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VIBRATION ISOLATION OF THE IMAX CINEMA, WATERLOO, LONDON

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1. INTRODUCTION

This paper describes the design and construction of measures adopted in the BFI IMAX Cinema, Waterloo, London to isolate it from vibration from an underground railway. The performance of the measures is appraised from vibration measurements taken before, during and after construction of the building. Other acoustic issues associated with the design and construction of this Cinema, such as sound insulation and room acoustics, have been described elsewhere(1).

The BFI IMAX Cinema is located in the centre of one of London's busiest roundabouts, at the southern end of Waterloo Bridge. Directly below the Cinema, only a few metres below ground level, there are two tunnels carrying London Underground's Waterloo and City Line trains.

The building is cylindrical in shape and generally of lightweight construction. It is steel framed with concrete floors and mainly plasterboard walls. The ground floor contains a restaurant, ticketing and public areas. The 500 seat auditorium makes up much of the remainder of the building from 1st floor to 6th floor. A glass gallery surrounds a large proportion of the building extending from 2nd floor level to roof level. Figures 1 and 2 show a cross section through the building in plan and section respectively.

2. DESIGN CRITERIA AND VIBRATION MEASUREMENTS

One of the basic acoustic design aims set by the IMAX Corporation is for external noise to be inaudible within the auditorium against a building services noise limit of NC 25. Noise and vibration measurements were taken at the site prior to development to assess the feasibility of achieving this criterion.

Prior to development, the site comprised a pedestrianised area known as the Bullring. Vibration levels on the pedestrianised area in the centre of the roundabout were found to vary significantly during Underground train events in the vicinity of the tunnels. It is believed that variations arose in part, at least, due to the complexity of the site and the variety of underground structures that exist, including a large brick sewer passing over the tunnels, and the presence of a sizeable British Telecom Chamber. There was also considerable variation among measurements made on different flagstones, some of which were not correctly bedded.

Below the flagstones, a solid 300mm thick layer of concrete was found to extend over a large proportion of the roundabout. Measurements on this concrete layer were found to be repeatable and were taken to provide a reasonable indication of the magnitude of ground borne vibration in the vicinity of the tunnels. Table 1 provides an indication of the magnitude of the vibration levels measured.

3. PREDICTION OF TRAIN NOISE LEVELS IN AUDITORIUM

Based on the vibration measurements obtained on site, generated during Underground train events, predictions were made of the noise levels likely to result in the auditorium assuming that no vibration isolation measures were introduced into the building. Account was taken of the change in vibration level that might result from coupling losses between the ground and foundations of the building, plus amplification of certain structural elements, such as the concrete raked seating structure. Noise levels were predicted using the following general formula:-

$$Lw = 30 + 10\log S + AL - 10\log f^2 + 10\log\sigma$$

where Lw = sound power level radiating from element, dB re 10^{-12}W
S = surface area of element, m^2
AL = average r.m.s. vibration acceleration level on surface of element, dB re $10^{-5}ms^{-2}$ *
f = frequency of sound. Hz
σ = radiation ratio

* Note that this reference value differs from the commonly used value of $10^{-6}ms^{-2}$.

Based on these calculations, it was predicted that the noise level in the auditorium during train events without special vibration isolation measures would rise to around NC 45.

Taking account of the above, the practical limitations of deploying vibration isolation measures, and the lightweight nature of the proposed construction, it was considered that noise from road and rail traffic could be controlled to around NC 25. At this level, train events may therefore just be audible at times against a quiet background but would most likely go unnoticed by the audience.

To convince the client, BFI, that this would be acceptable, a simulation was undertaken in the small IMAX cinema in the National Museum of Film & Photography in Bradford. The simulation involved replaying within the cinema the sounds that might arise from Underground trains below the proposed site, during film shows and between films. This confirmed the criterion of NC 25 as appropriate.

4. VIBRATION CONTROL MEASURES - DESIGN

It was assessed that to meet the agreed design criterion of controlling train noise levels to not more than NC 25, acceleration levels in the cinema building needed to be around 20 dB lower at low frequencies than those measured in the ground in the vicinity of the tunnels. It was considered that conventional building isolation devices alone, such as anti-vibration bearings, would not provide sufficient isolation in themselves to achieve this level of reduction.

Various methods of vibration control were considered including:-

i) control at source, such as treating the tracks in the Underground tunnels by replacing jointed track with welded track and/or using either base plate pads or a resilient layer beneath the track. This was found to be not practicable.

ii) double sleeving of piles, this was discounted in view of the relatively small reduction in vibration levels attainable and the cost and practicalities of piling much deeper.

iii) construction of trenches filled with sand, parallel to the tunnels. This was also discounted on the grounds of cost/practicality vs benefits.

iv) maintaining a 3 metre clearance between nearest piles and Underground tunnels together with an air gap between the building structure and local ground. This was implemented.

v) locating the building on anti-vibration bearings. This was implemented.

Since the aim of the above measures was to reduce structure-borne noise in the auditorium, the internal walls and parts of the ceiling surface of the auditorium were treated with Melatech foam to provide good low frequency absorption within the space. This was necessary in any event to meet IMAX Corporation's requirements to provide a "dead" acoustic space.

5. VIBRATION CONTROL MEASURES - PRACTICE

The unisolated ground slab was built as a series of deep beams and shallow slabs over the tunnel zone to introduce an air gap beneath it. Polystyrene was used as a temporary formwork beneath narrow beams and later removed. For wider beams, a novel approach was required. Clayboard was used as temporary shuttering for these beams. This was later collapsed by wetting, thus forming a physical gap beneath the slab and local ground.

