VIBRATION IN ONE-DIMENSIONAL STRUCTURES

INTRODUCTION

The control of vibration in beams or other one-dimensional structures is of considerable interest in practical engineering. With the majority of industrial machinery installations, it is this type of structure, for example, pipework vibrating at low frequencies and other mechanical linkages, which forms one of the main vibration paths which bypass isolator systems. It is well known that the introduction of discontinuities into these types of structures significantly affects the vibration transmission properties of the complete system. One such discontinuity representative of a vibration control device is the vibration neutralizer. In this study, neutralizers mounted asymetrically on one-dimensional structures are considered with particular reference to vibrational power transmission and wave-type conversion.

The performance characteristics of the device has been assessed using the concept of vibrational power transmission and for the purposes of this study the neutralizer is considered to be a simple lumped mass/spring single degree of freedom device attached to an infinite beam-like structure by means of a rigid, massless moment arm link. Both the neutralizer and the beam are assumed to be undamped and the beam is excited, into either flexural or axial motion, by a point harmonic force at a position remote from the point of attachment of the device.

ANALYSIS OF DISCONTINUITIES

For the purposes of this study, the vibration control performance of the discontinuity considered will be assessed using the concept of vibrational power transmission. Vibration power reflection and transmission coefficients are derived from a knowledge of the resulting reflected and transmitted wave amplitude coefficients which arise as a result of the action of the neutralizer. The mounting configuration of the neutralizer is shown in Figs. 1a and 1b respectively, for the condition of either flexural or axial wave field impinging.

Flexural Wave-Field Impinging

In this example, the device may be considered to have been mounted on the beam to control flexural vibration by producing a bending moment. However, if the device is mounted non-symmetrically, as in Fig. 1a, an additional axial force will be produced. That is to say, in addition to proportions of the impinging flexural and vibrational power being respectively reflected or transmitted as flexural waves, a proportion of the incident power will undergo wave-type conversion and be reflected and transmitted in terms of axial vibrational power.

By considering the conditions of continuity and equilibrium at the joint the resulting wave amplitude reflection and transmission coefficients can be found [1].

Axial Wave-Field Impinging

In the second example, the device may be considered to have been mounted on the beam to control axial vibration by producing an axial force. However, if the device is mounted non-symmetrically, as in Fig. 1b, an additional bending moment will be produced. Thus, a set of reflected and transmitted flexural waves are superimposed on the existing longitudinal wave-field in the beam. Once again, considering the conditions of continuity and equilibrium at the joint, the resulting wave amplitude reflection and transmission coefficients can be found [1].

RESULTS FROM COMPUTER SIMULATION

In order to assess the effect of the neutralizer in terms of the concept of vibrational power transmission the following expressions should be applied to convert wave amplitude ratios to power ratio coefficients.

Flexural Wave-Field Impinging

$$\text{Reflected Flexural Vibrational Power Ratio} = \left|\frac{A_{3f}}{A_{if}}\right|^2$$

$$\text{Transmitted Flexural Vibrational Power Ratio} = \left|\frac{A_{4f}}{A_{if}}\right|^2$$

$$\text{Reflected Axial Vibrational Power Ratio} = \gamma\left|\frac{A_{3l}}{A_{if}}\right|^2$$

$$\text{Transmitted Axial Vibrational Power Ratio} = \gamma\left|\frac{A_{4l}}{A_{if}}\right|^2$$

where $\gamma = 2Rk_f = 2\sqrt{R\ k_l}$
and R = radius of gyration of the beam
k_f = flexural wavenumber
k_l = axial wavenumber

Axial Wave-Field Impinging

$$\text{Reflected Axial Vibrational Power Ratio} = \left|\frac{A_{3l}}{A_{il}}\right|^2$$

$$\text{Transmitted Axial Vibrational Power Ratio} = \left|\frac{A_{4l}}{A_{il}}\right|^2$$

$$\text{Reflected Flexural Vibrational Power Ratio} = \frac{1}{\gamma}\left|\frac{A_{3f}}{A_{il}}\right|^2$$

$$\text{Transmitted Flexural Vibrational Power Ratio} = \frac{1}{\gamma}\left|\frac{A_{4f}}{A_{il}}\right|^2$$

