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Superposition of Polar Diagrams

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1. Introduction

In the setting up of large Public Address systems it is often desirable to have some idea of the directivity[1] of a loudspeaker array consisting often of many identical 'elements'. For large arrays, direct measurement of the directivity is often not practical and as such a method for predicting the directivity of loudspeaker arrays from the polar data of only a single element[2] is investigated here. In doing so this paper addresses the following question: Can one infer the directivity of a loudspeaker array from the linear superposition of polar data obtained from the measurement of a single element?

There are a number of 'sound' acoustic reasons why this approach could be flawed, these include, cabinet edge diffraction and the additional radiation impedance applied to each loudspeaker as a result of the acoustic field of the others.

Diffraction is a phenomenon caused by acoustic wave fronts diffracting round the cabinet edge much in the same way as coherent light diffracts through apertures. As with light, the diffraction of acoustic wave fronts causes interference patterns resulting in a type of spatial 'comb filtering'. The problem here is that mounting several loudspeakers in close proximity has the effect of increasing the baffle size and thus changing the way in which diffraction effects the polar response. Linear superposition of polar data will not account for this increase in baffle size and one would expect errors in the prediction of the array's polar response towards the sides, particularly at low frequencies.

When two loudspeakers are mounted in relatively close proximity, as is the case for the arrays discussed here, the incident sound field on each source has the effect of increasing the radiation impedance of that source. This has two effects, the first being an increase in the far field sound power output of each source at low frequencies. This is expected, and indeed accounted for by the superposition of the sources. The second is dependant on the type of source. In the case of direct radiating drive units a decrease of the diaphragm velocity can be observed whereas for horn loaded sources the effect is particularly complex.

The following section outlines the assumptions necessary and the fundamental methods used in both the measurement and the manipulation of the polar data.

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[1] The term 'Directivity' is used to describe the general directional characteristics of the loudspeaker or loudspeaker array and not only the directivity factor or directivity index. This holds throughout.
[2] The term 'Element' describes one of the sources mounted in the array. The elements are assumed throughout to be identical as in most practical cases they are.

2. Proposed Method

2.1 Assumptions and simplifications

Since this investigation aims only to assess the feasibility of array directivity prediction using linear superposition, a number of simplifications and assumptions are made. Firstly, the predictions are made for only two loudspeakers mounted side by side with the baffles flush and parallel. The loudspeakers are symmetrical about the vertical plane running from front to back of the enclosure such that a symmetrical horizontal polar response can be assumed. It is also assumed that the vertical response of the loudspeaker array remains unchanged when the loudspeakers are mounted in the configuration described above and as such only the horizontal plane is considered. The theory outlined below can be extended relatively simply to include the prediction of splayed arrays and full three dimensional polar data.

2.2 Theory of Superposition

Figure 1 shows a schematic of the loudspeaker and microphone configuration, s_1 and s_2 being the loudspeakers and $\hat{P}_m$ being the complex pressure amplitude as measured at the microphone position. R is the distance from the common centre of the two loudspeakers to the microphone.

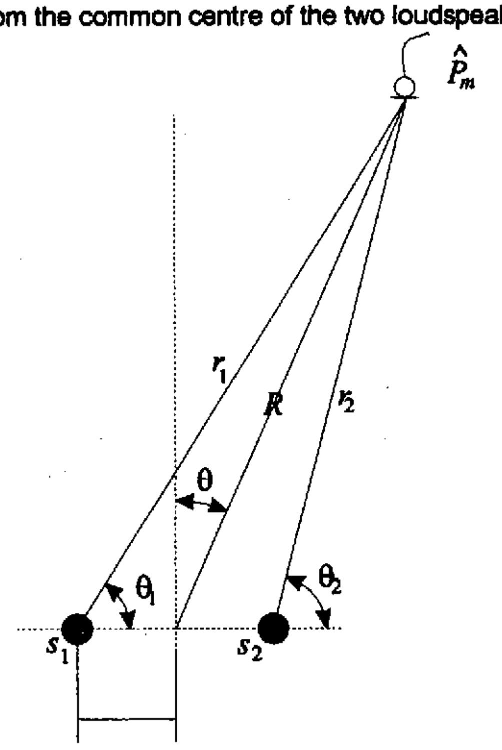

Figure 1: Far Field Superposition Principle Schematic.