The position chosen for the location of the anti-vibration bearings (AVB's) was beneath the first floor slab, on top of columns. Various types of AVB's were considered and some were tested with comparisons being made between elastomeric and steel spring types. The final choice was a pre-compressed steel spring arrangement with a system natural frequency of 3.5 Hz. The bearings were manufactured by GERB and the spring elements were retained in a damping fluid to reduce coil resonance effects. In addition to good performance capabilities, the pre-compression feature meant that the springs could be inserted in a rigid (unreleased) condition into the building. This allowed the building to be constructed without regard to complicated construction sequencing to avoid differential settlement. Towards the end of the contract, once the design load had been achieved, the springs were released.

Various practical approaches were taken to minimise bridging across the AVB's. These included:-

i) careful arrangement of building services to limit duct and pipe runs across ground/first floor interface,

ii) stairs built with physical break at landing level between ground and first floors,

iii) lift shafts built with soft joint to underside of first floor and lift car guide rails supported on a steel cradle suspended from the first floor structure,

iv) internal masonry walls at ground floor level provided with a fire resistant soft joint at their head, at the underside of the isolated first floor slab.

A general picture of how vibration levels varied during the construction of the IMAX building is shown in Figure 3. Table 1 provides similar information at the three octave band frequencies of 31.5 Hz to 125 Hz, those at which vibration levels from Underground trains were highest.

The following points arise from Table 1:-

a) In the vicinity of the tunnels, the vibration level measured in the original ground (slab) is similar to that measured on top of the piles.

b) A small reduction in vibration level occurred between that measured on the original ground and that measured on the ground slab beams of the IMAX, justifying the coupling loss allowance used in computations.

c) Vibration levels vary over the ground floor slab, depending on whether measurements are made on deep beams or shallow slab sections.

d) Only a small reduction in vibration level was measured in the ground slab following removal of the polystyrene and wetting of the clayboard.

e) Vibration levels in the first floor slab with AVB's in place but unreleased, are similar in magnitude to those on the unisolated ground floor.

f) Vibration levels on the first floor slab reduced significantly when the AVB's were released.

g) The performance of the AVB's appears better than expected with reductions of around 20 dB evident at 31.5 Hz.

6. FINAL CONDITIONS IN AUDITORIUM

Tests were undertaken on completion of the project to determine both final noise and vibration levels within the completed structure. The final vibration levels are reported in Table 1. With regard to noise in the auditorium, it was not possible during commissioning to hear or measure noise from any train events. In the empty auditorium, the underlying background noise level, measured as an L_{90} index, was found to lie around NC 20. The measured noise levels relate to when building services were operating and thus the results unfortunately do not reflect the full sound insulation and vibration isolation capabilities of the building. The results show however that the criterion of NC 25 has been achieved.

7. CONCLUSION

The IMAX Cinema is now open and operating successfully. The requirement for low background noise conditions within the auditorium has been achieved, with levels lying below NC 25. Train events are imperceptible in the auditorium despite trains passing in tunnel only a few metres below the building. Vibration measurements taken before, during and on completion of the project indicate that the spring anti-vibration bearings are performing at least as well as expected, and possibly better, and are largely responsible for reducing vibration levels to below design targets.

8. REFERENCES

(1) P Henson & J G Charles, *The IMAX Cinema, Waterloo, London - An Acoustic Challenge!* Proc. I.O.A. Vol 21 Pt 6 pp 171-178 (1999)

TABLE 1 - SUMMARY OF VIBRATION RESULTS AT IMAX CINEMA SITE BEFORE, DURING AND AFTER CONSTRUCTION

	GROUND SLAB						FIRST FLOOR SLAB			
	Outside tunnel line			Inside tunnel line			On first floor slab (deck)			
	Octave Band Centre Freq. Hz							Octave Band Centre Freq. Hz		
	31.5	63	125	31.5	63	125		31.5	63	125
Pre-Construction										
In ground				64	69	67				
On exist. slab				63	66	61				
Foundations										
On piles (sleeved)				60	64	58				
On piles (unsleeved)	59	64	56							
On pile cap	60	64	55							
On blinding	73	65	60	72	64	69				
Ground Slab, ½ built, polystyrene in place										
On beams				60	60	56				
On slab (deep)	54	43	<49	51	56	<50				
On blinding				69	69	67				
Ground slab, fully built, polystyrene removed, clayboard DRY										
On beams				58	58	53				
On slab (deep)				54	57	52				
Ground slab complete. Clayboard wetted. First floor in place, a little steel erected, springs in place but unreleased										
On beams				56	58	53	On deck	62	61	50
On slab (deep)	52	54	49	51	56	52	(average)			
On slab (thin)				62	68	59				
IMAX ½ built, springs part loaded, unreleased										
							On deck (average)	60	56	<51
IMAX complete, springs released										
On beams				54	59	58	On deck	42	<45	<41
On slab (thin)				58	65	63				

Notes: 1) All vibration measurements are maximum acceleration levels re: 10^{-5} ms^{-2}.
2) Vibration levels are average values, taken from measurements of train events.