In both of the examples considered, the neutralizer is assumed to be attached to an infinite beam-type structure with the approximate geometric and material properties of standard, empty, twoinch nominal bare carbon steel water pipe*. The natural frequency of the neutralizer system is assumed to be constant and tuned to a frequency of 75 Hz. In addition to this, the effect of the mounting configuration on the performance of the device is assessed by varying the applied moment arm, a, from 0 to 4 pipe diameters, whilst the neutralizer mass to beam mass per unit length ratio, m = 0.5. Firstly, we consider the case for a flexural wave-field impinging on the neutralizer discontinuity.

Note: The summation of the power coefficients shown for each respective example highlights small inaccuracies associated with the use of a computational matrix inversion routine to calculate values for respective wave amplitude coefficients.

As expected, in all cases, the proportions of transmitted and reflected axial power are equal due to the symmetry of the applied axial force about the plane of attachment of the discontinuity.

Now, if we consider specifically the effect of increasing the applied moment arm in conjunction with a fixed neutralizer mass ratio, Figs. 3a and 3b, as expected the result is an increase in the performance of the device. However, as the moment arm length is increased to enhance the performance, the levels of transmitted and reflected axial power are also enhanced to a point where the applied bending moment component becomes large compared to the applied axial force component and any further increase in moment arm does not significantly affect the levels of axial vibrational power produced. The example shown considers the respective vibrational powers for the conditions such that the neutralizer mass ratio, m = 0.5, and the moment arm is 3 and 4 pipeline diameters in length. In both cases, as a proportion of total impinging power, the levels of axial vibrational power are approximately 25%, but the level of transmitted flexural power is reduced from approximately 30% to 18%. However, the total transmitted power has only been reduced from approximately 55% to 33%, a result which clearly indicates the shortfalls in performance that can result from a condition of asymmetry in the mounting assembly.

In the second example we consider the situation whereby the device is impinged upon by an axial wave-field. Once again, the natural frequency of the neutralizer system is assumed to be constant and tuned to a frequency of 75 Hz, all other properties of the beam and neutralizer are as for that considered previously.

As expected, in all cases the proportions of transmitted and reflected flexural power are equal due to the symmetry of the applied bending moment about the plane of attachment of the discontinuity.

Comparison between proportions of incident power converted from axial to flexural, in this example, and that converted from flexural to axial, in the previous example, shows that for given neutralizer parameters respective proportions are always the same. That is to say, for a set neutralizer design, if x% of the impinging flexural vibrational power is converted to axial power then it follows that if the impinging wave-type were to be changed to axial motion then x% of the impinging axial power would be converted to flexural power.

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* At frequencies well below the ring frequency of the pipe only beam-like motion can occur and so the pipe can be considered to exhibit the dynamical properties of a simple beam [2].

Now, if we consider specifically the effect of increasing the applied moment arm in conjunction with a fixed neutralizer mass, Figs. 3a and 3b, as expected the result is a decrease in performance of the neutralizer. As the moment arm is increased the bending moment component becomes large in comparison to the applied axial force component and the capability of the device to generate flexural wave motion becomes superior to its capability to attenuate axial wave motion. The example considered, Fig. 3, shows the respective power amplitudes for a system whereby the neutralizer mass ratio, $m = 0.5$, and the moment arm is approximately 4 pipe diameters in length. The result is such that approximately 35% of the incident axial power is transmitted and only 15% reflected, and approximately 25% is each reflected and transmitted as flexural vibrational power. In this example, the total performance of the device is such that only 40% of the incident power is attenuated.

