) \ . \qquad (8)$$

For the problem of plane waves incident on the fluctuating layer of a medium for $\beta << 1$, the wave field is essentially stochastic. In contrast, for $\beta \geq 1$ the medium fluctuations become weak and the wave field is well described by the phenomenological theory of radiation transport. In the last case the field statistical moments exponentially decay inside the medium accordingly to the law $exp\left[-\sqrt{\beta(\beta+1)}D(H-z)\right]$ [4].

As can be seen from (8) the diffusion coefficient decreases both with decreasing frequency and increasing angle θ. For example, for $\theta = 0.5°$ and frequency $f = 10$ Hz and above, with the assigned parameters for the sound speed fluctuations the product becomes $HD \approx 8.3\times10^{-4}$. It is obvious, that in this situation the fluctuations are not essential. For $f = 1000$ Hz the product $HD \approx 8.3$ and in this case the spectra (3) will be independent on the impedance conditions at the bottom.

The parameter β decreases when the medium dissipation γ and angle θ are decreased. So for $\theta = 0.5°$ and $f = 1000$ Hz, if $\gamma = 10^{-5}$, then $\beta \approx 7$. Thus in this case the power dissipation effect considerably exceeds the scattering effect of waves on the random inhomogeneities. If we take $\gamma = 10^{-7}$, then $\beta \approx 0.07$ and wave field becomes considerably "stochastic". Further these examples will be demonstrated by exact calculations from Equations (1)and (2).

The method of averaging over fast oscillations allows the estimation of the spatial-time spectra (3). Following the Equations (1), (4) and (5) we can write:

$$\frac{\partial}{\partial H} < |\psi(z,H)|^2 > = < \left[iq\frac{1+\chi_H}{1-\chi_H} - iq^*\frac{1+\chi_H^*}{1-\chi_H^*} \right] |\psi(z,H)|^2 > \quad . \tag{9}$$

Averaging the right hand side over the fast oscillations of the function χ_H and neglecting the terms of second order in γ and ε, we obtain $\frac{\partial}{\partial H} < |\psi(z,H)|^2 > = -2\gamma\frac{k_0^2}{q_0} < |\psi(z,H)|^2 >$, $2\gamma\frac{k_0^2}{q_0} = \beta D$. From here

$$S_{pp}(\omega, æ; z) = S_a(\omega, æ) exp[-\beta D(H-z)] \quad . \tag{10}$$

Similarly, using expressions for the vertical oscillatory velocity and averaging all terms over the fast oscillations, we obtain

$$S_{pw}(\omega, æ; z) = S_a(\omega, æ) sin\theta\, exp[-\beta D(H-z)] \quad . \tag{11}$$

As it is seen from these formulae the spectra given in (10) and (11) are independent on the medium sound speed fluctuations under the diffusion approximation. In addition the high moments of the function $|\psi(z,H)|^2$, because of the linearity of equation (2), also are not dependent on the fluctuations. In contrast for the high moments of $Re\{pw^*\}$ this property is not conserved [2].

4. Results of statistical modeling

The spectra given by (3) have been calculated by the statistical modeling method [5]. For this, the model for $\varepsilon(z)$ has been taken to have "white noise" form (7), the frequency is taken to be $f = 1000$ Hz and the wave propagation angle is $\theta = 0.5°$. Two general cases were considered, corresponding to values of the stochastic parameter of $\beta = 7$ and $\beta = 0.07$. Figures 1 to 3 show the results both of the analytical estimations, (9) and (10), and of statistical modeling for the normalized (by the spectral function of surface sources, see (3) and the above expression for the sound pressure) functions

$$<(pp^*)^n> = <|\psi(z,H)|^{2n}>, (n=1,2), \qquad Re\{<pw^*>\} = k_0^{-1} < |\psi(z,H)|^2 Re\{-i\phi_z\}>.$$

These functions correspond respectively to the statistical moments of the plane wave field intensity, and the mean value of the vertical acoustic power flux density. Statistical averaging has been carried out over an ensemble of 1000 realizations. As can be seen from Figure 1, at $\beta = 7$ the results of the statistical modeling and the analytical calculation agree well, both for the first and for the second moment of the intensity. Medium parameter fluctuations do not influence the spectrum formation. However for small values of the stochastic parameter $\beta = 0.07$ (Figure 2), the values and behaviour of the first field intensity moment (curve 2) are considerably different from the results given by Equation (9) after it has been averaged over the fast oscillations (curve 1). Also, the second intensity moment (curve 2) exceeds by several orders the predictions of Equation (9) (curve 1). Hence it was necessary to use the logarithmic scale on the vertical axis to distinguish this curve 1 in the figure. The field spectrum in this case of small value of the stochastic parameter β is essentially defined by the random inhomogeneities of the medium and the diffusion approximation (9) is not valid here. It can be seen that the majority of the field energy is concentrated near the surface layer of the ocean, owing to scattering by the random inhomogeneities. Let us emphasize that this fact demonstrates the significant difference between the problems of wave excitation by mass sources (monopole type) [2, 4] and force sources. It is associated with the differences in the character of the boundary conditions for the same equations of linear acoustics which govern these problems.