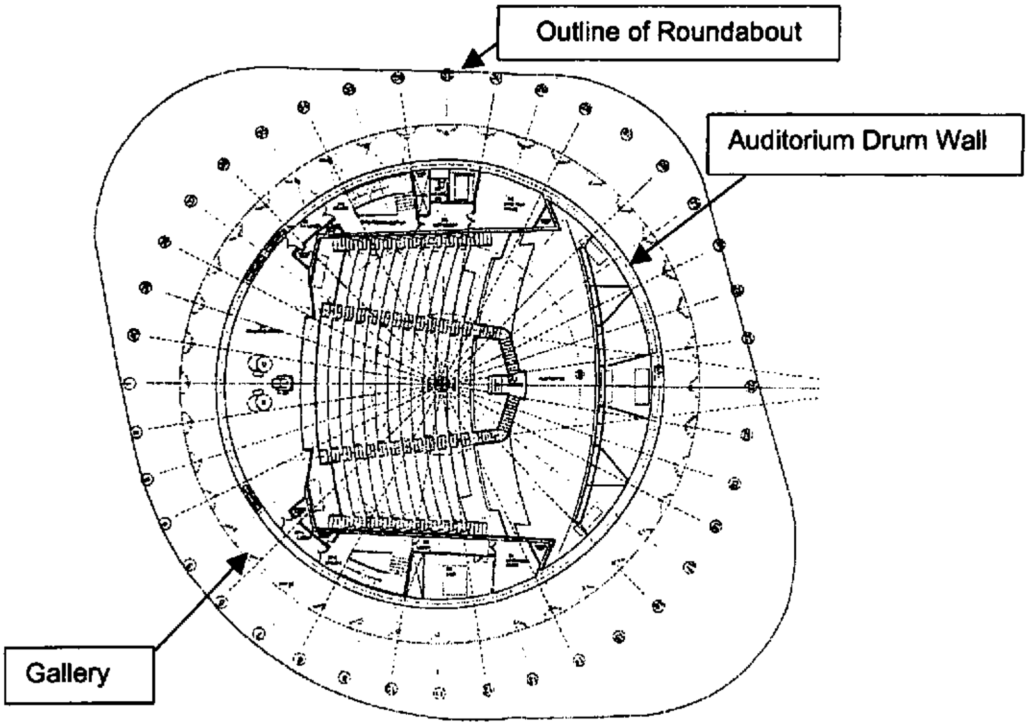

Figure 1 - Typical Auditorium Plan

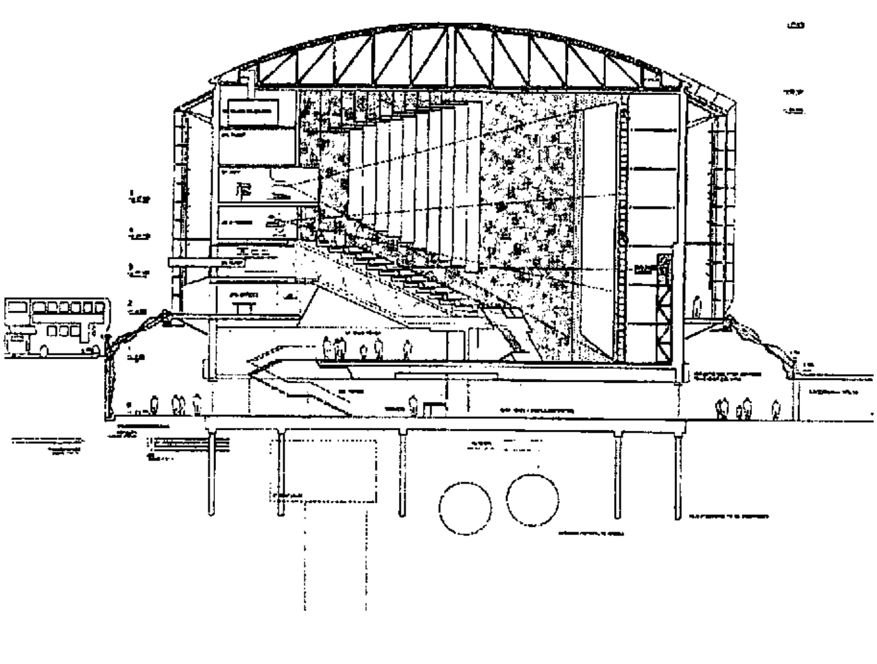

Figure 2 - Section through Auditorium

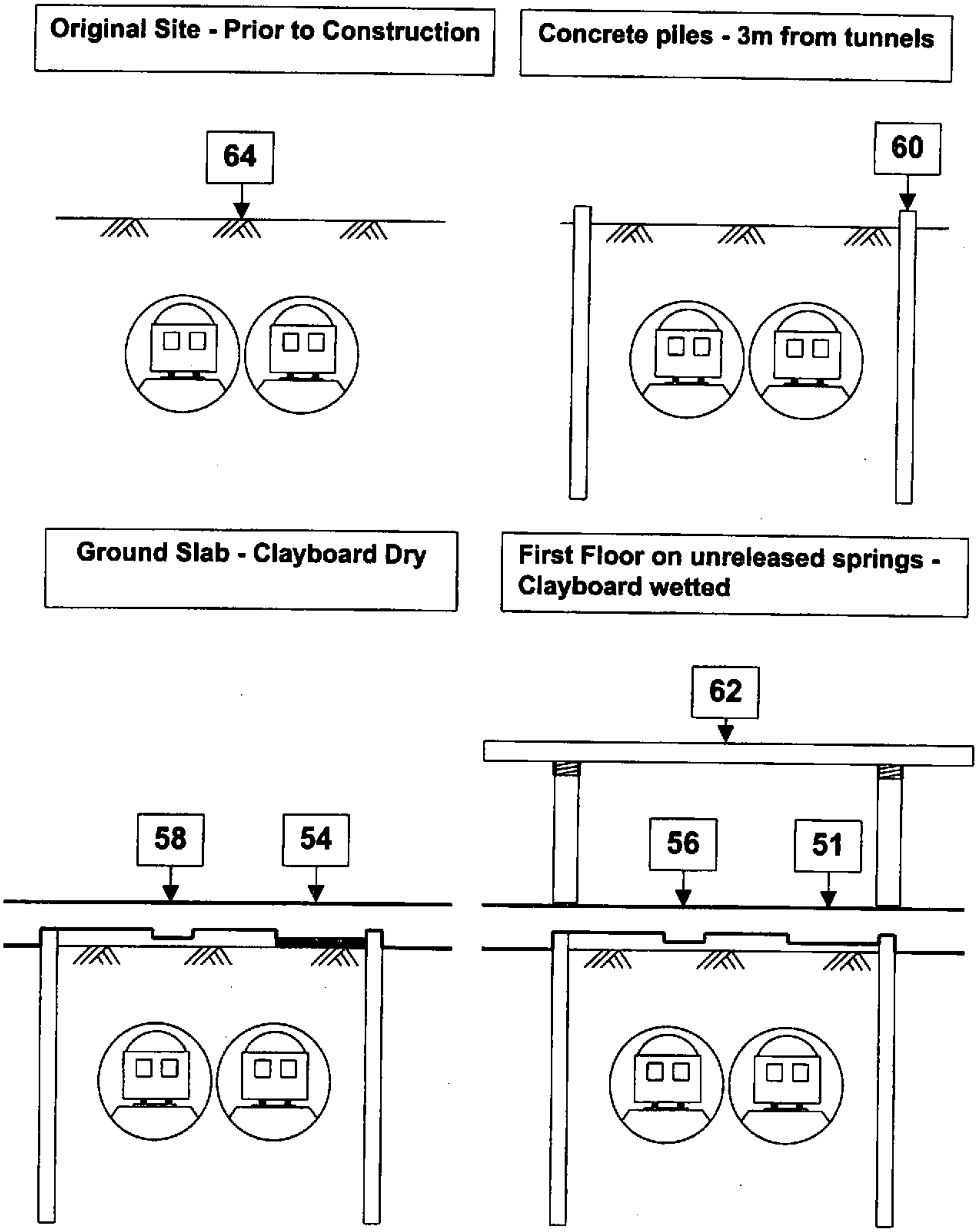

Figure 3 - Vibration from Underground Trains
Max. Acceleration Levels, dB re 10^{-5} ms^{-2}
at 31.5 Hz Octave Band