CONCLUSIONS

In this paper, neutralizers mounted asymmetrically on one-dimensional structures have been considered with particular reference to vibrational power transmission and wave-type conversion. From the simple parameter studies carried out, the following points are considered worthy of note.

(i) When the neutralizer is mounted on a beam, vibrating flexurally, in order to produce a bending moment, but a condition of asymmetry exists, the impinging flexural wave-field will cause the device to apply an additional axial force on the beam and an additional longitudinal wave-field will be introduced into the beam. Thus, as a result of the condition of asymmetry a proportion of the impinging flexural power will undergo a wave-type conversion at the discontinuity and be reflected and transmitted as axial vibrational power. If the rotational asymmetry is small, the device will act as a relatively efficient flexural vibration control device. However, its overall ability to attenuate vibrational power will be reduced due to the transmitted axial wave motion.

(ii) In a similar example to that considered in item (i) above, if the neutralizer assembly were to be impinged upon by an axial wave-field, simultaneously acting axial force and bending moment would be applied to the beam. It therefore follows that the impinging axial wave-field would induce flexural motion at the point of application of the device and a proportion of the impinging axial vibrational power would be respectively reflected and transmitted as flexural vibrational power. If the moment arm is small, i.e. the condition of axial asymmetry is small, the device will act as a relatively efficient axial vibration control device. However, similar to the case considered above the overall performance of the device will be reduced, when considered in terms of the total vibrational power transmission as a result of the generation of flexural motion at the discontinuity.

(iii) For the previous conditions considered, equal proportions of converted wave-type vibrational power are respectively reflected and transmitted by the discontinuity. This occurs as a result of the symmetric generation of either axial forces or bending moments about the discontinuity.

(iv) When considering the two examples of wave-type conversion, it is interesting to note that, for a given configuration and frequency, the proportion of impinging power that undergoes wave-type conversion is the same regardless of whether the impinging wave-field is flexural or axial. This can be explained by examination of the resulting motion of the beam at the point of attachment of the discontinuity which is the same for either impinging wave-type considered.

ACKNOWLEDGEMENT

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REFERENCES

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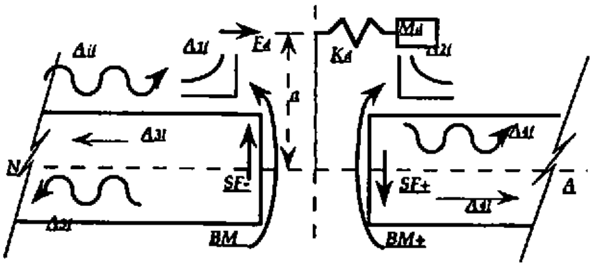


Figure 1a: Neutralizer mounted so as to induce both bending moment and axial force components – flexural wave-field impinging.
A_{if} = amplitude of the impinging flexural wave
A_{1f} = amplitude of the reflected flexural near-field wave
A_{2f} = amplitude of the transmitted flexural near-field wave
A_{3f} = amplitude of the reflected flexural travelling wave
A_{4f} = amplitude of the transmitted flexural travelling wave
A_{3l} = amplitude of the reflected axial wave

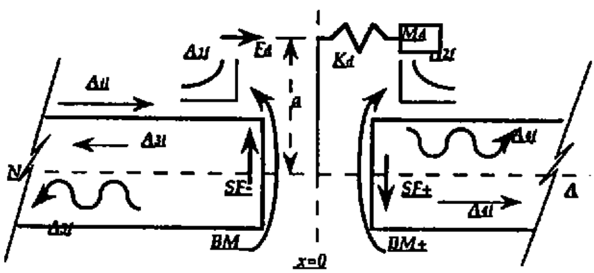
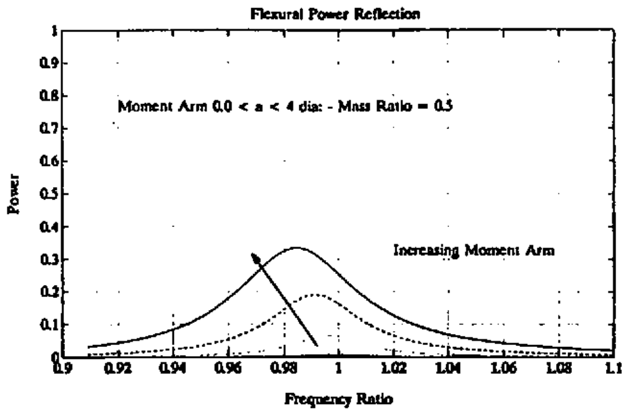


