

## TOWARDS A FORWARD LOOKING SYNTHETIC APERTURE SONAR

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### 1. INTRODUCTION

The concept of SAS (Synthetic Aperture Sonar) is generally studied in the side looking mode corresponding to route survey missions. Side looking SAS generates high quality sonar images. In the squint mode, the beam is no longer perpendicular to the sonar track but pointed forward. Theory shows that the azimuth resolution of squint SAS is equal to half the physical antenna length. By azimuth resolution we are referring to the resolution obtained perpendicularly to the observation direction. Applying the squint concept to the extreme leads to the forward looking SAS. This configuration is interesting in mine hunting because its azimuth resolution is much better than the azimuth resolution of a conventional sonar which is range and frequency dependent.

### 2. SQUINT SYNTHETIC APERTURE SONAR THEORY

#### 2.1 Synthetic antenna length

The squint SAS configuration is presented in Figure 1. The antenna moves along the y axis and observes a target T located in  $(R_c, 0)$ , where  $R_c$  is the closest sonar target range.  $S$ ,  $\theta$  and  $L_a$  are respectively the squint angle, antenna beamwidth and antenna length.

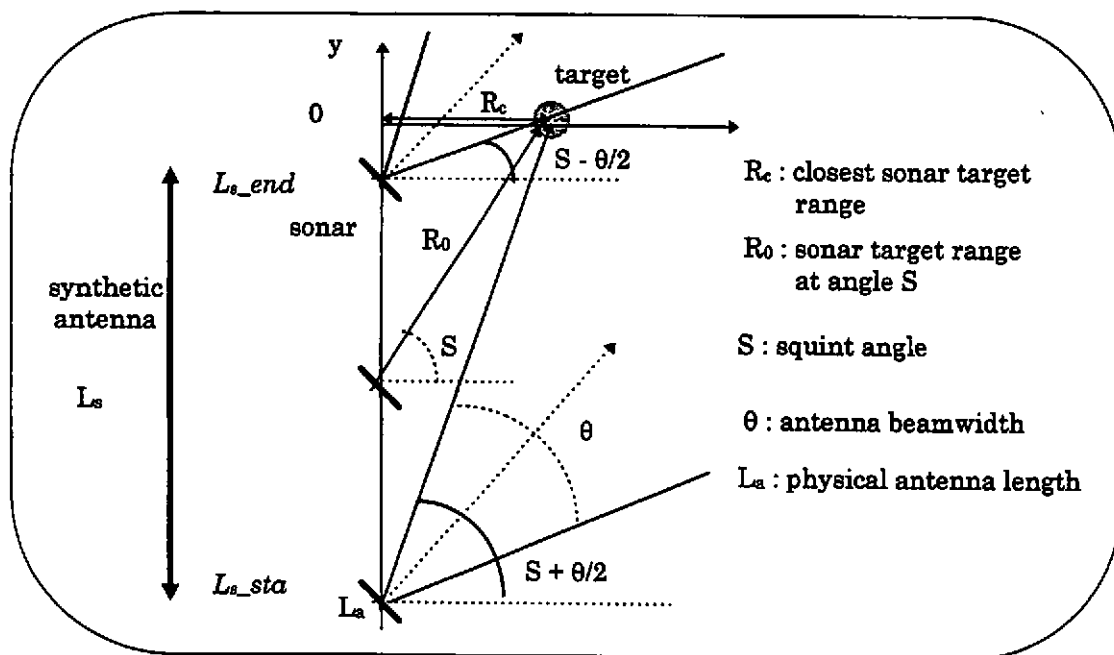


Figure 1 squint SAS

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The synthetic antenna corresponds to the sonar positions when the target is in the sonar beam. The synthetic antenna length is given by the two extreme sonar positions :

$$L_{s\_sta} = R_c \tan(S + \frac{\theta}{2}) \quad \text{and} \quad L_{s\_end} = R_c \tan(S - \frac{\theta}{2}) \quad (1)$$