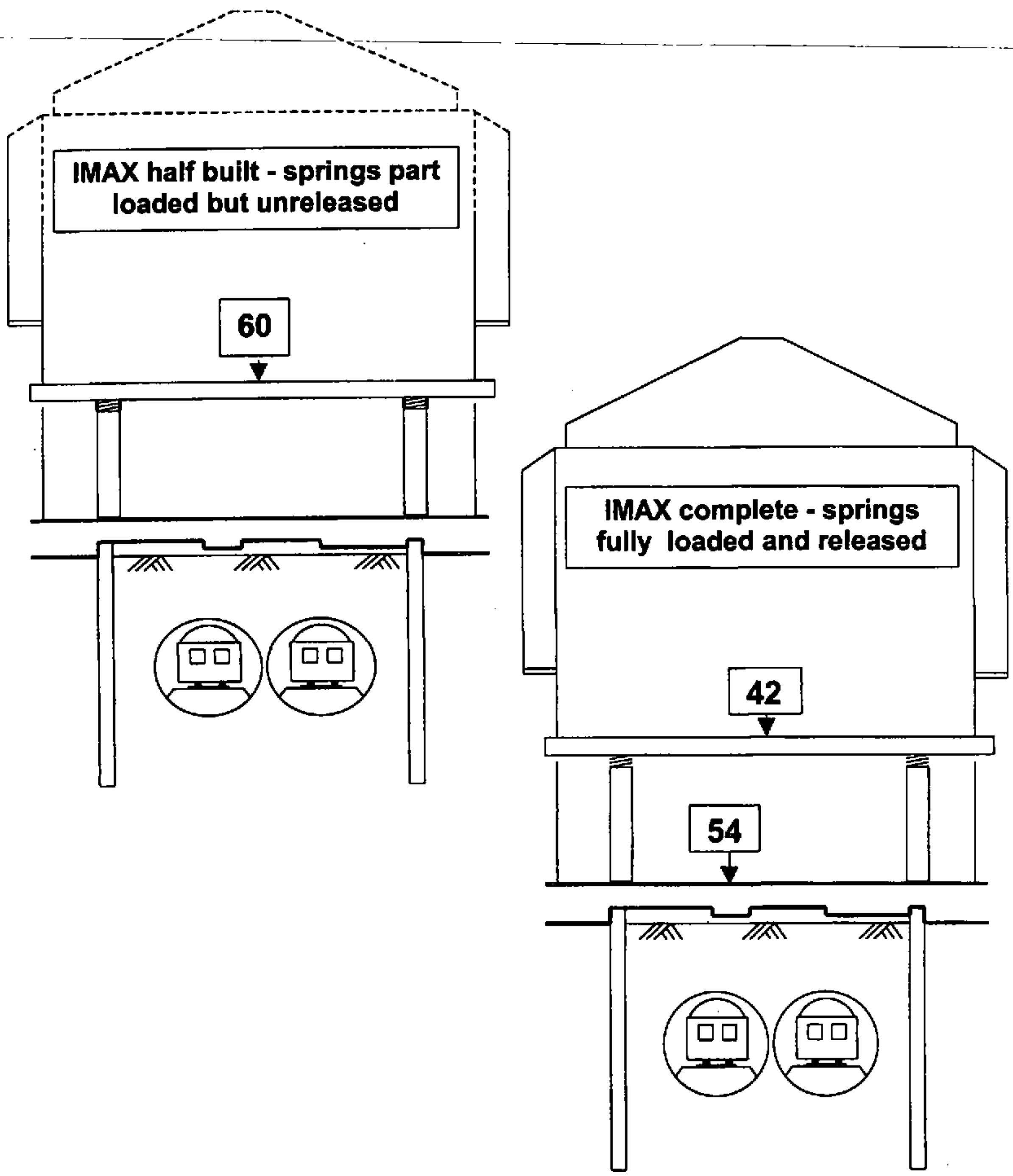

Figure 3 (cont.) - Vibration from Underground Trains
Max. Acceleration Levels, dB re 10^{-5} ms^{-2}
at 31.5 Hz Octave Band

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PROPAGATION OF LOCALISED FLEXURAL VIBRATIONS ALONG PLATE EDGES DESCRIBED BY A POWER LAW

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1. INTRODUCTION

Localised flexural vibrations propagating along sharp edges of elastic wedge-like structures are characterised by low propagation velocities (generally much lower than that of Rayleigh waves), and their elastic energy is concentrated in the area of about one wavelength from the edge. Such localised vibrations, also known as wedge acoustic waves, have been investigated in a number of papers (see, e.g. [1-14]) with regard to their possible applications to acoustic non-destructive testing of special engineering constructions and for better understanding vibrations of propellers, turbine blades and some civil engineering constructions. They may be important also for the explanation of many as yet poorly understood phenomena in related fields of structural dynamics, physics, environmental acoustics and may result in many useful practical applications. In particular, it is expected that these waves may play an important role in the dynamics of wedge-shaped off-shore structures (such as piers, dams, wave-breakers, etc.), and in the formation of vibration patterns and resonance frequencies of propellers, turbine blades, disks, cutting tools and airfoils. They may be responsible for specific mechanisms of helicopter noise, wind turbine noise and ship propeller noise. Promising mechanical engineering applications of wedge elastic waves may include measurements of cutting edge sharpness, environmentally friendly water pumps and domestic ventilators utilising wave-generated flows. Another possible application earlier suggested by one of the present authors [10] may be the use of wedge waves for in-water propulsion of ships and submarines, the main principle of which being similar to that used in nature by fish of the ray family.

Initially these localised flexural waves have been investigated for wedges in contact with vacuum [1- 6]. Later on, the existence of localised flexural elastic waves on the edges of wedge-like immersed structures has been predicted [7]. This was followed by the experimental investigations of wedge waves in immersed structures which considered samples made of different materials and having different values of wedge apex angle [8,9]. Recently, finite element calculations have been carried out [10] for several types of elastic wedges with the of apex angle varying in the range from 20 to 90 degrees. Also, the analytical theory based on geometrical-acoustics approach has been developed for the same range of wedge apex angle [11]. In the paper [12] dealing with finite element calculations of the velocities and amplitudes of wedge waves, among other results, calculations have been carried out of the velocities of waves propagating along the edge of a cylindrical wedge-like structure bounded by a circular cylinder and a conical cavity. In the paper [13] different cylindrical and conical wedge-like structures have been investigated using geometrical acoustics approach.

