

FLANKING PATH EFFECTS IN THE ACTIVE CONTROL OF STRUCTURAL POWER TRANSMISSION

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1. INTRODUCTION

An increasing amount of work has been published on the active control of vibration transmission in recent years. The first examples of commercial active mounts have appeared and the main applications include automotive suspension [1], seismic isolation for platforms of experimental arrangements or precision manufacturing machines [2], and more recently, the isolation of machines [3].

This paper presents a theoretical study of the active isolation of vibration transmission to a structure on which a machine is mounted. In particular, the problems generated by the presence of an uncontrolled flanking excitation acting on the receiver has been studied. The source is modelled as a rigid body, the receiver as a thin plate in which in-plane and out-of-plane waves can propagate and the active mounts as distributed systems in which both longitudinal and flexural waves can propagate. The control sources are modelled as axial forces applied at both ends of the mounts. The calculations have been performed using a matrix model based on input and transfer mobility terms which allows different control strategies to be studied. The effects of minimising the total power transmitted to the receiver through the mounts has been compared with more practical control strategies: the cancellation of out-of-plane velocities, the cancellation of out-of-plane forces and the cancellation of power due to out-of-plane velocities and forces at the junctions connecting the mounts to the plate.

2. GENERAL MODEL FOR MULTIPLE ACTIVE SUSPENSIONS

The dynamics of the system have been studied using a matrix model which considers the system to be divided into individual components (for a complete isolating system there are usually three components: the source,

the mounting system and the receiver) and each member is studied in terms of point and transfer mobilities or impedances [4-7]. As shown in

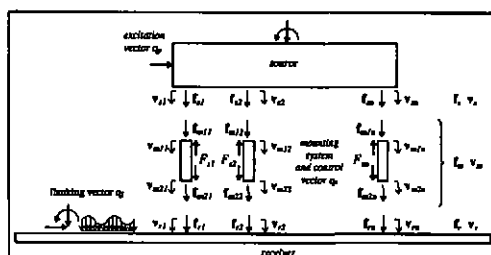


Fig. 1. Scheme of a general complete isolating system.

figure 1 the complete isolating system is divided into three flexible parts: the source, the mounting system, composed of n mounts, and the receiver. These parts are connected at a finite number of junctions. At each

junction, the motion and the forces transmitted are characterised by six parameters which are grouped in velocity and force junction vectors. The velocities and forces at both the source and receiver junctions can be expressed as linear functions of the vector of excitation and flanking forces \mathbf{q}_p and the vector of control forces \mathbf{q}_c ,

$$\mathbf{v}_{sr} = \mathbf{Q}_{pv} \mathbf{q}_p + \mathbf{Q}_{cv} \mathbf{q}_c \quad (1)$$

$$\mathbf{f}_{sr} = \mathbf{Q}_{pf} \mathbf{q}_p + \mathbf{Q}_{cf} \mathbf{q}_c \quad (2)$$

$\mathbf{q}_p = \{\mathbf{q}_e, \mathbf{q}_f\}$ contains the forces and moments acting either at the source or at the receiver while the control vector \mathbf{q}_c contains the control forces acting at the ends of the mounts. The expressions for the matrices \mathbf{Q} are not presented here but are described in detail in reference [7].

Using equations (1) and (2) it is possible to express the time-averaged power at a specific point in the system in terms of the excitation-flanking vector and the control vector by using the following equation

$$P = \frac{1}{2} \text{Re}(\mathbf{f}^H \mathbf{v}) \quad (3)$$

where \mathbf{f} and \mathbf{v} represent respectively the force and velocity at the position where the power is calculated. Power summarises the overall behaviour of the isolator more than any other parameter and therefore all the results presented in this paper are plotted in terms of power.

