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THE PRINCIPLE OF PERFECT COVERAGE

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INTRODUCTION

This Paper examines the effect of perfect loudspeaker coverage on direct-to-reverberant ratio within a space. It further determines the effect on other relevant sound system performance descriptors.

THE PRINCIPLE

The principle is based on the assumption that a perfect radiator, whose energy is solely confined within its radiation angles may be contrived to have variable dispersion characteristics such that it precisely covers the target or audience area.

The principle which will be demonstrated may be stated thus:

"If a radiator whose energy is confined within its radiation angle is contrived to provide uniform coverage over the required area, then the direct-to-reverberant ratio is independent of the distance between the source and receive area and further the direct-to-reverberant ratio is maximised and is a function of the acoustics of the space and the area to be covered."

THEORY

In the following derivation, conventional statistical room acoustics is assumed.

The acoustic field in a space is given by

$$L_p = L_w + 10Lg\left(\frac{Q}{4\pi r^2} + \frac{4}{R}\right) \qquad \text{[1]}$$

where: L_p = sound pressure level dB re 20µPa
L_W = sound power level dB re 10^{-12}Watt
Q = Source directivity
r = distance between source and receive positions (m)
R = Room constant (m^2)

given by: $R = \frac{S\bar{\alpha}}{1-\bar{\alpha}}$

where S = surface area of space (m^2)
$\bar{\alpha}$ = mean absorption coefficient

given by $\bar{\alpha} = \frac{\sum_1^n s_n \alpha_n}{\sum_1^n s}$

where: s and α are the surface areas and absorption coefficients of each surface respectively.

when: $\frac{Q}{4\pi r^2} >> \frac{4}{R}$

then the direct component predominates and equation [1] may be re-written as

$$L_{p,d} = L_w + 10Lg\left(\frac{Q}{4\pi r^2}\right) \qquad \text{[2]}$$

where: $L_{p,d}$ = direct sound pressure level dB re 20µPa

and when $\frac{4}{R} >> \frac{Q}{4\pi r^2}$

Then the reverberant field predominates and equation [1] may be rewritten as:

$$L_{p,r} = L_W + 10\mathrm{Lg}\left(\frac{4}{R}\right) \qquad \text{[3]}$$

Now the direct-to-reverberant ratio = $L_{p,d}$ - $L_{p,r}$

Hence substituting equation [3] and [2] we get

$$D/R = L_{p,d} - L_{p,r} = 10\mathrm{Lg}\left(\frac{Q}{4\pi r^2}\right) - 10\mathrm{Lg}\left(\frac{4}{R}\right)$$

which reduces to

$$D/R = 10\mathrm{Lg}\left(\frac{Q}{4\pi r^2}.\frac{R}{4}\right) \qquad \text{[4]}$$

To examine the implications on perfect coverage, consider fig. 1 below:

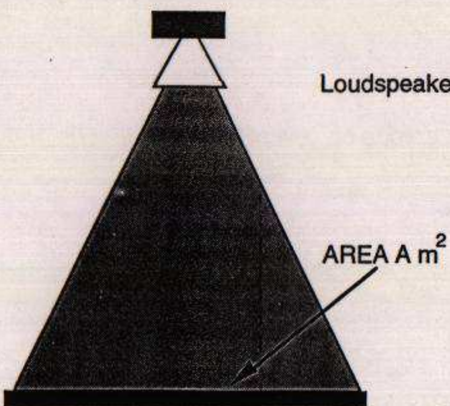

Fig. 1

In the above figure the loudspeaker L is required to have dispersion characteristics which precisely match the area to be covered A m^2 and in this example the Q of the loudspeaker is contrived to fit the coverage area.

If we assume that the coverage area is found on the surface of a sphere with radius r as shown in fig. 2 below:

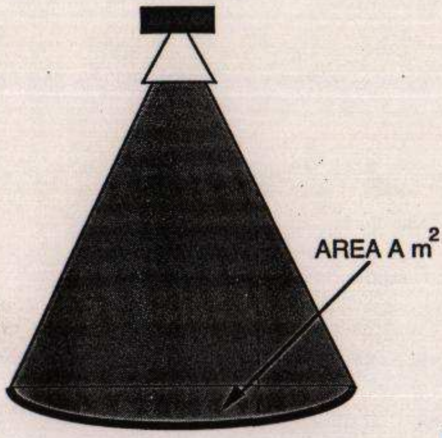

Fig. 2

then by definition $Q = \frac{4\pi r^2}{A}$

where: A = the area to be covered.

Hence substituting in equation [4] we get

$$D/R = 10Lg\left(\frac{1}{4\pi r^2}.\frac{R}{4}.\frac{4\pi r^2}{A}\right)$$

which reduces to

$$D/R = 10Lg\left(\frac{R}{4A}\right) \qquad \text{..........[5]}$$

From equation [5] above we may observe that the direct-to-reverberant ratio is independent of the distance between source and receive positions and is only a function of the space.

Note: The foregoing result is subject to certain assumptions:

(a) Perfect coverage is obtained.
(b) The loudspeaker is a perfect radiator.
(c) Statistical room acoustics are applicable.
(d) The coverage area is assumed to be on the surface of a sphere.

IMPLICATIONS OF THE PRINCIPLE

The apparent independence of system components and the emphasis on the acoustics of the space implies that provided that best practice has been observed, the ultimate system performance is a function of the acoustics of the space and of the area to be covered.

