

ACTIVE BOUNDARY CONTROL OF ENCLOSED SOUND FIELDS

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INTRODUCTION

It has been reported that active techniques can be usefully applied to the control of enclosed soundfields at frequencies where the modal density is low [1]. In this application, the active control source can be considered as a component of the boundary of the acoustic space, and the control action of the source can be described by specification of its surface impedance. In this sense, active control of the low frequency acoustics of enclosed spaces can be considered a boundary control technique. Such an interpretation is becoming increasingly relevant, as systems have been demonstrated which offer programmable surface impedance to incident sound [2]. It is the purpose of this paper to present a framework in which the active control of the low frequency acoustics of enclosed spaces can be described as a boundary technique and to define those secondary source surface impedances associated with a number of standard active noise control tasks.

A SIMPLE ACTIVE CONTROL APPLICATION

Consider the simple situation in which two simple sources excite sound in an enclosure. A primary source, located at P, has constant source strength Q_p . The secondary source, located at position S (where P and S are arbitrary and may be coincident), has controllable source strength, Q_s . Also within the enclosure is an ideal pressure microphone, located at M. In order for the secondary to exert useful control over the low frequency acoustics, the source strengths must be related by a linear, time invariant controller transfer function H :

$$Q_s(\omega) = H(\omega) \cdot Q_p(\omega) \quad (1)$$

Specification of different controllers, H, will result in different control

actions. Additionally, any controller, H , dictates the impedance at the secondary:

$$\frac{p_s}{Q_s} = Z_s + H^{-1} \cdot Z_{p,s} \quad (2)$$

in which p_s is the pressure at S , Z_s is the secondary source radiation load (measured when the primary is off) and $Z_{p,s}$ is that transfer impedance relating primary source strength to that pressure induced at S (measured when the secondary is off).

PRACTICAL CONTROLLER TASKS

The traditional and boundary control formulations associated with a number of practically important active noise control strategies are presented below.

Virtual earth at S . The simplest local control task is to establish a pressure null at S . The virtual earth at S is achieved when

$$Q_s \cdot Z_s = -Q_p \cdot Z_{p,s}$$

leading to the specification of the feedforward controller for the virtual earth case:

$$H(\omega) = \frac{-Z_{p,s}}{Z_s} \quad (3)$$

Substitution of Eqn. 3 into Eqn. 2 gives the anticipated result for the "short circuiting" action of the virtual earth controller:

$$\frac{p_s}{Q_s} = Z_s - Z_{p,s} \cdot \frac{Z_s}{Z_{p,s}} = 0 \quad (4)$$

Active cancellation at M . The control action required to cancel the pressure developed by the primary at the intermediate position M may be derived by setting the pressure at M to zero:

$$p_M = Q_p \cdot Z_{p,m} + Q_s \cdot Z_{s,m} = 0$$

in which $Z_{p,m}$ and $Z_{s,m}$ are the transfer impedances from P and S to M , respectively. This gives the required feedforward controller:

$$H(\omega) = \frac{-Z_{p,m}}{Z_{s,m}} \quad (5)$$

The impedance required at the secondary to cause pressure cancellation at M is obtained by substituting Eqn. 5 into Eqn. 2:

$$\frac{P_s}{Q_s} = Z_s - Z_{p,s} \cdot \frac{Z_{s,m}}{Z_{p,m}} \quad (6)$$

which reduces to the virtual earth case, Eqn. 4, when M and S are coincident.

Active de-reverberation at M. Rather than cancel the entire pressure at M, it is possible to configure the secondary source such that the residual pressure is that which the primary would generate when driving a free-field load; this technique is called active de-reverberation [3]. The controller for de-reverberation is derived by equating the pressure at M to that which the primary would produce in the absence of the room boundaries:

$$P_M = Q_s \cdot Z_{s,m} + Q_p \cdot Z_{p,m} = Q_p \cdot Z_{p,m}|_{FF}$$

in which $Z_{p,m}|_{FF}$ is the free-field transfer impedance between P and M. The feedforward controller for de-reverberation at M is:

$$H(\omega) = \frac{Z_{p,m}|_{FF} - Z_{p,m}}{Z_{s,m}} \quad (7)$$

Optimal cancellation. Whilst a controller can be derived to perfectly cancel the pressure at M, (Eqn. 5), that strategy which minimises the total acoustic potential in the enclosure is of interest in noise control applications. The total acoustic potential is:

$$E_p \propto |Q_p|^2 \left[\int_V |Z_{p,m}|^2 dV + \int_V |H|^2 |Z_{s,m}|^2 dV + 2 \Re \int_V Z_{p,m} [H Z_{s,m}]^* dV \right]$$

which is minimised when the controller assumes value:

$$H_{opt} = \frac{-\int_V Z_{p,m} Z_{s,m}^* dV}{\int_V |Z_{s,m}|^2 dV} \quad (8)$$

The secondary impedance which causes optimal cancellation is, therefore:

$$\frac{P_s}{Q_s} = Z_s - \frac{\int_V |Z_{s,m}|^2 dV}{\int_V Z_{p,m} Z_{s,m}^* dV} \cdot Z_{p,s} \quad (9)$$

