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# AN IMPROVED MODEL TO DESCRIBE AMPLITUDE AND PHASE BEHAVIOUR OF ACTUATORS

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## 1. INTRODUCTION

A theoretical model describing the behaviour of new miniaturised condenser transducers has been developed recently [1]. This model is convenient to describe any transducer which contains a thin fluid layer coupled to a vibrating membrane. It takes into account in a realistic way several effects which are partially or totally neglected in the conventional models. The purpose of the paper is to present this theoretical model and to give results when applied to a specific loudspeaker (the laterally radiating, piezoelectric loudspeaker).

## 2. ANALYTICAL MODEL

The goal of this analytical study is to describe the behaviour of a thin fluid layer trapped between a membrane and a rigid wall, taking into account several effects, such as the modal coupling between the membrane and the fluid, the usually considered inertia, compressibility, and shear viscosity, and also taking into account the heat conduction effects, usually neglected, since they could be not negligible at the highest frequencies of interest.

**Assumptions**

As the thickness of the fluid film is very small, the following assumptions are done (the notations are presented in Figure 1, for a given geometry):

- The pressure variation, p, is approximately constant through the thickness of the fluid film; it essentially depends on the coordinate **w**. Hence the z-component of the particle velocity **v** is much lower than the w-component; that is the flow is roughly assumed tangential to the walls: $v_z \approx 0$, and $\mathbf{v} = \mathbf{v_w}(\mathbf{w}, z)$.
- The velocity profiles will not be greatly dependant on the tangential coordinate **w**, and terms containing spatial variations in the w-direction can be neglected compared to terms containing spatial variations in the

z-direction normal to the walls, that is $|\partial/\partial w| << |\partial/\partial z|$.

- All quantities related to the z-coordinate, that is the particle velocity, $v_w(w, z)$, the temperature variation, $\tau(w, z)$, and the density variation, $\rho'(w, z)$, are replaced by their mean value across the section area of the fluid layer, denoted $< \cdot >$.

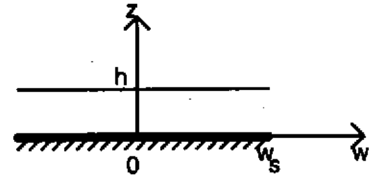



Fig. 1. A thin fluid layer trapped between a membrane and a rigid wall.

**Equations of motion of the fluid**

Assuming the approximations mentioned above, a complete set of linear equations, governing small amplitude disturbances of the fluid layer, can be written as follow :

- The Navier-Stokes equation, which reduces to the Poiseuille law for compressible fluid

$$(\partial_{ct} - l'_v \partial^2_{zz})\, v\,(w,z) = -(\rho c)^{-1}\, \mathbf{grad}_w\, p\,(w),$$

where the characteristic length $l'_v$ is defined as $l'_v = \mu / (\rho c)$, $\mu$ being the coefficient of shear viscosity .

- The conservation of mass equation (the thickness, h, of the fluid layer being considered as a first order differential element)

$$h\, \mathrm{div}_w <v(w,z)> + i\omega\xi(w) = -(i\omega/\rho)\, h <\rho'(w,z)>,$$

where $\xi$ is the amplitude of the membrane displacement which is the driving term for the fluid motion.

- The Fourier equation for conduction of heat

$$(\partial_{ct} - l_h \partial^2_{zz})\, \tau(w,z) = [(\gamma-1)/(\beta\gamma)]\, \partial_{ct}\, p(w),$$

where the characteristic length $l_h$ is defined as $l_h = \lambda_h/(\rho c C_p)$ ,$\lambda_h$ being the coefficient of thermal conductivity and $C_p$ the heat coefficient at constant pressure per unit of mass .

- The state equation expressing that the density variation, regarded as a function of the independent variables $p(w)$ and $\tau(w,z)$, is a total differential

$$<\rho'(w,z)> = (\gamma/c^2)\, [p(w) - \beta <\tau(w,z)>],$$

where $\gamma$ is the specific heat ratio and $\beta$ the increase in pressure per unit increase in temperature at constant density .

**Boundary conditions for the fluid film**

The tangential component of the velocity and the temperature variation vanish on the walls,

$$v(w,0) = v(w,h) = 0, \text{ and } \tau(w,0) = \tau(w,h) = 0,$$

and, for all points located at the output of the air gap, $\mathbf{w} = \mathbf{w_s}$, we assume an impedance-like boundary condition relating the pressure variation to the normal component $v_w$ of the particle velocity,

$$p(w_s) = Z < v(w_s,z) >.$$

**Equation of motion of the membrane and boundary conditions**

The equation governing the forced vibration of the membrane looks like the following: $L(\xi) = p_1(w,h) - p_2(w) + p_e$,

where $p_e$ is a force linked to the electrical part of the transducer, and the boundary condition at the periphery is accounted for assuming an impedance-like condition (clamped membrane for example).

