# COUPLED VOLUME MULTI-SLOPE ROOM IMPULSE RESPONSES: A QUANTITATIVE ANALYSIS METHOD

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## 1 INTRODUCTION

Several places, for example theaters, auditoria or even churches, present an interesting acoustic feature: a non-linear sound decay (Fig.1). This phenomenon is provided under specific conditions by various architectural volumes, which are acoustically linked to each other. Acousticians have been interested in understanding the relationship of acoustic fields in these volumes, and their interactions. Knowledge on these phenomena has led architects and acousticians to design concert halls based on the coupled volume principle in last decades[1], with more or less success. The concert halls built in Lucerne, Switzerland[2] and more recently in Suzhou, China[3] and soon in Paris, France[4] are relevant examples. Therefore precise analysis of sound energy decay in such places is necessary. Moreover standardization of an analysis method still does not clearly exist for non-linear sound energy decays[5]. Furthermore a fine knowledge of the sound energy decay is necessary to estimate the influence of variations of reverberation on the perception of a listener in such places[6]. While analytical models[7-10], that describe non-linear decays for step or impulse response, provide smooth decay curves, the ones from measurements or numerical simulations are generally more jagged than the latter, and hence more difficult to analyze. An accurate analysis method has to be robust in order to point out the relevant characteristics, despite the fluctuations of decay curves. This paper will first present sound decay models and different analysis methods developed in recent years, based on two different principles. Then a new method for estimating non-linear decay characteristics is proposed. Finally some problematic issues will be raised concerning the use of Schroeder backward integration[11] for non-linear decays.

## 2 PREVIOUS RESEARCH

Analytical models of sound energy decay in coupled rooms have been proposed[9,10], assuming diffuse sound field hypothesis. These models can be used to predict the behavior of sound fields in halls, but also to develop relevant tools for analyzing multi-slope decay curves. Among the methods used to quantify relevant non-linear decay curve parameters, two main procedures exist. On the one hand a direct method is used, combining calculations of equivalent reverberation times[12,13], and proposing several parameters to describe non-linear decays. This approach allows working with sound decays containing a maximum of two slopes. On the other hand a model-based method is used, which minimizes the error between a decay function estimation and the Schroeder integration function[14]. Relevant parameters are then extracted from the selected model of sound decay which allows more than two slopes[15-17].

### 2.1 Multi-slope decay models

Considering the case of N volumes – the sound source and receiver being placed in the same one – connected by apertures $S_{ij}$, an analytical model of sound energy decay in each room has been proposed[9], assuming diffuse sound field hypothesis.

$$E_i(t) = \sum_{j=1}^{N} a_{ij} e^{-2\delta_j t} \tag{1}$$

where constants $a_{ij}$ are the initial values of energy decay in each room and $\delta_j$ are the equivalent decay rates of sound pressure in each room, linked to others. This estimation of energy decay provides, under specific conditions, a bent decay curve which presents straight portions and can be described by relevant parameters: $T_j$ the equivalent reverberation times of the slopes, and the bending points coordinates *($MS_{tj}$ ; $MS_{Lj}$)* in time and decay level (Fig.1). A few years later, a more detailed analytical model of sound energy decay in the particular case of two coupled rooms has been proposed[10]. Considering two volumes – $V_1$ containing the sound source and receiver, and $V_2$ termed reverberation chamber – connected by an aperture $S_{12}$, assuming once again diffuse sound field hypothesis in each volume, a statistical estimation of sound energy is given as follows.

$$\begin{cases} E_I(t) = E_{I1}e^{-2\delta_I t} + E_{I2}e^{-2\delta_{II} t} \\ E_{II}(t) = E_{II1}e^{-2\delta_I t} + E_{II2}e^{-2\delta_{II} t} \end{cases} \quad (2) \quad where \begin{cases} \delta_{I,II} = \frac{1}{2}(\delta_1 + \delta_2) \mp \sqrt{\frac{1}{4}(\delta_1 + \delta_2)^2 - (1 - \kappa^2)\delta_1\delta_2} \\ \kappa^2 = \frac{S_{12}^2}{(A_1 + S_{12})(A_2 + S_{12})} \end{cases} \quad (3)$$