The synthetic antenna length can be expressed by :

$$L_s = L_{s\_end} - L_{s\_sta} = R_c \left( \tan(S + \frac{\theta}{2}) - \tan(S - \frac{\theta}{2}) \right) = \frac{2 R_c \tan(\frac{\theta}{2}) (1 + \tan^2 S)}{1 - \tan^2(\frac{\theta}{2}) \tan^2 S} \quad (2)$$

### 2.2 Doppler frequency and Doppler band

The Doppler frequency  $f_D$  is the difference between the received signal frequency and the transmitted signal frequency. The Doppler frequency is a function of  $V$  (sonar speed),  $\lambda$  (signal wavelength) and  $\alpha$  (angle between the sonar speed vector and the observation vector) (Figure 2).

$$f_D = \frac{2 \vec{V} \cdot \vec{u}}{\lambda} = \frac{2 V}{\lambda} \cos(\alpha) = \frac{2 V}{\lambda} \sin(\beta) \quad (3)$$

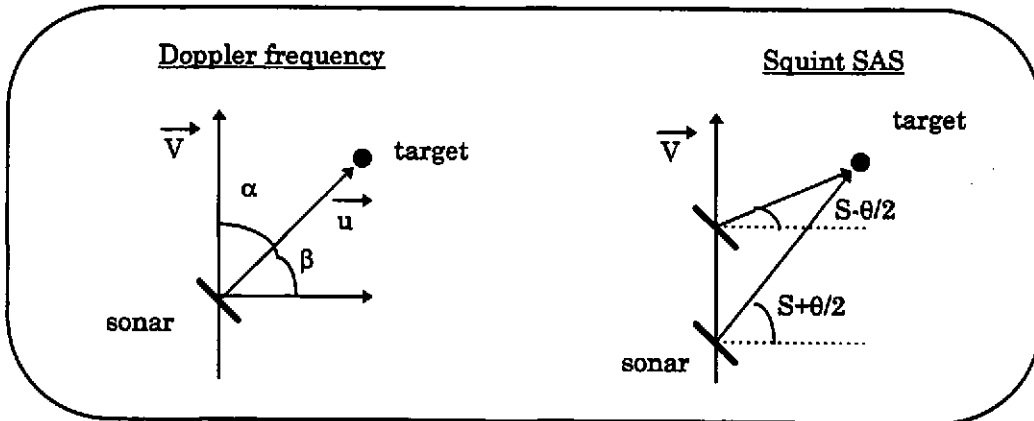


Figure 2 Squint SAS - Doppler frequency

For a squint SAS, the observation angle goes from  $S + \theta/2$  to  $S - \theta/2$ . The two extreme Doppler frequencies  $f_{D1}$  et  $f_{D2}$  are:

$$f_{D1} = \frac{2 V}{\lambda} \sin(S + \frac{\theta}{2}) \quad \text{and} \quad f_{D2} = \frac{2 V}{\lambda} \sin(S - \frac{\theta}{2}) \quad (4)$$

The Doppler band is the difference between these two Doppler frequencies.

$$B_D = f_{D1} - f_{D2} = \frac{4 V}{\lambda} \sin(\frac{\theta}{2}) \cos S \quad (5)$$

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### 2.3 Azimuth resolution

It has been shown that Doppler frequency is a linear function of time [1]. The SAS processing corresponds to a matched filter based on the linear frequency modulation.

The along track time resolution is then :

$$\Delta t = \frac{1}{B_D} = \frac{\lambda}{4 V \sin \frac{\theta}{2} \cos S} \quad (6)$$

The along track spatial azimuth  $r_{a_y}$  is equal to :

$$r_{a_y} = V \Delta t = \frac{\lambda}{4 \sin \frac{\theta}{2} \cos S} \quad (7)$$

The azimuth resolution  $r_a$ , perpendicular to the observation direction, is :

$$r_a = r_{a_y} \cos S = V \Delta t \cos S = \frac{\lambda}{4 \sin \frac{\theta}{2}} \quad (8)$$