Figure 1b: Neutralizer mounted so as to induce both bending moment and axial force components – axial wave-field impinging.
A_{il} = amplitude of the impinging axial wave
A_{3l} = amplitude of the reflected axial wave
A_{4l} = amplitude of the transmitted axial wave
A_{1f} = amplitude of the reflected flexural near-field wave
A_{2f} = amplitude of the transmitted flexural near-field wave
A_{3f} = amplitude of the reflected flexural travelling wave
A_{4f} = amplitude of the transmitted flexural travelling wave



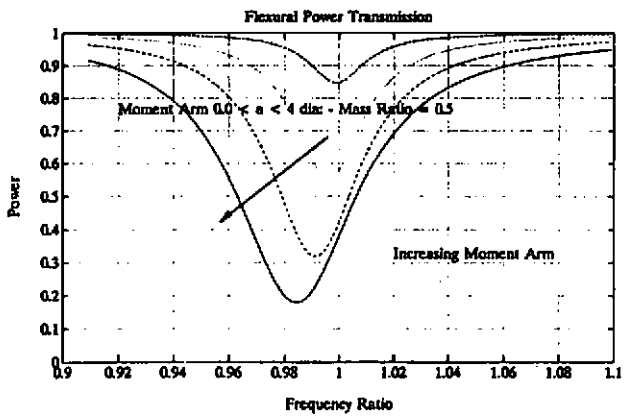


Figure 2a: Flexural power reflection and transmission coefficients for a flexural wave-field impinging a neutralizer mounted so as to induce both bending moment and axial force components.

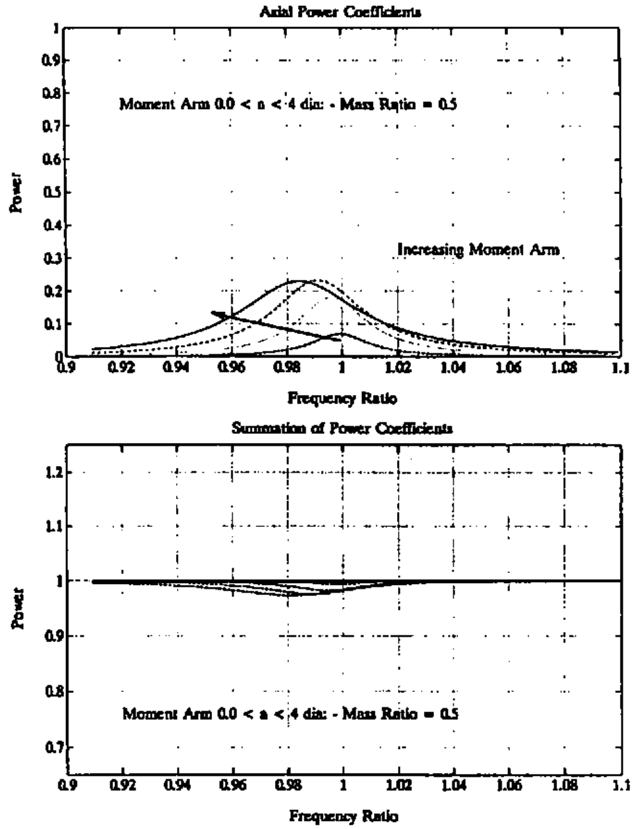


Figure 2b: Axial power coefficients and summation of all power coefficients for a flexural wave-field impinging a neutralizer mounted so as to induce both bending moment and axial force components.