with constants $E_{ij}$, whose analytical form can be expressed, are the initial values of energy decay in each room and $\kappa$ the coupling constant. The natural decay rates of sound pressure in each room, without coupling, depend on architectural parameters: $V_i$, $S_{12}$, $A_i$ the equivalent absorption area and c the speed of sound, such that $\delta_i = c(A_i + S_{12})/(8V_i)$. The corresponding reverberation times are $T_{1,2} = 6.9/\delta_{1,2}$. When the rooms are coupled, the decay rates become $\delta_{I,II}$ (eq.3). This estimation of energy decay can provide, in the less reverberant volume, a bent decay curve (Fig.1, Table 1) which can be described by four relevant parameters: $T_{early}$ and $T_{late}$ the equivalent reverberation times of the slopes, and the bending point coordinates *($DS_t$ ; $DS_L$)* in time and decay level.

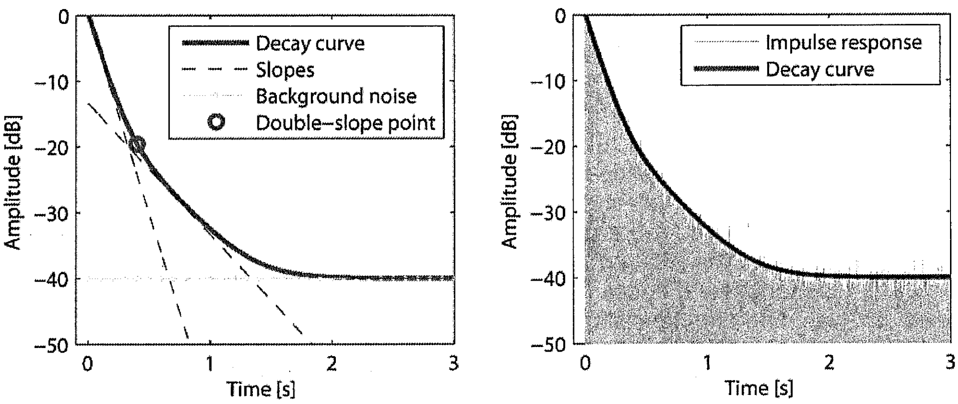


**Fig.1** *Double-slope decay curve (left); impulse response in dB scale (right)*

## 2.2 Analysis methods based on classical parameters

In the last decade, a number of studies have proposed to quantify double-slope sound decays with decay time ratios. For example[12] *T30/T15* and *T60/T15* have been used as metrics to quantify the degree of curvature between the early and late parts of double-slope sound decay. Both *T30/T15* and *T60/T15* provide the same kind of information, with the latter requiring a significant signal-to-noise ratio (SNR) of at least 65 dB, which is not always easy to obtain in real measurement conditions. These quantifiers give a general indication of the curvature of the decay curve, but are not able to uniquely characterize a decay curve. Furthermore there is no indication concerning the number of slopes in the decay. Note that ISO 3382-2 (2008), in annex B, suggests using *T30/T20* to quantify the degree of curvature of decay curves as the percentage of deviation from a perfectly straight line. This recommendation is proposed as an informative point, in the annex of the standard[5], and is not meant as a means of evaluating the decay curve, but only as an indication of deviation from linearity. Nevertheless, following this same idea, a previous study[13] proposed the ratio *LDT/EDT* with *EDT* (Early Decay Time) being the equivalent 60 dB decay time calculated by a least squares fit to the portion of the decay curve between 0 and -10 dB[5], and *LDT* (Late Decay Time) being similarly estimated between -25 and -35 dB. In contrast to *T30/T15*, *LDT/EDT* does not use overlapping decay regions. As such, the value of the latter will be higher for a given decay curve, which allows detecting smaller differences between the early and late parts of decays. However, the main weakness of *LDT/EDT* is the fact that the sound decay ranges used are fixed. If the bending point lies e.g. between -25 and -35 dB, estimation of *LDT* will be erroneous. Moreover, as other parameter ratios, this quantifier is not able to detect more than two slopes within the sound decay.