Similar behaviour is also observed for the mutual spectrum of power flux presented in Figure 3. For the small values of the parameter β the power flux decays more quickly with the depth (curve 2), than is predicted by the approximation (10) (curve 1).

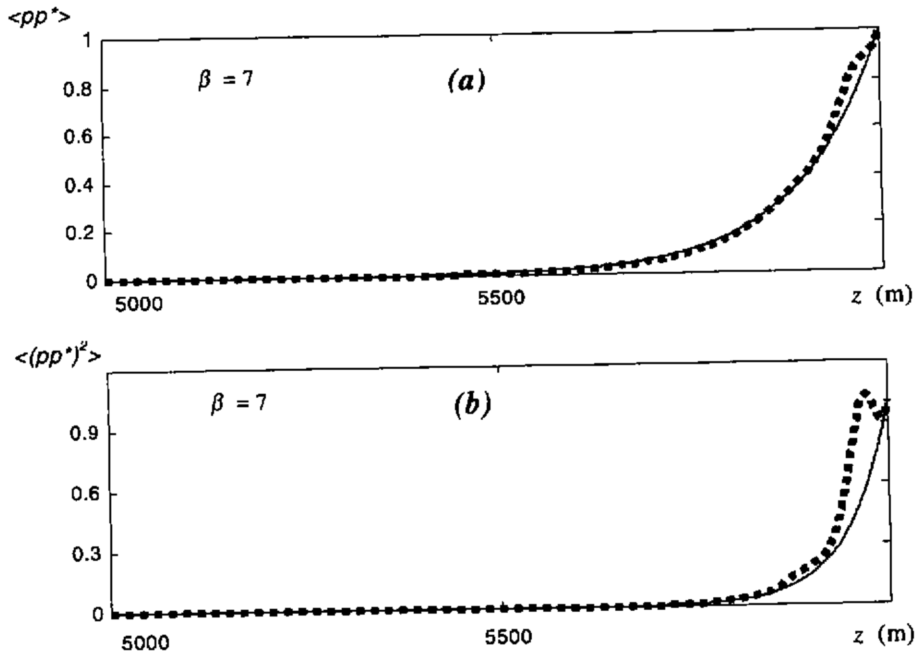

Figure 1. The normalized spatial-time intensity spectrum statistical moments of the noise field as a function of the depth z in the deep ocean (H = 6000 m) under the influence of c(z) random inhomogeneities. Stochastic parameter takes enough large value, β =7. (a) is the first statistical moment; (b) is the second moment. Solid curves correspond to predictions of the diffusion approximation and dotted curves correspond to results of statistical modeling.