3. CONTROL STRATEGIES

Pan et al [8-10] studied the dynamics of an active isolator by considering the power transmission. They considered the possibility of using the power transmitted from the source to the receiver as a cost function. In this paper the effectiveness of this new control strategy has been compared with three alternative approaches: the cancellation of out-of-plane velocities or forces at the receiver junctions and the minimisation of an estimate of

power evaluated using only the out-of-plane velocities and forces at the receiver junctions. The cost function for power minimisation is given by the following equation

$$J_p = \frac{1}{2} \text{Re}(\mathbf{f}_r^H \mathbf{v}_r) = \frac{1}{4} (\mathbf{f}_r^H \mathbf{v}_r + \mathbf{v}_r^H \mathbf{f}_r) \quad (4)$$

where \mathbf{f}_r and \mathbf{v}_r are the force and velocity vectors at the receiver junctions. For velocity and force cancellation the cost functions assume the following forms

$$J_v = \mathbf{v}_r^H \mathbf{v}_r, \quad J_f = \mathbf{f}_r^H \mathbf{f}_r \quad (5,6)$$

and in these cases the velocity and force vectors contain only the out-of-plane parameters.

4. ACTIVE CONTROL STRATEGIES EFFECTIVENESS

Fig. 2. shows the geometry of the system considered here. The source is a rigid aluminium mass whose dimensions are $0.52 \times 0.3 \times 0.25$ m. The two mounts are modelled as rings of rubber with two reactive control

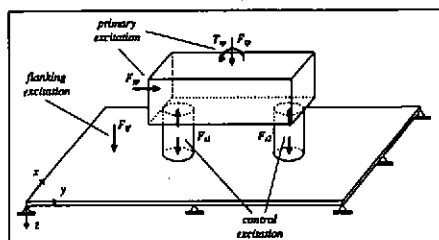


Fig. 2. Complete isolating system.

forces acting at the ends. The external and internal diameters and the height of the suspensions are respectively 6 cm, 3 cm and 10 cm. The aluminium receiver plate is 0.5 cm thick and has dimensions 1×1.5 m. It has been assumed that the rigid mass vibrates in a plane (y-z plane) and in this

way both the effects due to multi degree-of-freedom vibration and the effects due to several mounts are taken into account in the simulations.

The dynamics of the system at low frequencies is characterised by three rigid body modes which are respectively a transverse mode, an axial mode and a pitching mode. The transverse mode is characterised by a dominating transverse oscillation and a small pitching induced by the bending stress of the mount. The axial mode is given by a pure axial oscillation while the pitching mode is composed of a dominating pitching and a small transverse oscillation. The natural frequencies of these three modes are respectively 3 Hz, 4.2 Hz and 9 Hz as shown in Fig. 3. At higher frequencies the resonances of the distributed plate and mounts occur, in particular the first resonance due to longitudinal wave propagation in the two mounts occurs at 500 Hz. The control action for these four control strategies produces an interesting phenomenon. As it can be seen in figures 3 and 4 the three resonances of the rigid modes

are replaced by only two resonances at 1 and 6.5 Hz. This is due to the fact that the axial control forces completely decouple the axial oscillation of the source from the mounts and then the axial mode disappears. Also, because the axial vibration is now decoupled, the transverse and pitching modes are controlled only by the bending stiffness of the mounts and therefore their natural frequencies are reduced.

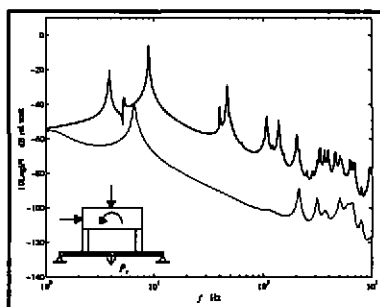


Fig. 3. Power transmitted into the plate without control (thick) and minimising the total power transmitted to the plate (thin).

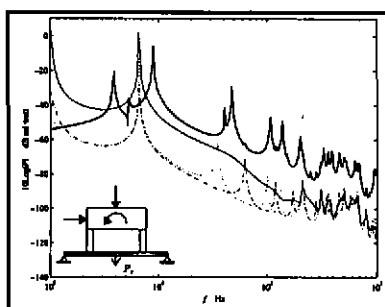


Fig. 4. Power transmitted into the plate without control (thick), minimising an estimate of the power transmitted to the plate (thin) and cancelling the out-of-plane velocities (dash-dotted) or forces (dotted).