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In the present paper, we report some new analytical results in the theory of localised flexural vibrations propagating along edges of free and immersed structures of "non-linear" shape (see Fig.1). The results are described with the emphasis on methodological aspects of using geometrical-acoustics approach for developing the theory. Some of these results have been recently delivered in the review paper [14]. Using the geometrical acoustics technique, the velocities of localised wedge modes are calculated for edges with a cross section described by a power-law relationship between the local thickness d and the distance from the edge x: $d = \varepsilon x^m$, where m is a positive rational number. It is shown that deviations of a wedge shape from the linear geometry ($m = 1$ and $\varepsilon = \Theta$, where Θ is the wedge apex angle) result in frequency dispersion of wedge modes. It is also shown that for $m \geq 2$ - in free wedges, and for $m \geq 5/3$ - in immersed wedges the velocities of localised waves tend to zero, unless there is a truncation on the wedge tip. In other words, localised waves do not propagate along free or immersed structures with $m \geq 2$ and $m \geq 5/3$ respectively. This phenomenon can be explained by trapping of flexural wave energy near the curved edges considered which represent acoustic 'black holes' for flexural waves. The discussion is given on possible use of these phenomena for vibro-isolation.

2. GEOMETRICAL-ACOUSTICS APPROACH

2.1 Linear Wedges in Vacuum

The approximate analytical theory of localised elastic waves in solid wedges can be based on the geometrical acoustics approach considering a slender wedge as a plate with a local variable thickness $d(x)$, where x is the distance from the wedge tip measured in the middle plane. In the case of "linear" wedge $d(x) = x\Theta$, where Θ is the wedge apex angle.

The velocities c of the localised modes propagating along the wedge tip (in y-direction) can be calculated from the following Bohr - Sommerfeld type equation [4-6]:

$$\int_0^{x_t} \left[k^2(x) - \beta^2\right]^{1/2} dx = \pi n . \qquad (1)$$

Here $k(x)$ is a local wavenumber of a quasi-plane plate flexural wave (as a function of the distance x from the wedge tip), $\beta = \omega/c$ is yet unknown wavenumber of a localised wedge mode, $n = 1, 2\ 3, \ldots$ is the mode number, and x_t is the so called ray turning point being determined from the equation $k^2(x) - \beta^2 = 0$.