With R finite, the expression for $\hat{P}_m$ is given by:

$$\hat{P}_m(k,\theta) = \hat{P}_1(k,\theta_1)e^{-jkr_1} + \hat{P}_2(k,\theta_2)e^{-jkr_2}$$

Where: $\hat{P}_1$ and $\hat{P}_2$ are the complex acoustic pressure amplitudes resulting from sources s_1 and s_2 respectively, and k is the wavenumber given by: $\frac{\omega}{c}$.

However, assuming that both sources are identical and measurements are made in the far field such that $R \rightarrow \infty$ and therefore $\theta_1 = \theta_2$, the expression reduces to:

$$\hat{P}_m(k,\theta) = 2\hat{P}(k,\theta)\cos(ka\sin\theta)$$

where $\hat{P} = \hat{P}_1 = \hat{P}_2$, based on the assumption that the sources are identical.

Measurements[3] are made of a single loudspeaker in order to obtain $\hat{P}$ at 1600 frequency points and an angular resolution of 5°. As mentioned earlier, the loudspeakers symmetry enables the measurements made in one half of the polar plane to be duplicated to give measurements for the opposite half plane. In order to provide a comparison, similar measurements were made for the simple array described in section 2.1 to obtain experimental data for $\hat{P}_m$. All measurements were made at a distance of 5m, more than ten times the horizontal dimension of the source, in order for the far field assumption to hold true. This measuring distance also minimises the effect of the natural acoustic centre of the source not being directly in line with the centre of rotation. Of course this applies only to the position of the acoustic centre on the line running from the front to the rear of the loudspeaker, as the position from left to right is defined by the model.

3. Results

The results are displayed below in three different formats. Polar diagrams of the superposed (dashed) and directly measured data (solid) are given at 500Hz intervals between 500Hz and 5000Hz. Polar waterfall plots give an overall view of both the superposed and directly measured data between 500Hz and 5000Hz. A similar plot of the directly measured data for a single loudspeaker is also included for interest. As a final indication of the success of the method, a plot of Directivity Index vs. Frequency is plotted for both superposed (dashed) and directly measured data (solid). It should be noted that the calculation of Directivity Index is made using only horizontal data and therefore only relevant for comparative purposes. No attempt has been made to infer the values which would normally be obtained from full spherical data. The 500Hz to 5000Hz frequency band is chosen since at frequencies below 500Hz, the response of all but the largest sources has little, if any dependence on direction. At frequencies above 5000Hz, the radiated sound field must obey the superposition principle, as it is not effected by the surroundings and more specifically edge diffraction.

[3] All measurements were made in the large anechoic chamber at the Institute of Sound and Vibration Research.