Optimal de-reverberation. The solution for de-reverberation at a point may be extended to define that controller which causes optimal cancellation of the reverberant field:

$$H_{de-rev} = \frac{-\int_V Z_{p,m} Z_{s,m}^* dV + \int_V Z_{p,m} |Z_{FF} Z_{s,m}^*| dV}{\int_V |Z_{s,m}|^2 dV} \quad (10)$$

such that the impedance of the secondary source configured for optimal de-reverberation is:

$$\frac{P_s}{Q_s} = Z_s - \frac{\int_V |Z_{s,m}|^2 dV}{\int_V Z_{p,m} Z_{s,m}^* dV - \int_V Z_{p,m} |Z_{FF} Z_{s,m}^*| dV} \cdot Z_{p,s} \quad (11)$$

A practical absorber. For some values of controller, H , the secondary source will be absorbing acoustic power. This absorption can be used to mimic traditional passive acoustic treatments by providing an energy dissipation mechanism. Although a highly efficient active absorber is found generally to cause an increase in the acoustic potential in the space, some configurations have been identified which usefully control the low frequency modes. These configurations are of particular interest in the context of reproduced sound [3] and may be useful as a noise control technique [1]. A useful absorber is obtained when the secondary source offers surface impedance equal to the negative conjugate of its free field radiation load:

$$\frac{P_s}{Q_s} = -Z_s^* |_{FF} \quad (12)$$

Under this configuration the equivalent feedforward controller, H , can be identified as:

$$H(\omega) = \frac{-Z_{p,s}}{Z_s + Z_s^* |_{FF}} \quad (13)$$

Maximum power absorption. The acoustic absorber configured to absorb maximum power from an enclosed field is of little practical merit for noise control, but it is described here for completeness. The acoustic power at the secondary W_s for any controller H is:

$$W_s \propto [\operatorname{Re}[Z_s] |H|^2 + \operatorname{Re}[Z_{p,s} \cdot H^*]] |Q_p|^2 \quad (14)$$

where $\operatorname{Re}\{x\}$ denotes the real part of x .

The maximum power absorption configuration $H_{\max,W}$ is obtained by differentiating Eqn. 14 with respect to H and equating the derivative to zero, giving:

$$H_{\max,W}(\omega) = \frac{-Z_{p,s}}{2\operatorname{Re}[Z_s]} \quad (15)$$

The impedance which should be presented by a secondary source designed to absorb maximum power is given by substituting Eqn.11 into Eqn.2, giving the familiar result:

$$\frac{p_s}{Q_s} = Z_s - Z_{p,s} \cdot \frac{2\operatorname{Re}[Z_s]}{Z_{p,s}} = -Z_s^* \quad (16)$$

SUMMARY

The active control of enclosed sound fields can be usefully interpreted as a boundary control technique, with the active source forming an element of the boundary of the space. Those secondary source surface impedances associated with various control actions are summarised in Table 1. Of those conditions reported in table 1, the practical absorber is of particular interest as it requires only instrumentation of the field local to the secondary source and has been shown elsewhere to exert useful global control in a number of different applications.

REFERENCES

- [1] Darlington P & Avis M R (1996) "Noise control in resonant soundfields using active absorbers" Proc. Internoise '96
- [2] Nicholson G C & Darlington P (1993) "Active control of acoustic absorption, reflection and transmission" Proc. IOA 15, 403-409
- [3] Avis M R & Darlington P (1995) "Modifying low frequency room acoustics 1: active de-reverberation" Proc IOA 17(7), 77-86

Table 1 Summary of feedforward controllers and secondary impedances

	H	$p/Q_s (-Z_s + H^{-1}Z_{p,s})$
Virtual Earth	$\frac{-Z_{p,s}}{Z_s}$	0
Canceller @M	$\frac{-Z_{p,m}}{Z_{s,m}}$	$Z_s - Z_{p,s} \frac{Z_{s,m}}{Z_{p,m}}$
De-Reverb. @M	$\frac{Z_{p,m} _{FF} - Z_{p,m}}{Z_{s,m}}$	$Z_s - Z_{p,s} \frac{Z_{s,m}}{Z_{p,m} - Z_{p,m} _{FF}}$
Optimal Canceller	$\frac{-\int_V Z_{p,m} Z_{s,m}^* dV}{\int_V Z_{s,m} ^2 dV}$	$Z_s - \frac{\int_V Z_{s,m} ^2 dV}{\int_V Z_{p,m} Z_{s,m}^* dV} Z_{p,s}$
Optimal De-Reverb.	$\frac{-\int_V (Z_{p,m} - Z_{p,m} _{FF}) Z_{s,m}^* dV}{\int_V Z_{s,m} ^2 dV}$	$Z_s - \frac{\int_V Z_{s,m} ^2 dV \cdot Z_{p,s}}{\int_V (Z_{p,m} - Z_{p,m} _{FF}) Z_{s,m}^* dV}$
Practical Absorber	$\frac{-Z_{p,s}}{Z_s + Z_s^* _{FF}}$	$-Z_s^* _{FF}$
Max. Power Absorber	$\frac{-Z_{p,s}}{2\Re[Z_s]}$	$-Z_s^*$