It is useful to re-arrange and examine equation [5] in order to fully appreciate the implications.

Consider equation [5]

$$D/R = 10\mathrm{Lg}\left(\frac{R}{4A}\right)$$

A sound system designer when designing for maximum speech intelligibility attempts to maximise the D/R ratio. A salient value of D/R is when $L_{p,d} = L_{p,r}$ i.e. at one critical distance.

In logarithmic terms, D/R = 0dB.

Hence for unity D/R; $\frac{R}{4A} = 1$ or for positive D/R $\frac{R}{4A} \geq 1$

But $R = \frac{S\overline{\alpha}}{1-\overline{\alpha}}$

Hence for unity D/R: $\frac{S\overline{\alpha}}{1-\overline{\alpha}} \geq 4A$

$$\text{or} \quad \frac{S}{A} \geq \frac{4(1-\overline{\alpha})}{\overline{\alpha}} \qquad \text{[6]}$$

A knowledge of the total surface area and the mean absorption coefficient provides an immediate estimation if the criteria of D/R = 0dB can be met.

In most acoustically difficult situations $\bar{\alpha} << 1$.

Hence equation [6] may be rewritten as:

$$\frac{S}{A} \geq \frac{4}{\bar{\alpha}}$$

$$\text{or} \qquad S\bar{\alpha} \geq 4A \qquad \text{[7]}$$

We may readily see therefore that as the area to be covered increases, a corresponding increase in absorption is required.

USEFUL PERSPECTIVES

Examination of some simple cases illustrates the degree of difficulty in achieving D/R ≥ 0dB.

Consider a space formed as a cube.

Then if the plan area is the area to be covered (see fig. 3 below):

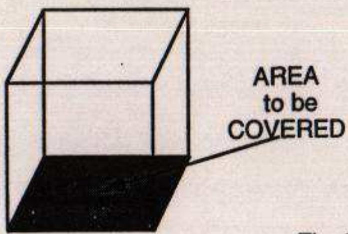

Fig. 3

Then S = 6A.

Hence substituting in equation [6] we get

$$\frac{6A}{A} \geq \frac{4(1-\bar{\alpha})}{\bar{\alpha}}$$

From the foregoing equation we can see that $\overline{\alpha} \geq 0.4$ for a cube to achieve unity D/R.

The concept may be extended to rectangular spaces (see fig. 4 below):

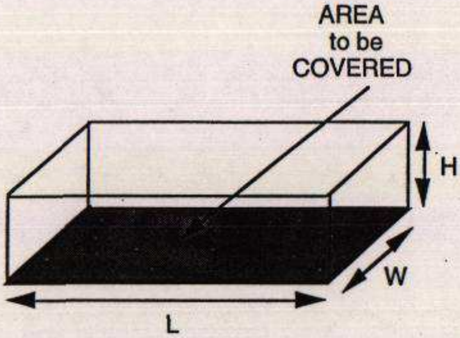

Fig. 4

Consider the space in fig. 4 above where:

L = Length (m)
W = Width (m)
H = Height (m).

But $RT = \dfrac{0.16V}{S\overline{\alpha}}$ or $S\overline{\alpha} = \dfrac{0.16V}{RT}$

where: RT = Reverberation Time (secs.)
V = Volume of space (m^3)
S = Surface area of space (m^2)
$\overline{\alpha}$ = mean absorption coefficient.

Then substituting in[7] we get

$$\frac{0.16V}{RT} \geq 4A$$

or $$\frac{V}{A} \geq 25RT \qquad \text{[8]}$$

But $V = AH$ and substituting in [8]

We get $H \geq 25RT$ ....[9]

or equation [9] may be written in a general format as

$$H \geq 25RT(1-\overline{\alpha}) \qquad \text{[10]}$$

The above gives a ready quick assessment of the likelihood of success.

EFFECT OF PERFECT COVERAGE ON %ALCONS

Although not necessarily a robust indicator of speech intelligibility, it is worth noting the effect of perfect coverage on %ALcons.

Consider the simple %ALcons expression

vis: $$\%AL_{cons} = \frac{200D^2RT^2}{VQ} \qquad \text{[11]}$$

If perfect coverage is obtained then $Q = \dfrac{4\pi D^2}{A}$

where: A = Area to be covered.

Substituting in equation [11] we get

$$\%AL_{cons} = \frac{200D^2RT^2A}{4\pi D^2V}$$

which reduces to

$$\%AL_{cons} = \frac{50}{\pi} \cdot \frac{A}{V} \cdot RT^2 \qquad \text{[12]}$$

For a rectangular space V = AH.

Hence substituting in equation [12] we get

$$\%AL_{cons} = \frac{50}{\pi} \cdot \frac{A}{AH} \cdot RT^2$$

which reduces to

$$\%AL_{cons} = \frac{16}{H} \cdot RT^2 \qquad \text{..........[13]}$$

For $\%AL_{cons} \leq 10\%$

We get $\quad 10 \geq \frac{16}{H} \cdot RT^2$

Hence: $\quad H \geq 1.6RT^2 \qquad$ ....[14]

where: $\quad H$ = Height of space in (m).

CONCLUSIONS

The principle of perfect coverage does provide a useful quick indicator of sound system performance.

It should be noted that this treatment may be extended to include multiple source distributed systems.