3.1 Polar Diagrams

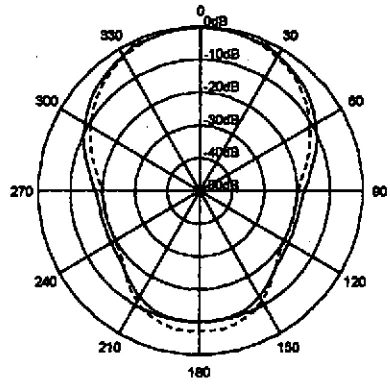

Figure 2: 500Hz

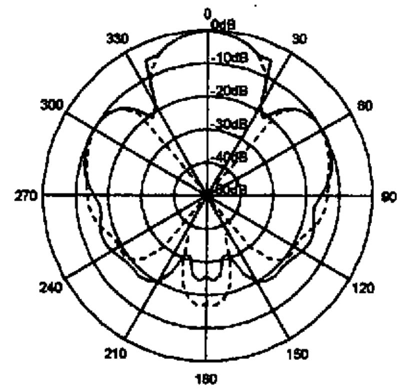

Figure 3: 1000Hz

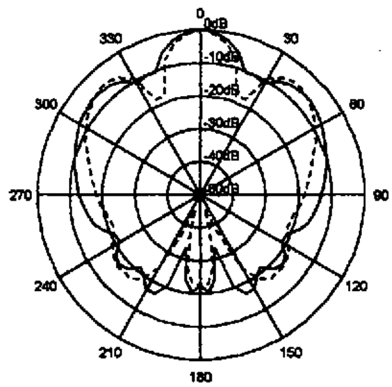

Figure 4: 1500Hz

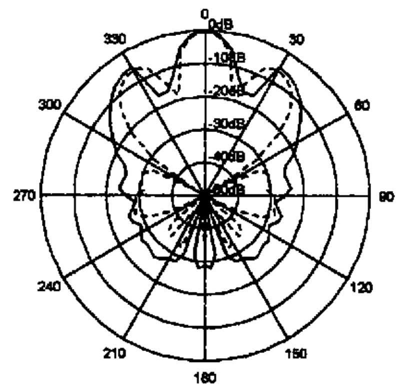

Figure 5: 2000Hz

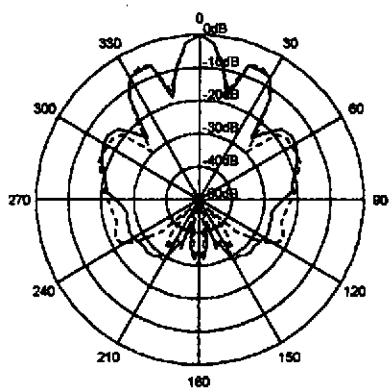

Figure 6: 2500Hz

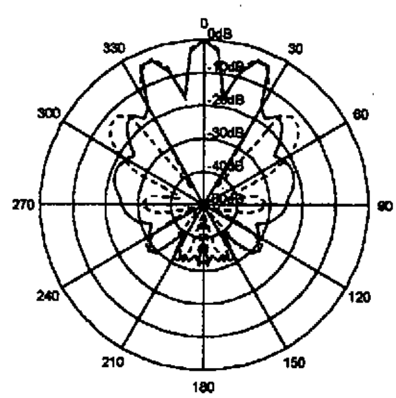

Figure 7: 3000Hz

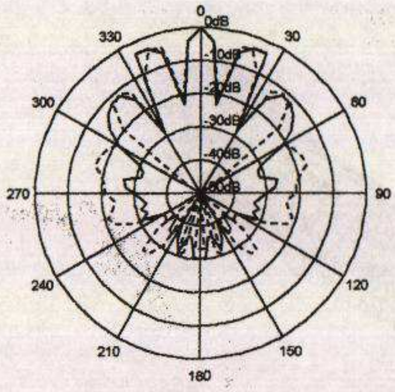

Figure 8: 3500Hz