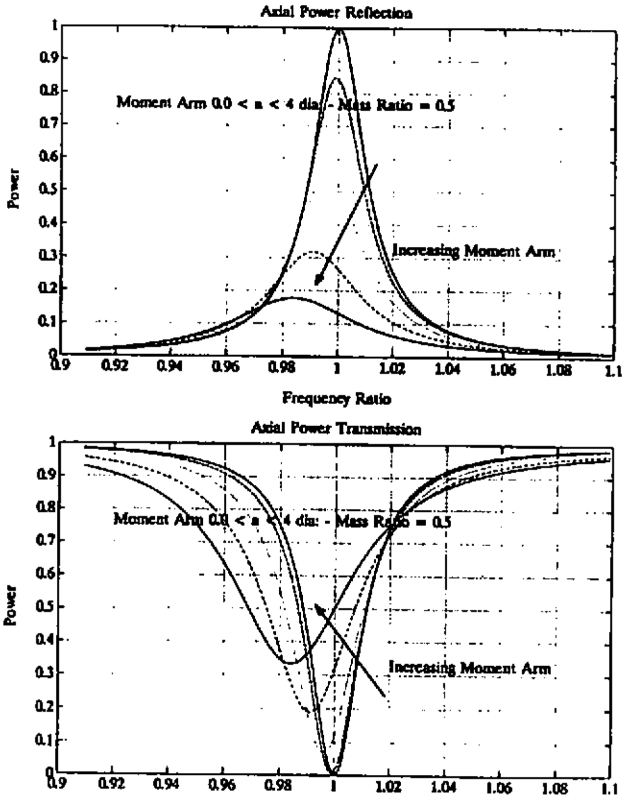


Figure 3a: Axial power reflection and transmission coefficients for an axial wave-field impinging a neutralizer mounted so as to induce both axial force and bending moment components.

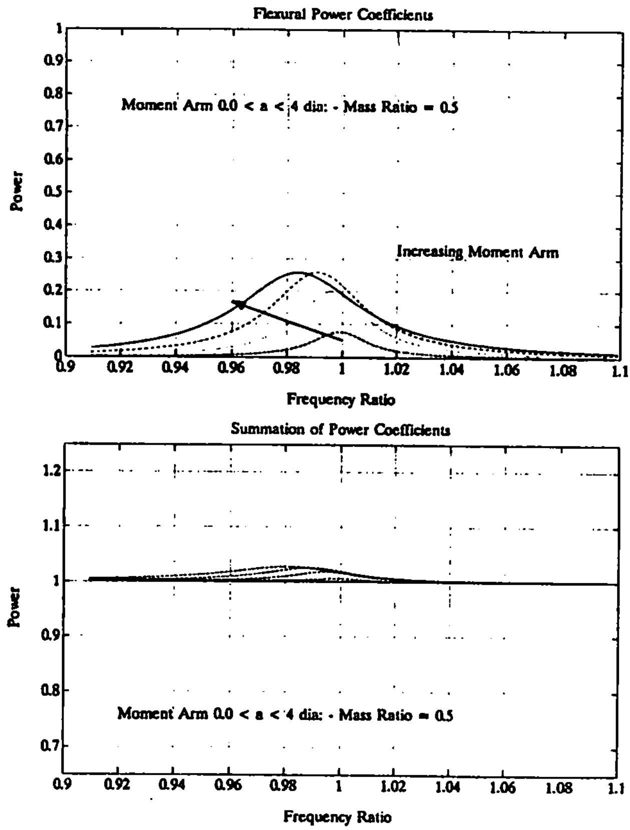


Figure 3b: Flexural power coefficients and summation of all power coefficients for a wave-field impinging a neutralizer mounted so as to induce both bending moment and axial force components.