In the case of linear wedge in vacuum

$$k(x) = 12^{1/4}\, k_p^{1/2} (\theta x)^{-1/2}, \qquad (2)$$

where $k_p = \omega/c_p$ is the wavenumber of a symmetrical plate wave, $c_p = 2c_t\,(1 - c_t^2/c_l^2)^{1/2}$ is its phase velocity, and c_l and c_t are longitudinal and shear velocities in wedge material. Hence $x_t = 2\sqrt{3}k_p/\Theta\beta^2$, and the solution of eqn (1) yields the extremely simple analytical expression for wedge wave velocities [4-6]:

$$c = c_p n \Theta/\sqrt{3}. \qquad (3)$$

The expression (3) agrees well with the other theoretical calculations [1-3] and with the experimental results [1]. Note that, although the geometrical acoustics approach is not valid for the lowest order wedge mode ($n = 1$) [5], in practice it provides quite accurate results for wedge wave velocities in this case as well.

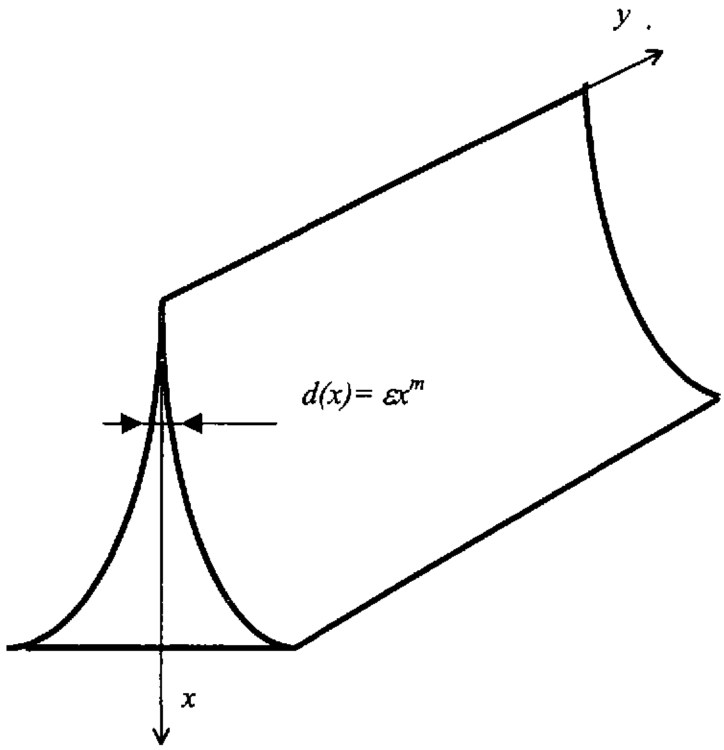

Fig. 1. Elastic wedge-like structure of non-linear shape

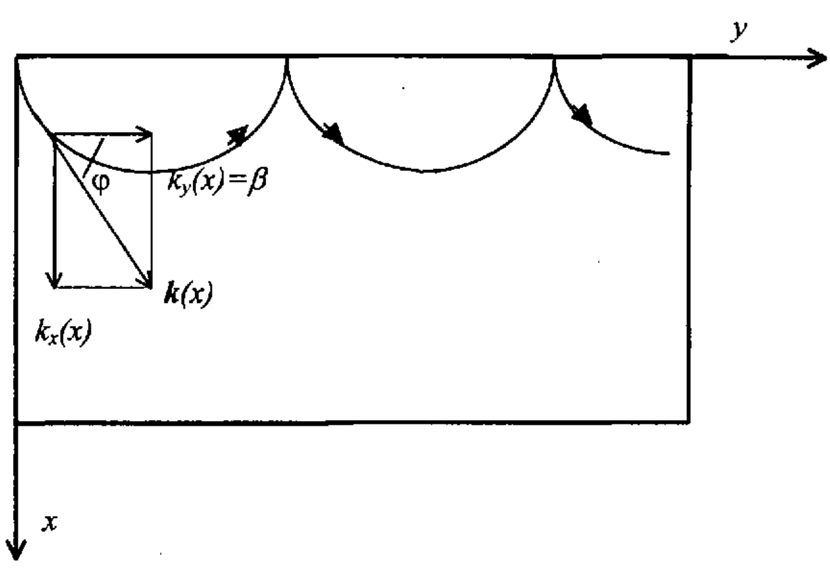

Fig. 2. Localised wedge modes as successively reflecting flexural waves.

For the purpose of this paper, it is convenient to consider changing the integration over x in eqn (1) to the integration over the angle φ between the vector $k(x)$ and its horizontal projection $k_y(x) = \beta$ [15]. Using the obvious relationships (see Fig.2)

$$k(x)\cos(\varphi) = \beta, \tag{4}$$

$$k_x(x) = \beta \tan(\varphi) \tag{5}$$

and expressing x from eqn (4) as function of the angle φ, with β being a parameter: $x = f(\varphi,\beta)$, one can rewrite eqn (1) in the form

$$\beta \int_{\pi/2}^{0} \tan(\varphi) f'(\varphi,\beta) d\varphi = \pi n\,, \tag{6}$$

where $f'(\varphi,\beta) = \partial f(\varphi,\beta)/\partial\varphi$. This form of the velocity equation is useful if the dependence of $k(x)$ is such that there exists an explicit analytical relationship $x = f(\varphi,\beta)$. This is obviously the case for the above mentioned wedge of linear geometry. It is easy to check that for such a linear wedge eqn (6) combined with eqns (4) and (5) gives the same result (3) that follows from eqn (1). As we will see below, for wedges of non-linear cross-section, the use of eqn (6) brings more advantages, essentially simplifying the derivation.