## 2.3 Model-based methods

An alternative method has been proposed in a previous study based on least squares principle to quantify relevant non-linear decay curve characteristics[14]. This method has been subsequently developed as an analysis method using a Bayesian probabilistic framework[15-17], allowing access to the relevant decay curve parameters and the number of slopes within the sound decay. Both methods are based on an analytical model of sound decay, containing several parameters which determine a unique decay curve.

$$F(\boldsymbol{A},\boldsymbol{B},t_k) = \sum_{j=1}^{m} A_j G_j(B_j,t_k),\ 0 \le t_k \le L \qquad where\ G_j(B_j,t_k) = \begin{cases} \exp(-B_j t_k) & ,j = 1,\dots,m-1 \\ L\text{-}t_k & ,j = m \end{cases} \quad (4)$$

The first $(m\text{-}1)$ terms of $G_j$ correspond to the different slopes within the decay curve. The term $(L\text{-}t_k)$ stands for the constant background noise. A collection of parameters is tested to approach the decay curve considered. Those parameters are set to minimize the error between so-called decay function estimation and Schroeder integration function. To do so, the first study mentioned uses the least squares method while the second one performs Bayesian analysis[15-17]. The relevant parameters are then extracted from the selected model. The latter approach has the important advantage that both the number of slopes and the value of decay time of each individual slope can be cohesively estimated within one inference framework. A drawback of this Bayesian parameter estimation is its complexity, which makes this method not easy to implement.

# 3 "MARCHING LINE" METHOD

Theoretically, a multi-slope decay function is composed of different damped exponentials[9] and a constant background noise. To describe a multi-slope decay curve, one needs to know several relevant parameters: the decay rates of the different straight portions and the coordinates (time; level) of the bending points. The method proposed here is based on a direct comparison between the decay curve and linear regressions. It provides the number of slopes with equivalent reverberation times, and the coordinates of bending points between two consecutive slopes. The input decay curve can be calculated by different manners, as discussed in section 4.

## 3.1 Starting point

Taken a measured or simulated impulse response, a first estimation of starting point of associated decay curve corresponds to the maximum of the squared impulse response. A backward integration curve is then calculated and normalized to 0 dB. To avoid errors due to strong early reflections whose amplitude might be superior to direct sound, refinement is performed by catching the last point of this curve superior to -0.1 dB, defining the starting point of decay curves.

## 3.2 Adjustable values

Some options are available to make the linear regressions closer to the decay curve and thus the results more accurate. The following adjustable values are part of the algorithm as the input options.
$L_{Begin}$: decay level to start the analysis. Default = -5 dB.
$L_{End}$: decay level to stop the analysis. Default is 10 dB over the background noise level.
$L_{BetweenSlopes}$: decay level between the end of a linear regression and the beginning of the next one, in order to avoid curved part of the decay around a bending point. Default = 10 dB.
$L_{Step}$: decay level between each iteration to evaluate a larger linear regression. Default = 1 dB.
$L_{DevMax}$: RMS deviation maximum threshold between decay curve and linear regression.

## 3.3 Stepwise linear regressions

To estimate equivalent reverberation times of linear portions of the decay curve, linear regressions are performed. Since reverberation time estimation requires a decay of at least 10 dB, the first linear regression is calculated for points between $L_{Begin}$ and $(L_{Begin} - 10\ dB)$. RMS deviation between the decay curve and the regression is calculated and if this value is inferior to $L_{DevMax}$, a new iteration begins with the lower limit $(L_{Begin} - 10\ dB - L_{Step})$. If $D$ is the decay curve and $LR$ is the last linear regression calculated between $t_u$ and $t_l$ , the upper and lower limits respectively, the RMS deviation is calculated as follows.

$$RMS_{dev} = \sqrt{\frac{1}{N}\sum_{t=t_u}^{t_l}[LR(t) - D(t)]} \quad \text{Where N is the number of points between } t_u \text{ and } t_l. \quad (5)$$

If the decay curve presents at least two different slopes, the RMS deviation will strongly increase around the bending area. Iterations are stopped when the RMS deviation value is superior to $L_{DevMax}$. An equivalent $T_{60}$ is estimated using the last linear regression. Next slope is then searched in the same manner, starting from the decay curve point corresponding to $(D(t_l) - L_{BetweenSlopes}\ dB)$.