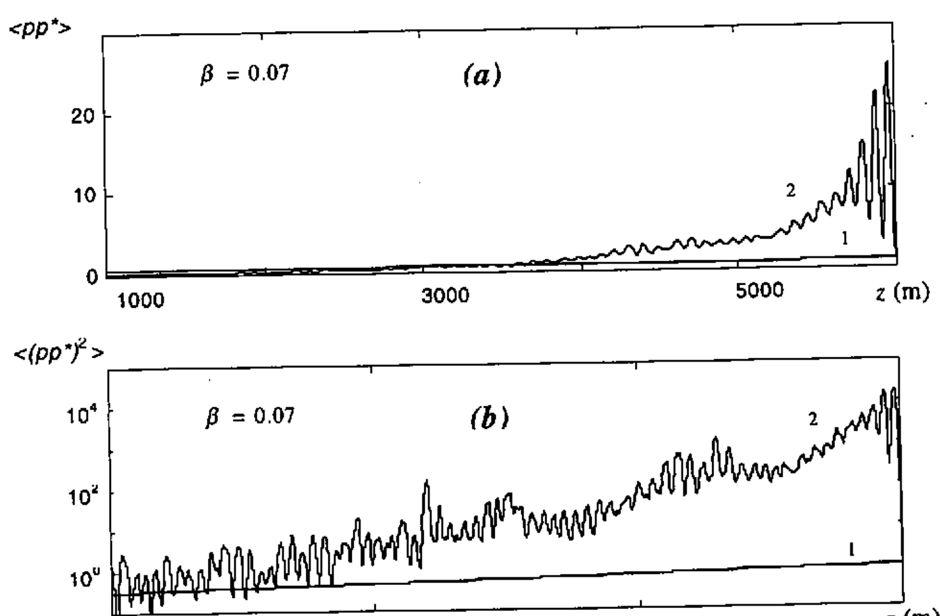

Figure 2. This figure depicts dependencies similar to those shown in to Figure 1, but in the case of there being a small value for the stochastic parameter, β = 0.07. The curves labeled 1 correspond to predictions of the diffusion approximation, whereas the curves labeled 2 correspond to results of statistical modeling.

The calculations have been carried out also for the various impedance conditions on the bottom. Among them, for example, we have considered the case of a homogeneous liquid halfspace with the sound speed and density parameters that are equal to the ones inside the ocean layer: $\alpha_1 = \alpha_2 = 1$, $\gamma_1 = 0$. However for all cases there are no essential variations in spectra behaviour. This can readily be understood in terms of the underlying physics, since we have used a "statistically thick" ocean layer ($HD \approx 8.3$).

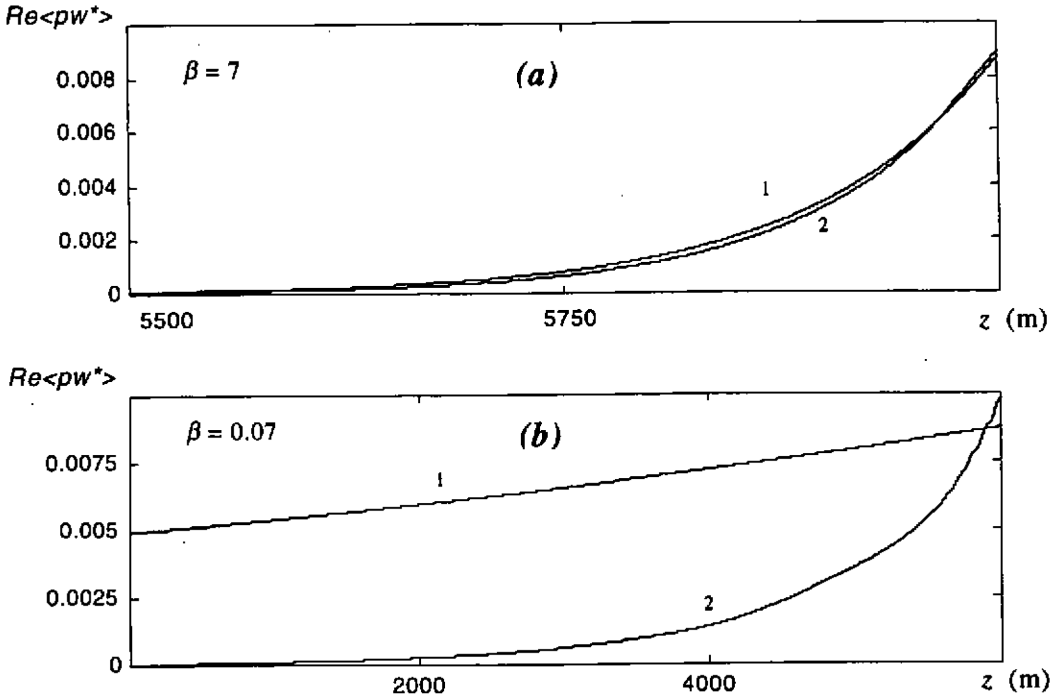

Figure 3. The normalized spatial-time spectrum of average power flux density (its z-component) of the noise field as a function of the depth z in the deep ocean (H = 6000 m) under the influence of c(z) random inhomogeneities. (a) is the case of stochastic parameter $\beta = 7$; (b) is the case of stochastic parameter $\beta = 0.07$. The curves labeled 1 correspond to predictions of the diffusion approximation, and the curves labeled 2 correspond to results of statistical modeling.

5. Final remarks

In conclusion we summarise that only accurate statistical modeling gives the complete pattern of the underwater noise phenomenon in the case of fluctuating sound speed parameter. It allows the description for the regions of stochasticity where the diffusion approximation becomes inapplicable, and we have demonstrated these results here. It should be noticed that during this consideration of the problem, only the influence of the medium sound speed fluctuations have been taken into account by us. However the consideration of the random fluctuations of the density and its gradients does not in principle present any difficulties, both as regards the study of the analytical problem and the method of statistical modeling.

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