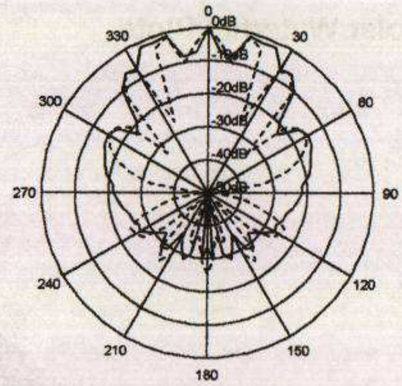

Figure 9: 4000Hz

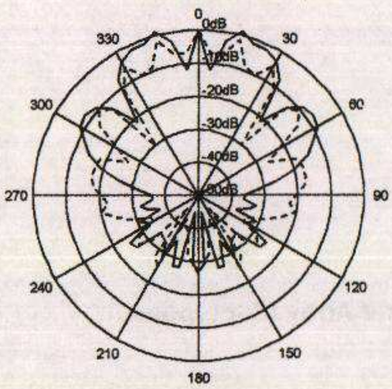

Figure 10: 4500Hz

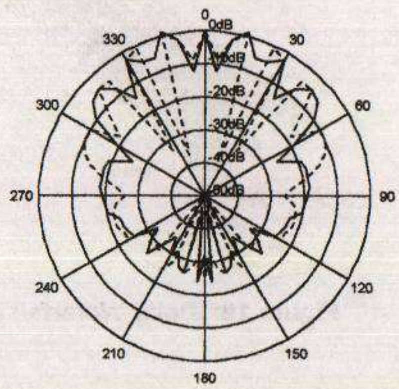

Figure 11: 5000Hz

3.2 Polar Waterfall Plots

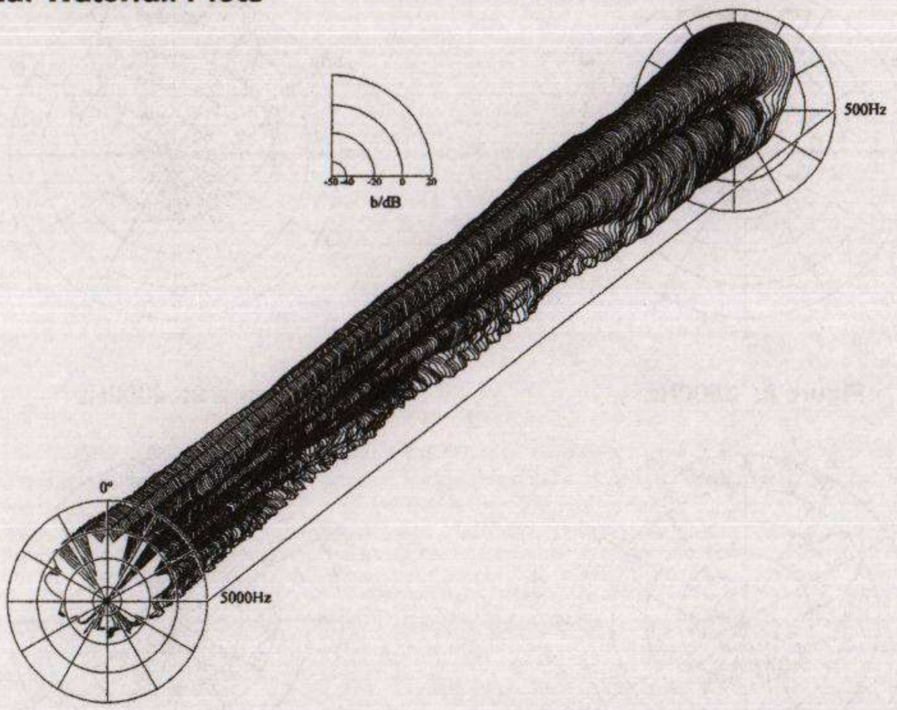

Figure 12: Polar Waterfall for Loudspeaker Array (Superposed).

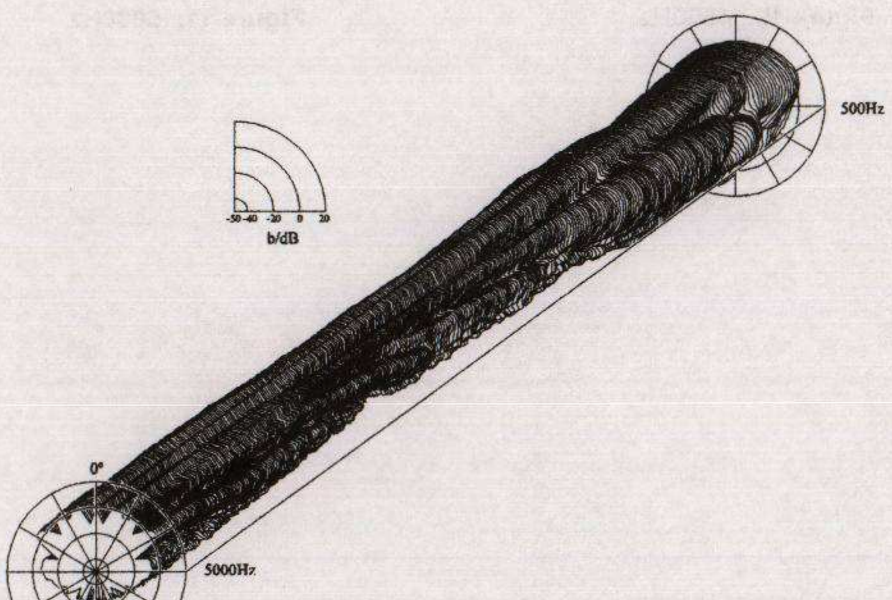

Figure 13: Polar Waterfall for Loudspeaker Array (Directly Measured).