### 3.4 Bending points coordinates estimation

As soon as two consecutive linear regressions are obtained, one can calculate their intersection point $M_i$, whose coordinates are $(t_i, L_i)$. The corresponding $i^{th}$ bending point is assumed to be the nearest point from $M_i$ within the decay curve. Distance between $M_i$ and each point of the decay curve is then calculated, as follows.

$$dist(j) = \sqrt{(t_j - t_i)^2 + (D(t_j) - L_i)^2} \quad (6)$$

The so-called bending point is the point within the decay curve corresponding to the minimum of vector *dist*. Since two consecutive linear regressions are needed to estimate a bending point, if $m$ slopes are detected, $(m-1)$ bending points will be found.

### 3.5 Optimized algorithm version

If the user knows that only two slopes are present within the decay curve, e.g. in case of two volumes with homogeneously distributed absorbent material, a specific algorithm can be performed. The first slope is detected as previously explained. To detect the second one, linear regressions are calculated from the end of decay curve, i.e. the point corresponding to $L_{End}$. The upper limit for those regressions is set at $(L_{End} + 10\ dB)$ and is stepwise incremented with $L_{Step}$ for the upper limit, until the RMS deviation is superior to $L_{DevMax}$. This algorithm version assumes that the decay curve presents only two slopes and requires a precise estimation of the background noise level to correctly set $L_{End}$, but it avoids the need for setting $L_{BetweenSlopes}$, which makes parameters estimation more precise.

## 4 BACKWARD INTEGRATION OR ENVELOPE

The accepted analysis method for energy decays is the backward integration of squared impulse response[11]. While this robust method produces a decay curve equivalent to averaging an infinite number of trials, its dependency to the upper limit of integration can lead to a significant drawback when considering multi-slope decays. An important point is that this method provides a decay curve corresponding to a step response of the room, since the impulse response is integrated. Analytical models allow comparisons between decay curves corresponding to step and impulse responses. As a result, the bending point is found to be translated upwards along the first slope.

### 4.1 Backward integration of multi-slope decay

The backward integration of squared impulse response proposed by Schroeder is defined for the time range [t;∞]. Assuming f the impulse response recorded at a receiving point, the ensemble average of the squared sound decay is obtained as follows.

$$\langle s^2(t)\rangle = \int_t^{\infty}[f(x)]^2dx \quad (7)$$

When using this method, upper limit of integration becomes a finite value $t_{max}$. This limit has to be chosen carefully because if $t_{max}$ is too large, the background noise will corrupt the decay curve by overestimating energy level, and if $t_{max}$ is too small, information is lost and the SNR is reduced. In single-slope case, a slightly too large value of $t_{max}$ is acceptable because the decay rate of the curve is not changed. Concerning multi-slope case, different values of parameters can be found, as mentioned in previous studies[14,15], depending on the upper limit of integration and the SNR, which provides an ambiguous decay rate for the late slopes (Fig.3, left; table 1). Therefore using backward integration can lead to uncertain results in specific cases.

## 4.2 Envelope approach

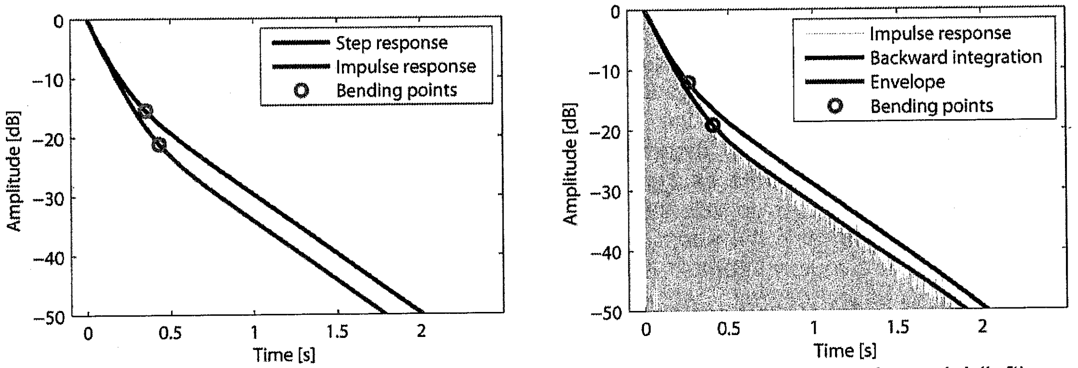


**Fig.2** *Theoretical step & impulse response decay curves from Cremer's model (left); Simulated impulse response in dB scale, analyzed by backward integration & envelope (right)*