The total power minimisation acts as a benchmark against which the other three control strategies can be gauged. Comparing the results of figures 3 and 4 the three alternative cost functions present some problems in particular at low frequencies. In fact, at certain frequencies, for example at the resonances of the transverse and pitching modes after control, the power transmitted when the system is controlled is higher than the power transmitted by the passive mounts.

This behaviour is particularly evident when only the out-of-plane component of power is minimised. It has been found that in this case the failure of the control strategy is due to a particular phenomenon of "power circulation" [7]. The cost function can assume negative values at some frequencies indicating that the power absorption by the secondary sources is being maximised and the control sources tend to suck power from the receiver through axial vibrations and this power is supplied to the plate by the transverse and angular vibrations of the mounts.

5. EFFECTS DUE TO AN UNCONTROLLED FLANKING SOURCE

In this section the effects of an uncontrolled flanking excitation acting on the receiver are analysed. The flanking excitation is considered to be a harmonic excitation whose amplitude has been set so that it delivers around 15 dB less power than the source. As shown in figure 5, the power

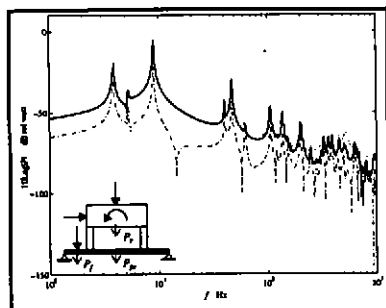


Fig. 5. Power input (dotted) and transmitted into the plate (thin) by the source, power input by the flanking source (dash-dotted) and total power input into the plate (thick).

input due to the flanking force is so low in comparison to the power input due to the primary source that it can generally be neglected. Figures 6 and 7 shows that both of the control strategies involving minimisation of power transmitted by the source are unable to control the system since they produce an increase in the power transmitted into the receiver at certain frequencies. This is due to the same phenomenon of power circulation seen before.

The cost functions which minimise the total power or an estimate of the power transmitted by the source to the receiver through the mounts can assume negative values indicating power absorption by the control sources.

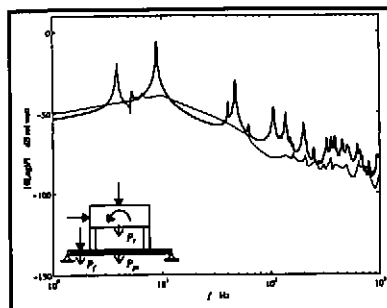


Fig. 6. Power transmitted into the plate without control (thick) and minimising the total power transmitted to the plate (thin).

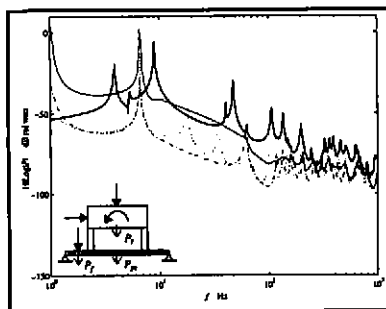


Fig. 7. Power transmitted into the plate without control (thick), minimising an estimate of the power transmitted to the plate (thin) and cancelling the out-of-plane velocities (dash-dotted) or forces (dotted).

In this case the power absorbed by the control sources is supplied from the flanking source. The two control strategies of velocity and force cancellation are not affected by this problem and, however, perform almost in the same way as in the case without the flanking source.

6. CONCLUSIONS

In this paper the study of a multi degree-of-freedom active isolator has been presented. The effectiveness of minimising the total power transmitted to the receiver has been compared with the minimisation of an

estimate of the power calculated from only the out-of-plane velocities and forces and with the cancellation of out-of-plane force or velocity at the receiver junction. In general it has been shown that the control of total or estimated power is very delicate since the presence of even a very weak flanking excitation acting on the receiver can prejudice the control. This is due to the phenomenon of power circulation, in which power absorbed by the secondary sources is generated from an increase in power supplied by uncontrolled paths. The control of only the out-of-plane component of the power gives particularly bad results. The traditional control approaches of minimising force or velocity are not as effective as total power minimisation, if implemented perfectly, but seem to be useful since they are not sensitive to the problem generated by the presence of a flanking path.

ACKNOWLEDGEMENTS

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