### 2.4 Small antenna beamwidth application

Considering a small antenna beamwidth we have :  $\tan(\frac{\theta}{2}) \approx \sin(\frac{\theta}{2}) \approx \frac{\theta}{2}$  and  $1 - \tan^2(\frac{\theta}{2}) \tan^2 S \approx 1$

The previous expressions (2) (5) (8) can be simplified into :

2.4.1 Synthetic antenna length :

$$L_s \approx 2 R_c \frac{\theta}{2} (1 + \tan^2 S) = \frac{R_c \theta}{\cos^2 S} \quad \text{and} \quad \theta = \frac{\lambda}{L_a}$$

so

$$L_s = \frac{R_c \lambda}{L_a \cos^2 S} = \frac{R_0 \lambda}{L_a \cos S} \quad (9)$$

For a squint SAS, the synthetic antenna length is range  $R_c$  linear and inversely proportional to the squared cosinus of the squint angle  $S$ . The synthetic antenna length, and consequently the sonar recurrences number to integrate for SAS processing, strongly increase with the squint angle.

2.4.2 Doppler band :

$$B_D = \frac{2V}{\lambda} \theta \cos S \quad \text{so} \quad B_D = \frac{2V}{L_a} \cos S \quad (10)$$

The Doppler band is smaller for squint SAS than for a side looking SAS.

2.4.3 Azimuth resolution :

$$r_a = \frac{\lambda}{4 \sin \frac{\theta}{2}} \approx \frac{\lambda}{2 \theta} \quad (11)$$

The azimuth resolution is equal to the ratio of the signal wavelength over twice the transmission antenna beamwidth [2]. Replacing the antenna beamwidth with its expression function of the antenna length, we get the final result : the azimuth resolution of SAS processing is equal to half the transmission antenna length. This result is independent from the squint angle.

$$r_a = \frac{L_a}{2} \quad (12)$$

### 2.5 Spatial sampling

In order to avoid aliasing or azimuth ambiguities, the spatial sampling or PRF (Pulse Repetition Frequency) has to be greater than or equal to the Doppler band (Shannon theorem).

$$PRF \geq B_D = \frac{2V \cos S}{L_a} \quad (13)$$

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Since the Doppler band is smaller for squint SAS than for side looking SAS (10), this enables the use of smaller PRF [3]. The maximum spatial sampling or ping to ping sonar displacement is then :

$$\Delta_s = \frac{V}{PRF} = \frac{L_a}{2 \cos S} \quad (14)$$

For instance, at a squint angle of 60°, the maximum sonar speed can be doubled compared to side looking SAS.

### 3. SIMULATIONS

#### 3.1 Azimuth resolution

Squint SAS simulations have been done for testing the previous results. These simulations are based on basin experiments conducted in CPE Lyon.

The parameter of the first simulations was the squint angle. Three values of squint angle have been studied: 0, 40 and 80 degrees. The transmitted signal frequency is 800 kHz. The sonar antenna is a single transducer 2 cm long. The antenna beamwidth is 5.3 degrees. The target is a 2.6 cm diameter sphere. The spatial sampling is set to a quarter of the antenna length.

The basin dimensions (2m\*1m\*1m) and the synthetic antenna length increase with squint angle have required an adaptation of the sonar target range  $R_c$  as follows :

$$R_c = \sqrt{X_c^2 + h^2} \quad \text{and} \quad X_c = X_0 \cos S \quad (15)$$

with  $X_0$  being the closest sonar target abscissa and  $h$  the antenna height above the basin bottom

The raw images and the SAS processed images for the squint angles of 0, 40 and 80 degrees are presented in Figures 3.a to 3.f. We notice that the synthetic antenna length increases with the squint angle. The parabolic migration of the target echoes is transformed into a linear migration.

In each case, the target echoes of the raw images are relocated in the target coordinates ( $R_c, 0$ ) on the processed images. The azimuth resolution is equal to half the antenna length : 1 cm. This result is independent from the squint angle. The sidelobes direction is perpendicular to the observation direction.