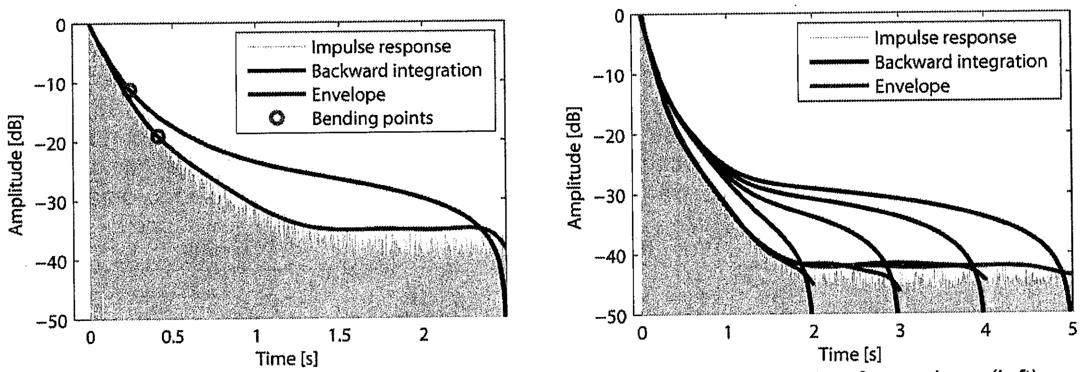


**Fig.3** *Simulated IR with low SNR, analyzed by backward integration & envelope (left); Backward integrated and envelope decay curves for 4 upper time limits (right)*

| | | $T_{early}$ (s) | $T_{late}$ (s) | $DS_t$ (ms) | $DS_L$ (dB) |
|---|---|---|---|---|---|
| Fig.1 | | 1.00 | 3.00 | 417 | -20.0 |
| Fig.2 left | Step | 1.12 | 2.97 | 340 | -15.2 |
| | Impulse | 1.10 | 2.96 | 430 | -21.1 |
| Fig.2 right | BIC | 1.16 | 3.07 | 324 | -12.1 |
| | Env | 1.18 | 3.13 | 455 | -19.5 |
| Fig.3 left | BIC | 1.22 | 3.18 | 318 | -11.1 |
| | Env | 1.19 | 3.11 | 460 | -20.1 |

| | | $T_{early}$ (s) | $T_{late}$ (s) | $DS_t$ (ms) | $DS_L$ (dB) |
|---|---|---|---|---|---|
| Fig.3 right | BIC (tmax=2) | 1.25 | 3.16 | 316 | -13.7 |
| | Env (tmax=2) | 1.20 | 3.10 | 428 | -19.9 |
| | BIC (tmax=3) | 1.26 | 3.36 | 333 | -14.2 |
| | Env (tmax=3) | 1.19 | 3.04 | 416 | -19.6 |
| | BIC (tmax=4) | 1.26 | 3.72 | 352 | -14.7 |
| | Env (tmax=4) | 1.16 | 3.03 | 405 | -18.8 |
| | BIC (tmax=5) | 1.26 | 4.03 | 367 | -15.0 |
| | Env (tmax=5) | 1.18 | 3.06 | 413 | -19.4 |

**Table 1** *Analysis parameters values for Fig.1, 2 & 3*

An alternate approach to obtain a decay curve is using the envelope of the impulse response, which provides a different decay curve from the backward integration method. On the one hand, the level of bending points is lower than found with backward integration method, corresponding to the difference between step and impulse response cases[10] (Fig.2; table 1), which are analytically linked by a derivative relationship. On the other hand, the obtained decay curve appears to be much less affected by residual background noise length at the end of the impulse response as compared to the integration method (Fig.3, right), which provides an increased SNR for analysis (Fig.3, left). This approach has been implemented in two steps. First a direct envelope is calculated, in the same manner as a backward integration: starting from the end of impulse response, the algorithm comes back to the beginning, step by step, keeping the highest level between a point and the next one. As a result, a decreasing curve is obtained. To smooth this curve, a polynomial interpolation is calculated, with sufficiently high order, e.g. 10, to correctly fit the latter. This method tends to avoid problems during analysis due to fluctuations in measured impulse responses, by replacing a jagged curve by a much smoother decay curve.