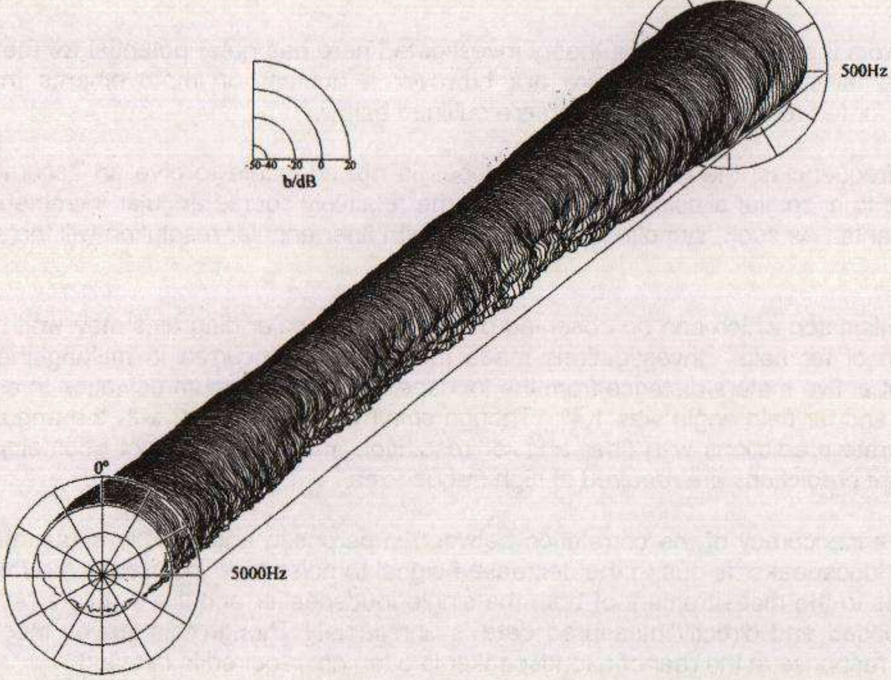

Figure 14: Polar Waterfall for Single Loudspeaker (Directly Measured)

3.3 Directivity Index vs. Frequency

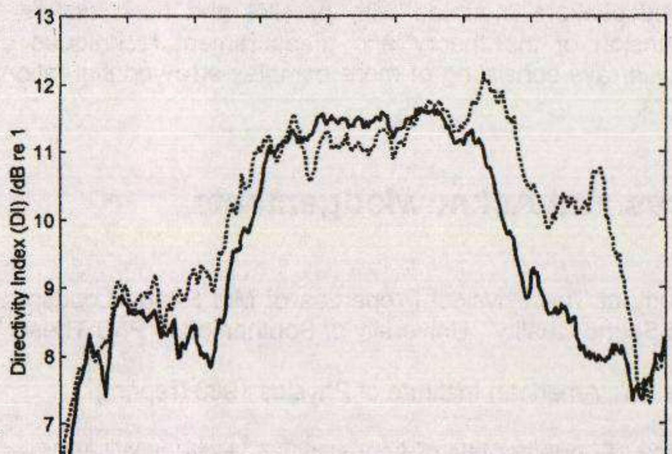

Figure 15: Plot of Directivity Index vs. Frequency Calculated from Predicted (dashed) and Measured (solid) Data.

4. Conclusions and Discussion

It is clear from the results, that the theory investigated here has great potential for the prediction of loudspeaker array directivity. There are however a number of improvements that would be suggested for further investigation which are outlined below.

At higher frequencies, the definition of the lobes is not adequate to give an accurate prediction. This is due to a 'spatial aliasing' resulting from the relatively course angular increment used in the measurements. As such, sampling the response with finer angular resolution will increase the lobe definition.

The lobe mismatch which can be observed on some of the polar diagrams may well be due to the assumption of far field. Investigations made into the error incurred in making the assumption proved that at five meters distance from the loudspeaker, the maximum deviation in angle between the actual and far field angle was 1.4°. Though small when compared with the angular increment more accurate predictions with finer angular resolution, may require exact geometry to be used, particularly if predictions are required at high frequencies.

The relative inaccuracy of the correlation between superposed and directly measured data at the rear of the loudspeaker is due to the decreased signal to noise ratio incurred in the measurements. This applies to the measurement of both the single loudspeaker and the simple array and so both the superposed and directly measured data is unreliable. Though this leaves inaccuracies, the directional response at the rear of a loudspeaker is often not required in detailed.

The application of this method to three dimensional data, would involve only the expansion of the geometry and of course the acquisition of all the required data.

And so to answer the question posed in the introduction. It would certainly appear that linear superposition of polar data can give a reliable prediction of the directivity of a loudspeaker array consisting of two loudspeakers mounted with parallel and flush baffles. This can only instil confidence that expansion of the theory and measurement techniques used here, will allow accurate prediction for arrays consisting of more complex array configurations possibly containing many splayed elements.

5. References and Acknowledgements

K. R. Holland, "A Study of The Physical Properties of Mid Range Loudspeaker Horns and Their Relation to Perceived Sound Quality", University of Southampton, PhD Thesis, 1992.

L. L. Beranek, "Acoustics", American Institute of Physics 1986 (reprint).

L.A. Kinsler & A.R. Frey, "Fundamentals of Acoustics", 3rd Edition, Wiley Press, 1982.

K. Weltner, J. Grosjean, P. Schuster, W.J. Weber, "Mathematics for Engineers and Scientists", Stanley Thorne, 1995.

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