2.2 Immersed Linear Wedges

To apply the geometrical acoustics approach for calculating the velocities of localised modes in a wedge immersed in liquid one has to make use of the expression for a plate wave local wavenumber $k(x)$ which takes into account the effect of liquid loading [7,11]. The starting point to derive $k(x)$ for this case is the well known dispersion equation for the lowest order flexural mode in an immersed plate.

For shortness, we dwell in this paper only on the case $\rho_f/\rho_s \approx 1$ typical for light solid materials in water and limit our analysis by a subsonic regime of wave propagation ($k > \omega/c_f$), where ρ_s and ρ_f are respectively the mass densities of solid and liquid, and c_f is the velocity of sound in liquid. For the sake of simplicity, we impose even a more severe restriction on wave velocities considering very slow propagating plate flexural modes ($k \gg \omega/c_f$) and using the approximation of incompressible liquid. Then, for $kd \ll 1$ typical for thin plates, the wavenumber $k(x)$ in the case of linear wedge $d = d(x) = x\Theta$ has the form

$$k(x) = \left[\sqrt{6}\,\frac{c_l}{c_t}\,\frac{1}{\sqrt{c_l^{\,2} - c_t^{\,2}}}\,\frac{\sqrt{\rho_f}}{\sqrt{\rho_s}}\,\frac{\omega}{(x\theta)^{3/2}} \right]^{2/5}. \tag{7}$$

Substituting eqn (7) into (1) or (6) and performing some simple transformations, one can derive the following analytical expression for wedge wave velocities c [11]:

$$c = c_t A^{-5/2} D^{-3/2} (\pi n)^{3/2} \Theta^{3/2}, \tag{8}$$

where $A = 6^{1/5}(\rho_f/\rho_s)^{1/5}(1 - c_t^2/c_l^2)^{-1/5} = 6^{1/5}(\rho_f/\rho_s)^{1/5}\,[2(1-\sigma)]^{1/5}$ is a nondimensional parameter which depends on the relation between the mass densities ρ_f/ρ_s and on the Poisson ratio σ, and $D = \int_0^1 (x^{-6/5} - 1)^{1/2} dx = 2.102$. Comparison of eqns (3) and (8) shows that the effect of liquid loading results in significant decrease of wedge wave velocities in comparison with their values in vacuum, especially for small angles Θ.

3. WAVES IN WEDGES DESCRIBED BY A POWER LAW

In what follows we generalise the above theory by introducing a power law relationship between the local thickness d and the distance from the tip x: $d = \varepsilon x^m$, where m is any positive rational number. Then, for free and immersed wedges, eqns (2) and (7) should be replaced by the following expressions respectively:

$$k(x) = 12^{1/4}\, k_p^{1/2} (\varepsilon x^m)^{-1/2}, \tag{9}$$

$$k(x) = \left[\sqrt{6}\, \frac{c_l}{c_t} \frac{1}{\sqrt{c_l^2 - c_t^2}} \frac{\sqrt{\rho_f}}{\sqrt{\rho_s}} \frac{\omega}{(\varepsilon x^m)^{3/2}} \right]^{2/5}. \tag{10}$$

Substituting eqns (9) and (10) into eqn (6) and using (4), (5), one can obtain the general relationships for wedge wave velocities of localised elastic modes propagating in non-linear free and immersed wedges respectively:

$$c = \frac{\omega^{(m-1)/(m-2)} P^{1/(m-2)} F_m^{\;m/(m-2)}}{(2\pi m \varepsilon^{1/m})^{m/(m-2)}}, \tag{11}$$

$$c = \frac{\omega^{3(m-1)/(3m-5)} B^{2/(3m-5)} G_m^{\;3m/(3m-5)}}{(2\pi m \varepsilon^{1/m})^{3m/(3m-5)}}. \tag{12}$$

Here

$$P = \frac{12^{1/2}}{c_p} = \sqrt{3}\, \frac{c_l}{c_t} \frac{1}{\sqrt{c_l^2 - c_t^2}}, \tag{13}$$

$$B = \frac{24^{1/2}}{c_p} \frac{\sqrt{\rho_f}}{\sqrt{\rho_s}} = \sqrt{6}\, \frac{c_l}{c_t} \frac{1}{\sqrt{c_l^2 - c_t^2}} \frac{\sqrt{\rho_f}}{\sqrt{\rho_s}}, \tag{14}$$

and

$$F_m = (4/m) \int_0^{\pi/2} \sin^2\varphi \cos^{(2-2m)/m} \varphi\, d\varphi, \tag{15}$$

$$G_m = (10/3m) \int_0^{\pi/2} \sin^2\varphi \cos^{(5-6m)/3m} \varphi\, d\varphi. \tag{16}$$

It is clearly seen from eqns (11) and (12) that deviation of a wedge shape from linear geometry ($m = 1$ and $\varepsilon = \Theta$) results in dispersion of wedge wave velocities. For linear wedges, as expected, the velocity c is independent of ω and reduces to the earlier derived expressions (3) and (8).