The alternative to the backward integrated curve (BIC) proposed here is then a smooth decreasing curve, whose SNR is weekly dependent on background noise (Fig.3), which is helpful in problematic cases of low SNR or truncated IR where RT estimations can be erroneous[14,15]. In the context of multi-slope decay curves, even more than in single-slope cases, RT estimations are needed over classically defined parameters and thus larger SNR are required. The main drawback of this room impulse response treatment is that the decay curve obtained does not stand for an ensemble average, as would be a backward integrated curve. Hence a compromise is pointed out between the ability to accurately detect the background noise level when SNR is low and to get numerous impulse responses, in order to allow estimating relevant, precise decay curve parameters. Furthermore one should raise the question of the pertinence of these methods relative to human sound perception. In a musical context it has been shown that the late reverberation time ("stop chord" response) is associated with perceptions of reverberance while the early decay time is more associated with the sense of "running reverberation". In the context of multi-slope decays, the problem is compounded. The transition points, and their definition, must be addressed.

# 5 EXPERIMENTAL RESULTS

Three different IRs (Table 2, Fig.4) have been analyzed, using both the model-based method and the new Marching Line method. Two of them have been synthetized and one has been measured in a real concert hall with reverberation chambers. Results from both methods are compared, as well as results from the new method, using backward integration and the envelope approach (Table 3). The latter uses first the general algorithm without an a priori knowledge of the number of slopes within the decay curve. If two slopes are detected, the optimized algorithm is used to obtain more accurate results.

| | | $T_{early}$ (s) | $T_{late}$ (s) | $DS_L$ (dB) | Noise Level (dB) |
|---|---|---|---|---|---|
| Synthesis parameters | IR1 | 2.00 | 5.00 | -29.0 | -60 |
| | IR2 | 1.00 | 3.00 | -19.5 | -35 |

**Table 2** *Synthesis parameters values for IR 1 & 2*

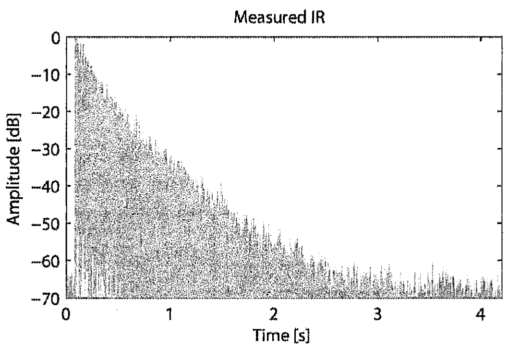


**Fig.4** *IR3 measured in a concert hall with reverberation chambers*

## 5.1 Synthesized impulse responses

The synthesized IRs are produced by applying a designed decay curve onto white noise, according to synthesis parameters (Table 2). The first IR presents a 60 dB SNR, with two distinct slopes. The second IR is built in the same manner, with a much lower SNR, so that the second slope is ambiguously melt with background noise.

Results (Table 3) show a strong similarity for *IR1*, especially in estimation of $T_{early}$ and $T_{late}$ by model-based method and by new method with envelope approach. *IR2* analysis provides more scattered values, particularly for $T_{late}$, which underlines difficulty of accurately estimating a second-slope reverberation time when SNR is weak. The relative difference is calculated between controlled synthesis parameters and corresponding analysis parameters estimated with both methods. Low differences are obtained by new Marching Line method with envelope approach, as well as model-based method, except for the second RT of *IR2*, which has very low SNR. New method has less accurate results when analyzing BIC: relative differences are up to 15%. Analysis of synthesized IRs show that the new method is more accurate when used with the envelope approach than backward integrated curve. Model-based method is very precise but can be sensitive to SNR since using BIC. Hence its estimation of $T_{early}$ is accurate but $T_{late}$ can be misestimated because of high background noise level.