**Solutions**

This set of equations allows the calculation of the acoustic field in the fluid layer and the coupled displacement of the membrane using an exact modal solution.

## 3. APPLICATION TO A PIEZOELECTRIC LATERALLY RADIATING LOUDSPEAKER

**The laterally radiating, piezoelectric actuator**

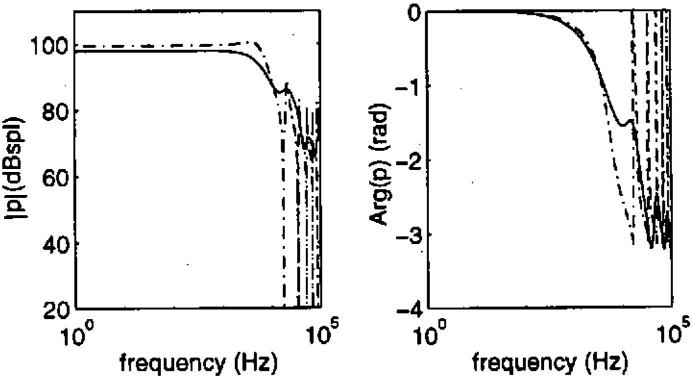
The laterally radiating, piezoelectric loudspeaker considered here is a device having a piezoelectric membrane and a rigid foil, both rolled around a cylindrical core, as for a paper electric condenser, in a spiral shape. A radial displacement of the membrane is induced by applying an electrical tension between its two faces. The membrane acts on each of the loading fluid layers in such a way that it produces a lateral acoustical flow at one of their ends (the other end being closed). Thus, the actuator is expected to supply an important volume velocity, having regard to its size [2].

**Results and discussion**

Figures 2a and 2b respectively show the amplitude and the phase of the acoustic pressure calculated, versus frequency, provided by the loudspeaker whose membrane is one turn rolled only. The loudspeaker is used here as an earphone, i.e. it is loaded on one end by a small cavity which represents the external ear (the other end being loaded by infinite space). The numerical values of the parameters used are: supplied voltage 10 V, radius of the membrane 3 mm, length of cylinder 10 mm, fluid layer thickness 50 µm, volume of the loading cavity 1 cm$^3$. The dashed line represents result obtained from a conventional model (based on equivalent lumped elements network) and the full one result obtained from the model presented above.

Figure 3 shows the amplitude of the acoustic pressure, versus frequency, provided by an earphone, designed to clearly emphasise only the influence of the shear viscosity effects and heat conduction effects.

These results show the ability of the model to accurately describe the behaviour of the actuator, and the capability of the actuator to provide a large volume flow in the low frequency range (where active noise control is efficient). In addition, the influence of the different dampings introduced in the modelisation can be clearly underscored.




2 a 2 b

Fig. 2. The amplitude (2a) and phase (2b) of the acoustic pressure provided by a laterally radiating, piezoelectric loudspeaker.
—— : Analytical model; -·-·-·- : Classical model.

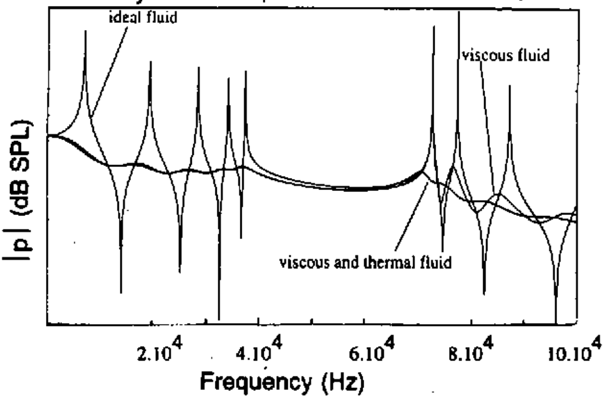



Fig. 3. The influence of shear viscosity effects and heat conduction effects on the acoustic pressure, versus frequency, of a laterally radiating, piezoelectric loudspeaker.

## References

[1] M Bruneau, A M Bruneau, Z Skvor, P Lotton, Acta Acustica, 2, 1994 (1994).
[2] Z Škvor, F Kadlec, A M Bruneau, P Lotton, Acta polytechnica, 32, 59 (1992).