It is interesting to notice that for $m \geq 2$ (in the case of free wedges) and for $m \geq 5/3$ (in the case of immersed wedges) the integrals in (15) and (16) become divergent. The analysis shows that the corresponding wave velocities determined by eqns (11) and (12) tend to zero for $m \to 2$ and $m \to 5/3$. This is in agreement with the earlier investigation of wedge modes in a quadratic wedge (m = 2) in contact with vacuum [16]. In the latter case the velocities of all wedge modes are equal to zero, unless there is a truncation of the wedge tip. For $m > 2$ and $m > 5/3$ not only the integrals in (15) and (16), but also the expressions (11) and (12) diverge, indicating that they are not valid for these cases. Nevertheless, it is quite reasonable to assume that for $m > 2$ and $m > 5/3$ the wedge wave velocities are zero as well. The reason for this will be clear from the analysis in the next section.

4. ELASTIC WEDGES AS ACOUSTIC BLACK HOLES

Let us compare the above mentioned conclusions for localised wedge modes with the results for flexural wave propagation in normal direction towards the wedge tip. In the case of quadratic wedge in vacuum ($m = 2$) such analysis has been first performed by Mironov [17] who noticed that in this case the incident wave requires indefinite time to reach the tip and thus never reflects back. He has suggested to use such wedges as absorbers of vibration energy which are equivalent to astronomical "black holes". As we will see below, this property relates not only to quadratic wedges in vacuum, but to a wider class of wedges in contact with both vacuum and liquid if their profiles satisfy the conditions $m \geq 2$ (for free wedges) and $m \geq 5/3$ (for immersed wedges), i.e. the above mentioned conditions associated with zero velocities of wedge modes in free and immersed wedges.

Indeed, let us consider propagation of a plane flexural wave in normal direction towards the tip of a wedge described by a power law and calculate the integrated wave phase Φ resulting from the wave propagation from a certain point x to the wedge tip ($x = 0$):

$$\Phi = \int_0^x k(x)dx \, . \qquad (17)$$

Substituting the corresponding expressions for $k(x)$ in the cases of free and immersed wedges (eqns (9) and (10) respectively) into eqn (17), one can prove that the integral in eqn (17) diverges for $m \geq 2$ and $m \geq 5/3$ for free and immersed wedges respectively. This means that the phase Φ becomes infinite under these circumstances and the wave never reaches the edge. This gives a very simple interpretation to the wedge wave velocities being equal to zero for such wedge profiles. Indeed, wedge waves in geometrical acoustics interpretation are plate flexural wave propagating from the ray turning point to the wedge tip and reflecting back to the turning point. When the wave propagating along the ray trajectory approaches the edge, it falls on the wedge tip almost at the right angle. Then, if the power-law profile of free or immersed wedges is characterised by $m \geq 2$ and $m \geq 5/3$ respectively, the wave is being trapped near the edge in the same way, as it was described in the case above. In the light of this, it does not propagate along the edge, and its velocity is equal to zero.

In practice, the use of non-linear wedges as vibration energy absorbers is limited by technological difficulties with manufacturing of perfect non-linear profiles. Real wedges always have truncated edges. And this strongly affects their performance as vibration absorbers. For ideal non-linear wedges, it follows from eqn (17) that even an infinitely small material damping characterised by imaginary part of $k(x)$ is sufficient for all the wave energy to be absorbed. However, for truncated wedges the lower integration limit in eqn (17) must be changed from 0 to a certain value x_0 describing the truncation. In the case of normal incidence of a quasi-plane flexural wave this results in drastic reduction of wedge absorbing properties that reveals in the increase of the associated reflection coefficient from zero to $R_0 = \exp(-2\int_{x_0}^{x} \mathrm{Im}\, k(x)dx)$ [17]. According to the calculations made in [17] for a quadratic wedge in vacuum having realistic values of truncation x_0 and material quality factor Q, the corresponding value of R_0 was about 0.7. This means that relatively little absorption actually took place.

The use of localised wedge modes can significantly improve the situation. Since, according to the geometrical acoustics approach [4-6], wedge waves can be interpreted as quasi-plane flexural waves propagating along a curvilinear trajectory and experiencing multiple reflections from the free edge and ray turning points (see Fig. 2), the resulting wave reflection coefficient R taking into account the integrated wave attenuation can be approximated as

$$R = (R_0)^N, \qquad (19)$$

where N is the number of edge reflections on the distance from the source to the point of observation. The realistic values of N can be up to 10. Therefore, the use of localised vibration modes in the example described above may result in the reflection coefficient R being as low as $0.7^{10} = 0.028$. The absorbing properties of non-linear wedges can be even more enhanced by covering their surfaces with highly absorbing materials. This problem, however, requires a special consideration that goes beyond the scope of this paper.

5. CONCLUSIONS

Using the geometrical acoustics technique, the analytical expressions for phase velocities of localised wedge modes have been derived for edges with a cross section described by a power-law relationship between the local thickness d and the distance from the edge x: $d = \varepsilon x^m$, where m is a positive rational number.

It was shown that deviations of a wedge shape from the linear geometry ($m = 1$ and $\varepsilon = \Theta$, where Θ is the wedge apex angle) result in frequency dispersion of wedge modes.

For profiles with $m \geq 2$ - in the case of free wedges, and with $m \geq 5/3$ - in the case of immersed wedges the velocities of localised waves tend to zero, unless there is a truncation on the wedge tip. This implies that localised waves do not propagate along free or immersed structures with $m \geq 2$ and $m \geq 5/3$ respectively. This phenomenon can be explained by trapping of flexural wave energy near the curved edges considered which represent acoustic 'black holes' for flexural waves.

The discussion was given on possible use of wedge waves propagating along edges of free and immersed non-linear wedges with $m \geq 2$ and $m \geq 5/3$ respectively for isolation of flexural vibrations.

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