| | | $T_{early}$ (s) | $T_{late}$ (s) | $DS_t$ (ms) | $DS_L$ (dB) |
|---|---|---|---|---|---|
| **IR 1** | Bayesian | 2.01 | 5.02 | 817 | -21.2 |
| | ML (BIC) | 2.11 | 4.92 | 861 | -22.1 |
| | ML (Env) | 2.01 | 4.95 | 988 | -27.1 |
| **IR 2** | Bayesian | 1,00 | 2.54 | 206 | -9.7 |
| | ML (BIC) | 1.16 | 3.18 | 225 | -10.3 |
| | ML (Env) | 1.01 | 2.9 | 343 | -19.5 |

**Table 3** *Analysis parameters from Bayesian analysis & Marching Line (ML) methods*

## 5.2 Measured impulse response

The measured IR (Fig.4). has been recorded in a real concert hall with reverberation chamber. One can note that the slope is not the same at the top and bottom of *IR3*. The difficulty is estimating the point where the transition happens. Results for the different methods are provided in Table 4. Model-based method found three slopes within the corresponding decay curve, with the final bending point appearing at -45 dB in an IR with a background noise level of -60 dB, or -50 dB for BIC on which the calculation is performed, which may be questionable due to noise corruption. The Marching line method offers different results depending on the parameter details, however, characteristics of the two first slopes are quite similar between methods, with RT values in the same range for the model-based method and the new method with envelope approach. These results are less homogeneous than those of synthesized IRs, maybe because the measured IR is more jagged. Moreover the difference between the initial and final decay rates is weaker in this example than in synthesized IRs, which makes the analysis more difficult.

An idea of sensibility of the Marching Line method can be estimated by varying input main parameter $L_{DevMax}$ and compare output results. Small variations can lead to small changes when applied to a regular, simple IR like *IR1*. Nevertheless analysis of ambiguous *IR3* leads to important changes (Table 4) since 2 to 4 slopes are found for small variations of input parameter. This shows the dependence of the method on the nature of IR. This raises the question of a threshold to determine several slopes within a decay curve. A further study could try to propose a perceptual criterion which would give a suitable number of slopes, e.g. based on "just noticeable difference" applied to coupled volume reverberations.

| | T 1 (s) | T 2 (s) | $DS_t$ 1 (ms) | $DS_L$ 1 (dB) | T 3 (s) | $DS_t$ 2 (ms) | $DS_L$ 2 (dB) | T 4 (s) | $DS_t$ 3 (ms) | $DS_L$ 3 (dB) |
|---|---|---|---|---|---|---|---|---|---|---|
| Bayesian | 1.19 | 2.63 | 258 | -10.9 | 4.82 | 1857 | -44.61 | | | |
| $L_{DevMax}$ (dB) | | | | | | | | | | |
| 0.04 | 1.21 | 2.10 | 286 | -12.9 | 2.65 | 898 | -30.7 | 3.23 | 1360 | -41.4 |
| 0.05 | 1.23 | 2.17 | 306 | -13.7 | 3.00 | 1080 | -35.1 | | | |
| 0.09 | 1.26 | 2.27 | 345 | -15.0 | | | | | | |

**Table 4** *Analysis parameters from Marching Line method with envelope approach & Bayesian method on measured IR3*

# 6 CONCLUSION

Among the coupled volume acoustic issues, analysis accuracy has an important place. A quantitative approach is necessary and thus quantifiers which exactly describe a multi-slope decay curve are required. This study proposes to use equivalent reverberation time of each straight portion of energy decay curve and the bending points coordinates, dynamically estimated, contrary to fixed parameters previously used with multi-slope curves. Before using quantifiers, one has to choose a means to obtain a relevant decay curve. The backward integration of the squared impulse response is commonly used, but can present major drawbacks in certain cases such as with low

SNR. Furthermore the choice of upper time limit of integration can influence the quantifier values. Therefore an envelope approach is proposed, which yields larger SNR with smoothed decay curves and a lower dependence on upper time limit truncation. Comparison of model-based Bayesian analysis method and the new Marching Line method made on three different impulse responses has shown a general similarity, decreasing for ambiguous cases of low SNR and low RT differences. Further discussions can be conducted on the limiting decay level for estimating a slope different from the background noise, as well as criterion to limit the number of slopes within decay curves.

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