#### 3.2 Azimuth ambiguities

The objective of the second series of simulations was to study the azimuth ambiguities. The transmitted signal frequency is 800 kHz. The sonar antenna is a single transducer 1 cm long. The antenna beamwidth is 10.7 degrees. The target is a 2.6 cm diameter sphere. The closest sonar target range is set at 50 cm. The squint angle is constant and equal to 40 degrees.

Two sonar displacement  $\Delta_s$  values have been selected. The first  $\Delta_s$  value is equal to  $L_a$  the physical antenna length (Figure 4.a). After SAS processing, the target echoes are relocated in the focalisation point ( $R_c, 0$ ) and two grating lobes appear (Figure 4.b). We note the asymmetry of the azimuth cut (Figure 4.c) due to squint SAS processing [4].

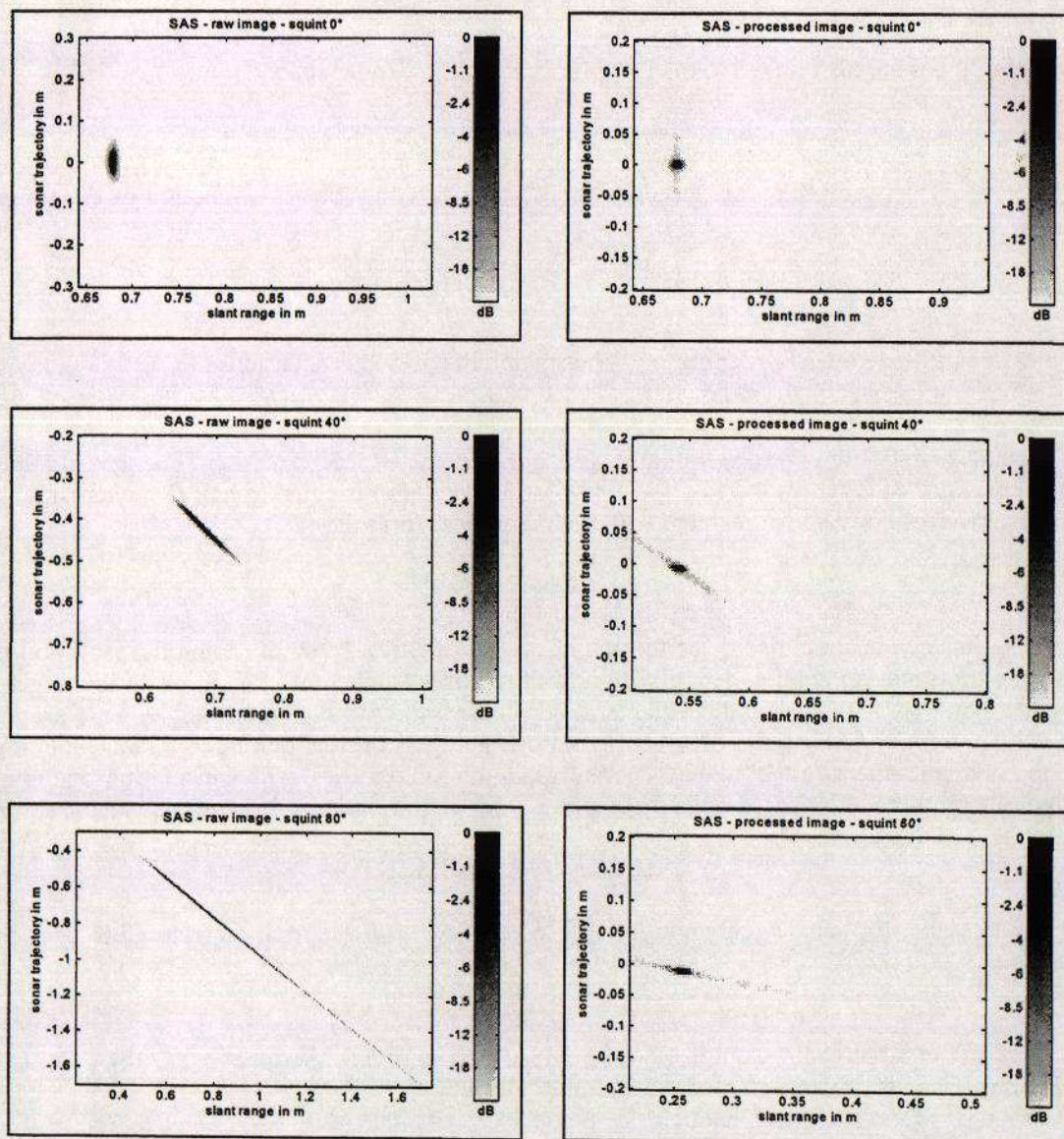
The squint SAS beam pattern directivity expression is :

$$D_s(\theta) = \frac{\sin\left(\frac{2\pi L_s}{\lambda} (\sin \theta - \sin S)\right)}{\sin\left(\frac{2\pi \Delta_s}{\lambda} (\sin \theta - \sin S)\right)} \quad (16)$$



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$$X_c = X_0 \cos S - X_0 = 65 \text{ cm} - h = 25 \text{ cm} - L_a = 2 \text{ cm} - f_0 = 800 \text{ kHz} - \theta = 5,3^\circ - \Delta_S = L_a/4$$



Figures 3.a to 3.f Squint SAS - azimuth resolution -  $0^\circ \leq S \leq 80^\circ$



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The observation direction  $\theta_k$  of the  $k^{\text{th}}$  grating lobe is :

$$\sin \theta_k = \sin S + k \frac{\lambda}{2 \Delta_s} \quad (17)$$

The grating lobes are also linked to the Doppler centroid  $f_{DC}$ , sonar speed  $V$  and  $PRF$  [5] :

$$\Delta_s = V T = \frac{V}{PRF} \quad \text{and} \quad f_{DC} = \frac{2V}{\lambda} \sin S \quad \sin \theta_k = \frac{\lambda}{2V} (f_{DC} + k PRF) \quad (18)$$

The interval  $d_{\theta k}$  between the main lobe and the  $k^{\text{th}}$  order grating lobe is a function of the sonar target distance.

$$d_{\theta k} = R_0 \tan (S - \theta_k) = \frac{R_0 \tan (S - \theta_k)}{\cos S} \quad (19)$$

The location of the two first order grating lobes match the previous equations (18) (19) (figure 4.c) :

$$k = +1 \quad \theta_{+1} = 47,4^\circ \quad d_{\theta+1} = -8,5 \text{ cm} \quad k = -1 \quad \theta_{-1} = 33,3^\circ \quad d_{\theta-1} = 7,6 \text{ cm}$$

The second spatial sampling  $\Delta_s$  value is set to  $L_a/(2 \cos S)$  (i.e.  $0.65 L_a$ ). The grating lobes are effectively canceled (Figures 4.d to 4.f). The azimuth resolution is equal to half the antenna length, 0.5 cm. These two results are in agreement with the theoretical formulas (12) (14).

### 4. CONCLUSION

This paper has demonstrated some of the squint SAS properties. First, the azimuth resolution is equal to half the transmission antenna length. This result requires a coherent integration of the target echoes along the synthetic antenna length which strongly increases with the squint angle. This property has been validated for various squint SAS configurations going from side looking to forward looking SAS. Second, synthetic aperture technique generates azimuth ambiguities. The cancelation of the azimuth ambiguities requires an adapted spatial sampling. This requirement is less restrictive for squint SAS than for side looking SAS.

### 5. ACKNOWLEDGMENT

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### 6. REFERENCES

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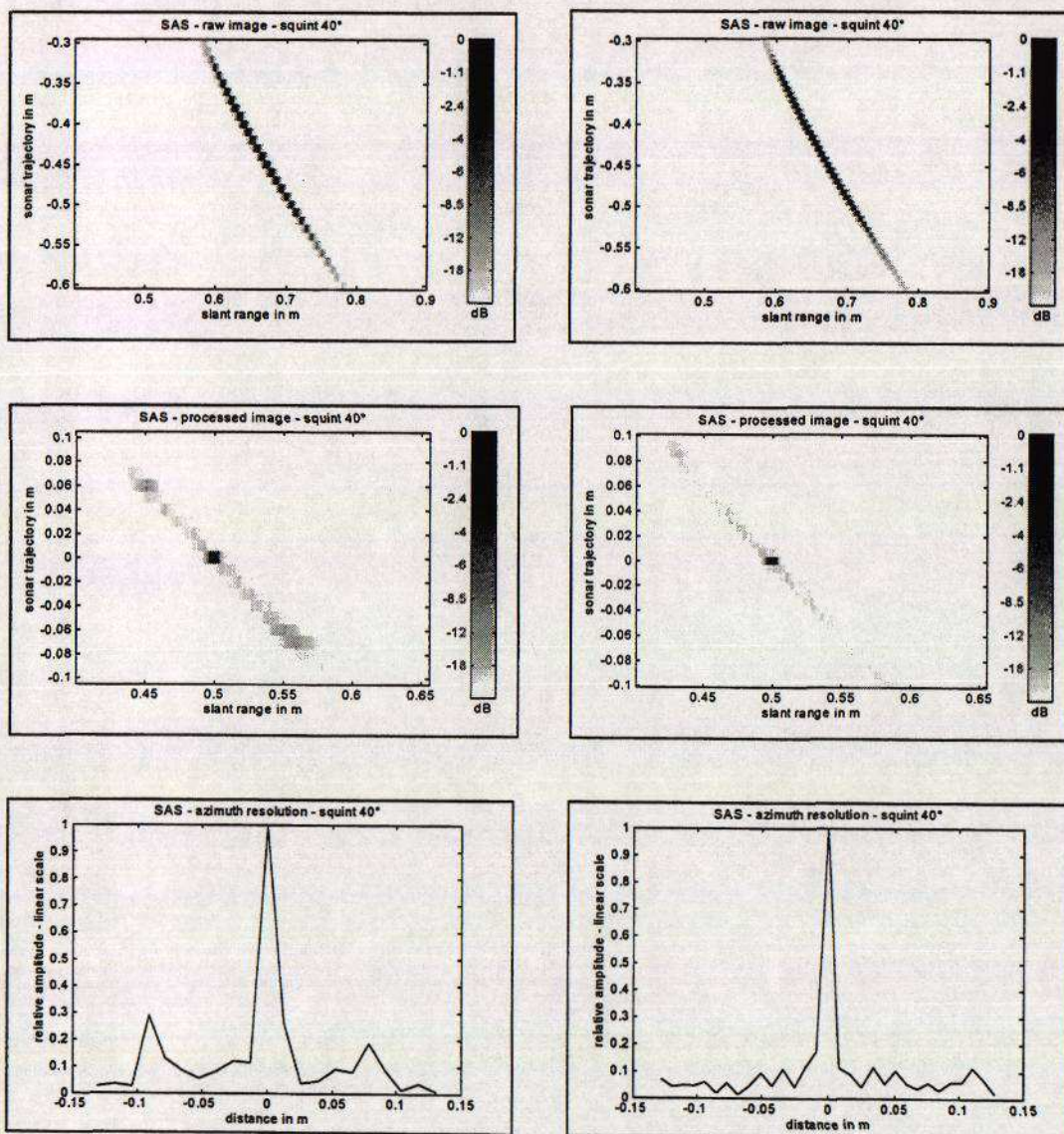


# TOWARDS A FORWARD LOOKING SYNTHETIC APERTURE SONAR

$$R_c = 50 \text{ cm} - L_a = 1 \text{ cm} - f_0 = 800 \text{ kHz} - \theta = 10,7^\circ$$

$$\Delta_s = La$$

$$\Delta_s = La / (2 \cos S) = 0,65 La$$



Figures 4.a to 4.f Squint SAS - Azimuth ambiguities -  $S = 40^\circ$



